



Massachusetts  
Institute of  
Technology



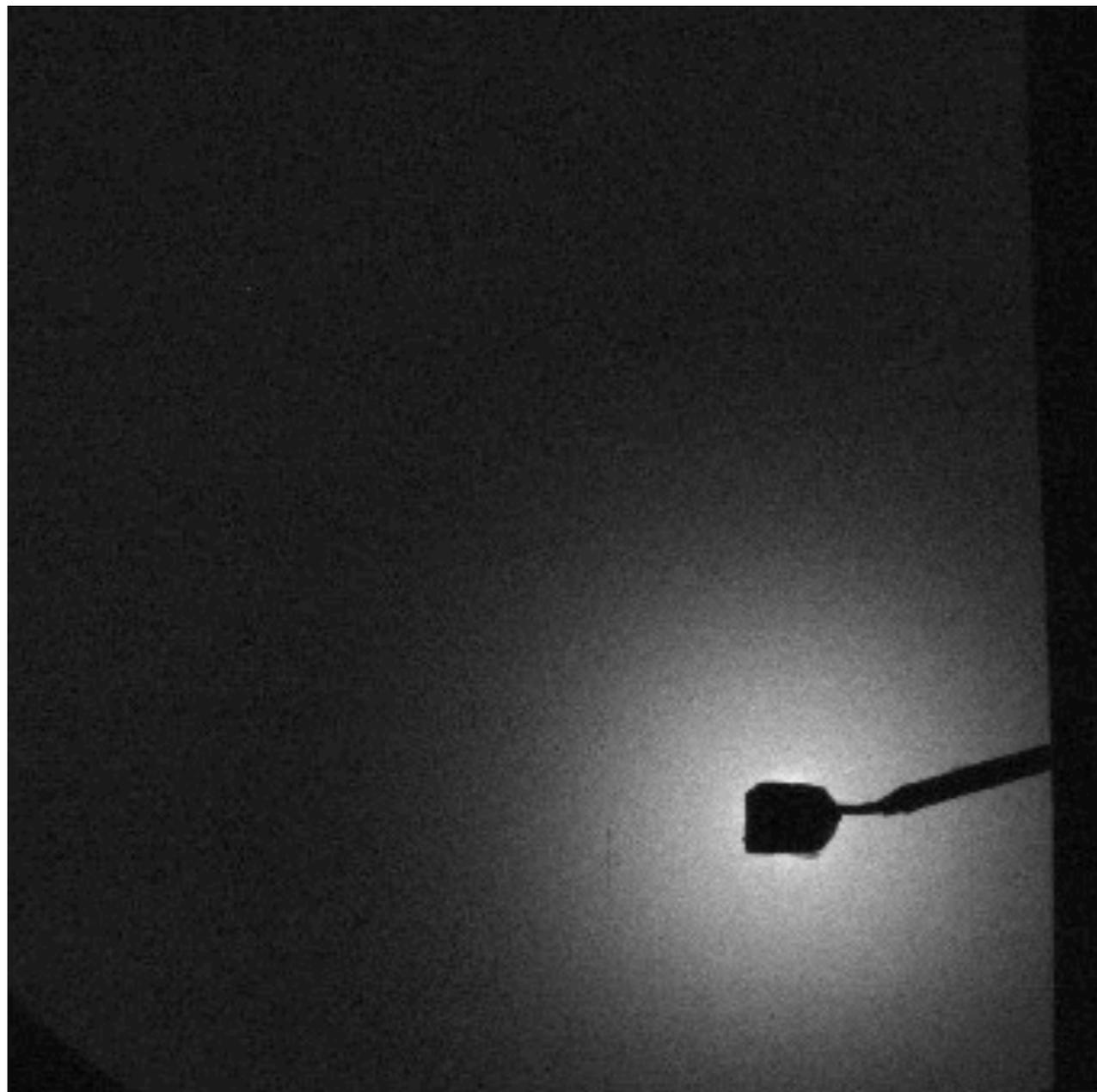
# Small-Angle X-ray Scattering (SAXS)

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MIT Department of Chemistry

2014 Penn State Bioinorganic Workshop  
May 28 - June 4, 2014

# What do you see?

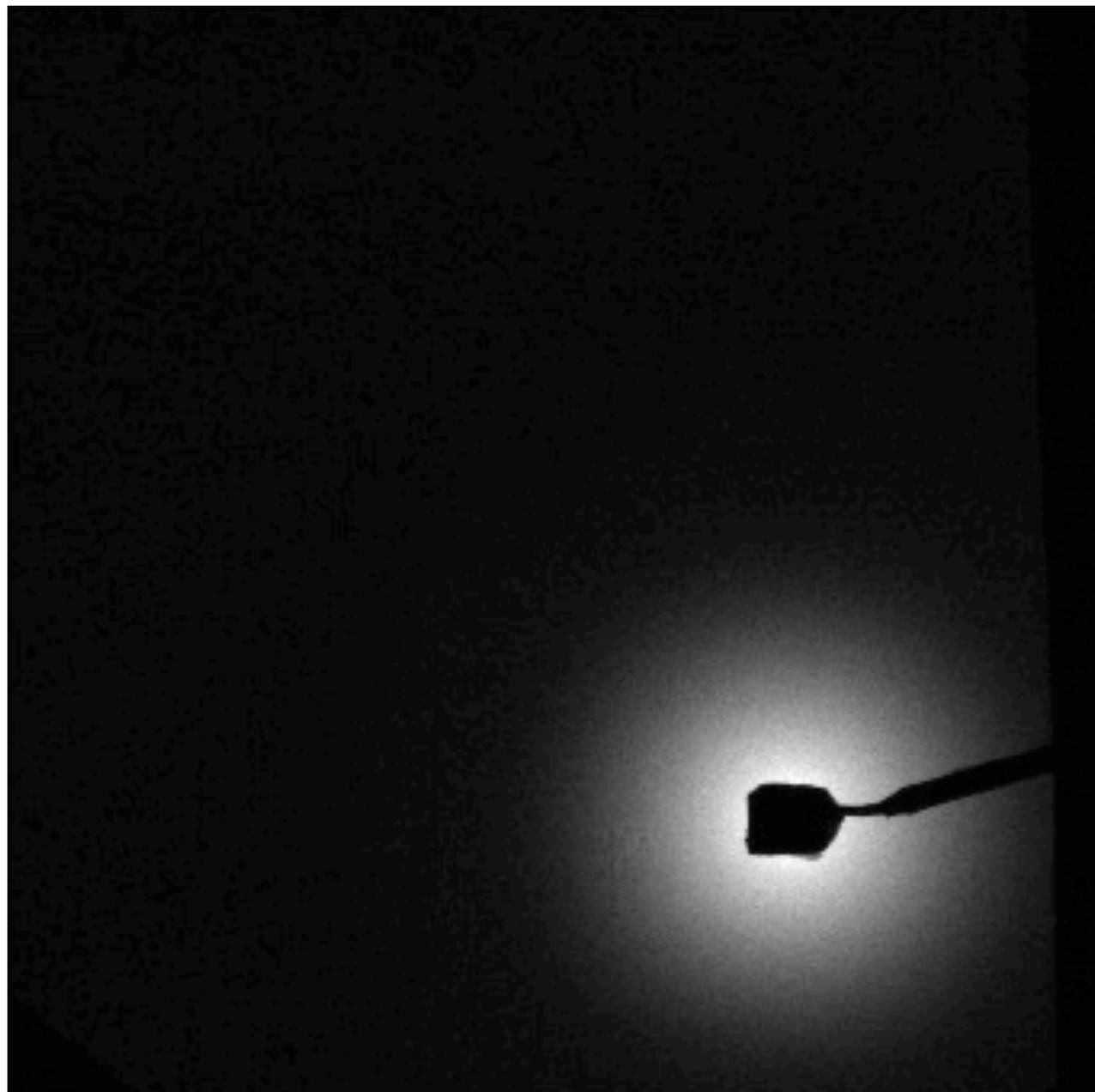
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Ando, et al. (2012). JACS, 134(43), 17945–17954.

# What do you see?

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Ando, et al. (2012). JACS, 134(43), 17945–17954.

# Outline

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- Why use SAXS?
- Scattering theory
- Basic interpretation of SAXS data
- Experimental requirements
- An advanced example: ribonucleotide reductase
- Variations

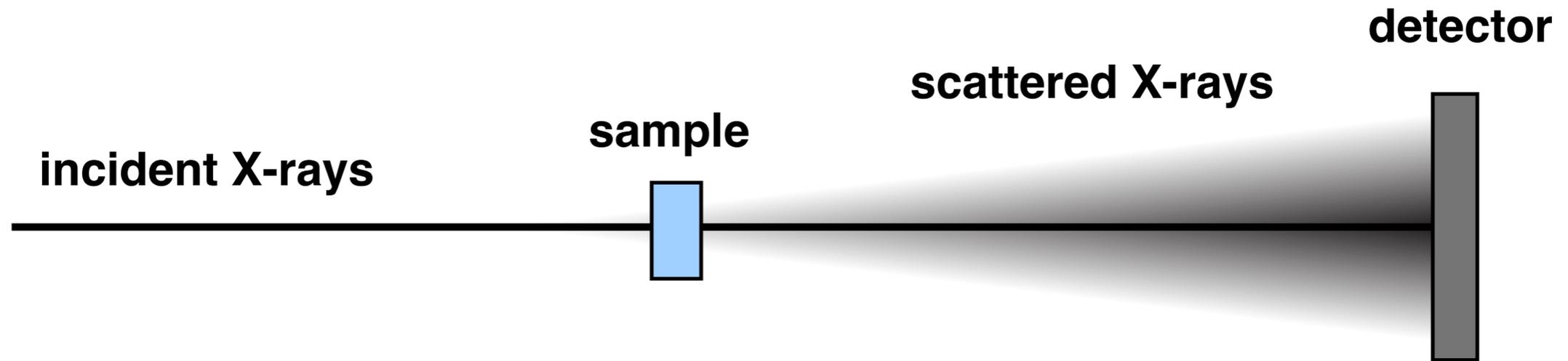
# Is this you?

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- I have a protein that won't crystallize.
- I have a crystal structure, but I want to know what forms in solution.
- I want to know the oligomerization state or the stoichiometry of a complex.
- I want to know what happens to my protein in the absence or presence of a cofactor or substrate.
- I want to know if my samples are properly folded.
- I want structural information on my protein under the same conditions used for activity assays.

# SAXS probes nanometer length scales

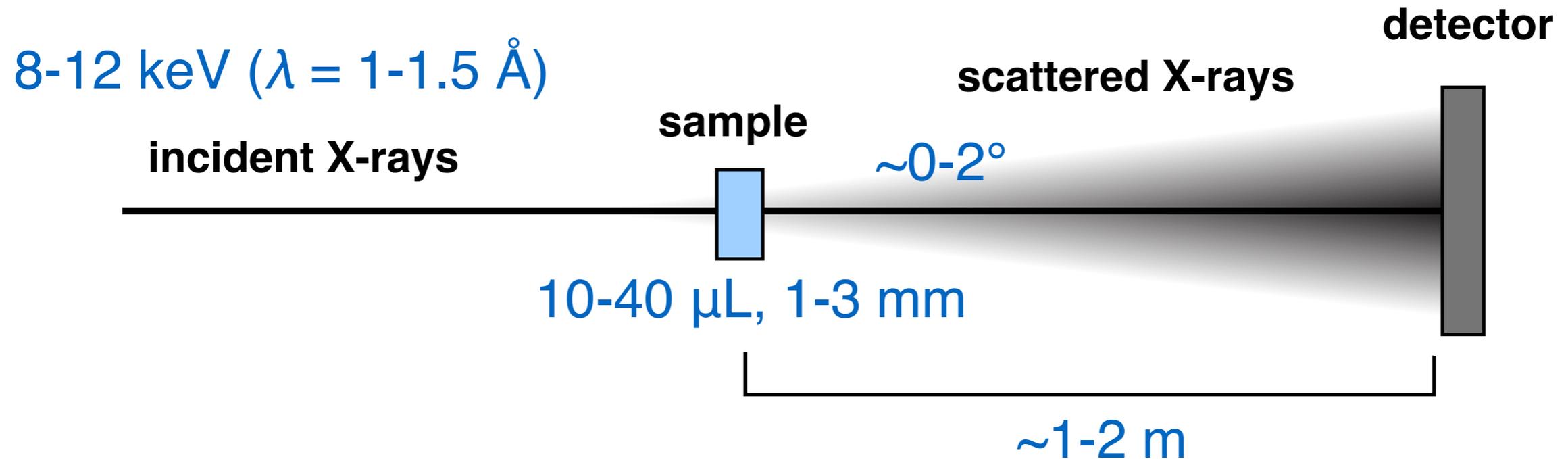
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## Samples:

- biomolecules in solution
- lipid membranes
- cellular organelles
- polymers
- emulsions
- liquid crystals
- alloys
- fibers

# SAXS probes nanometer length scales

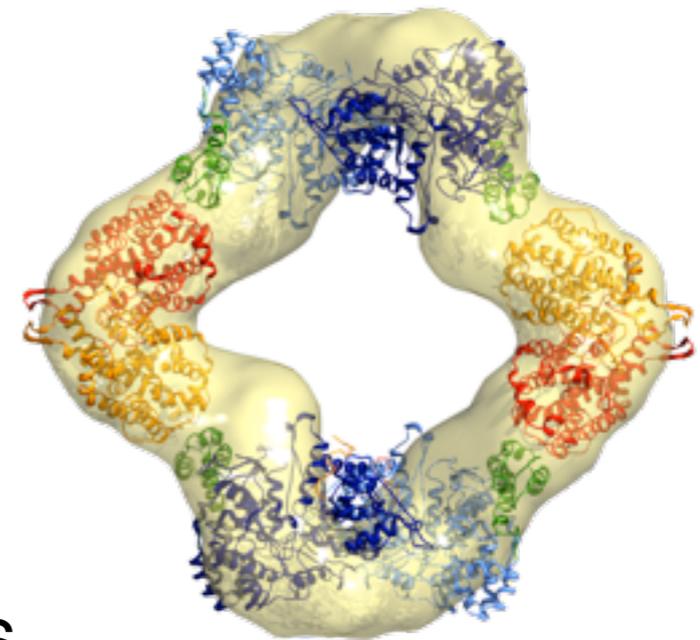


## Samples:

- biomolecules in solution  $\longrightarrow$  “solution SAXS”, “bioSAXS”

## Information:

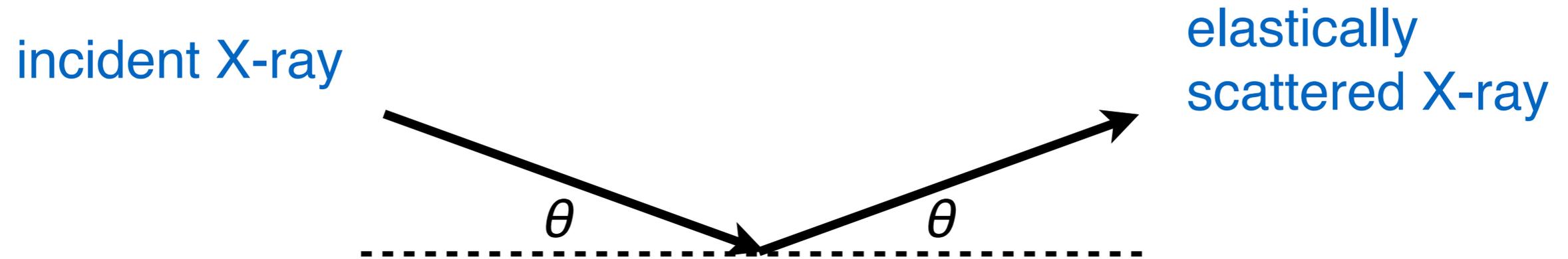
- mass, spatial size
- foldedness, packing, flexibility
- overall shape
- stoichiometry and shape of complexes
- conformational changes and #states as functions of time, ligands, solution conditions



# Part 1: Basic Theory

# Scattering angles

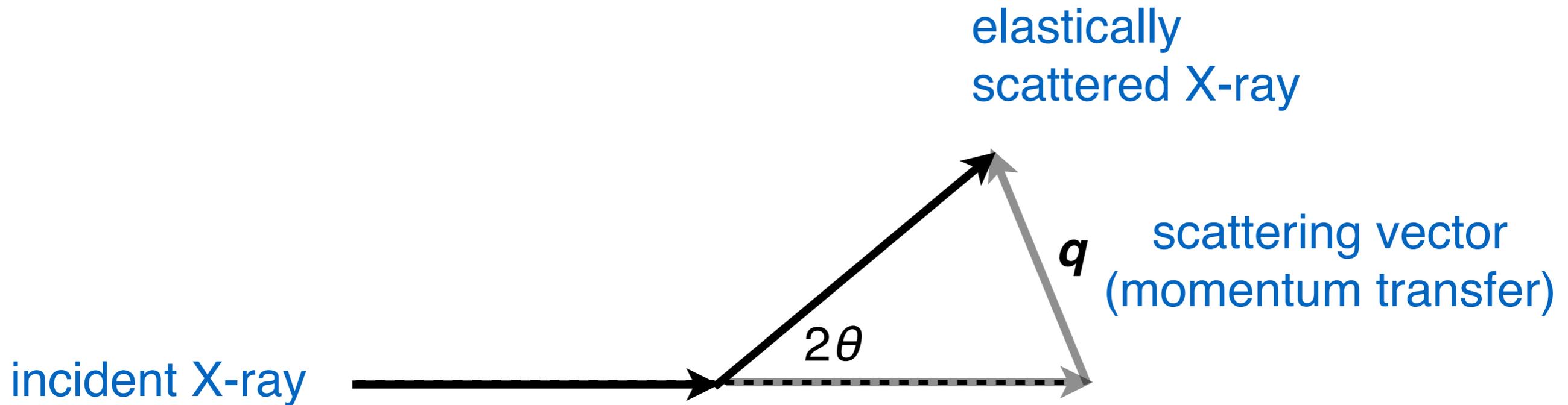
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By crystallography conventions, the scattering angle is defined as  $2\theta$ .

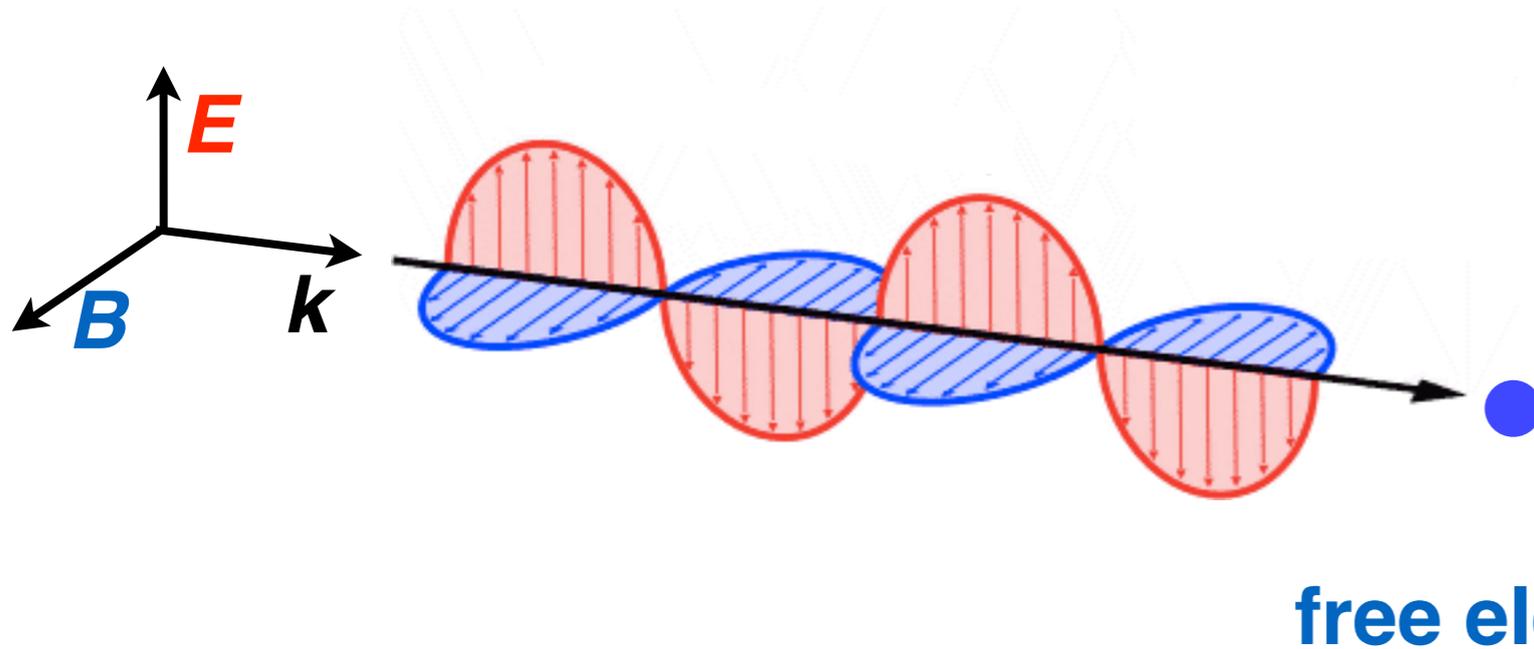
# Scattering angles

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By crystallography conventions, the scattering angle is defined as  $2\theta$ .

# Scattering from a single electron



Driving force:

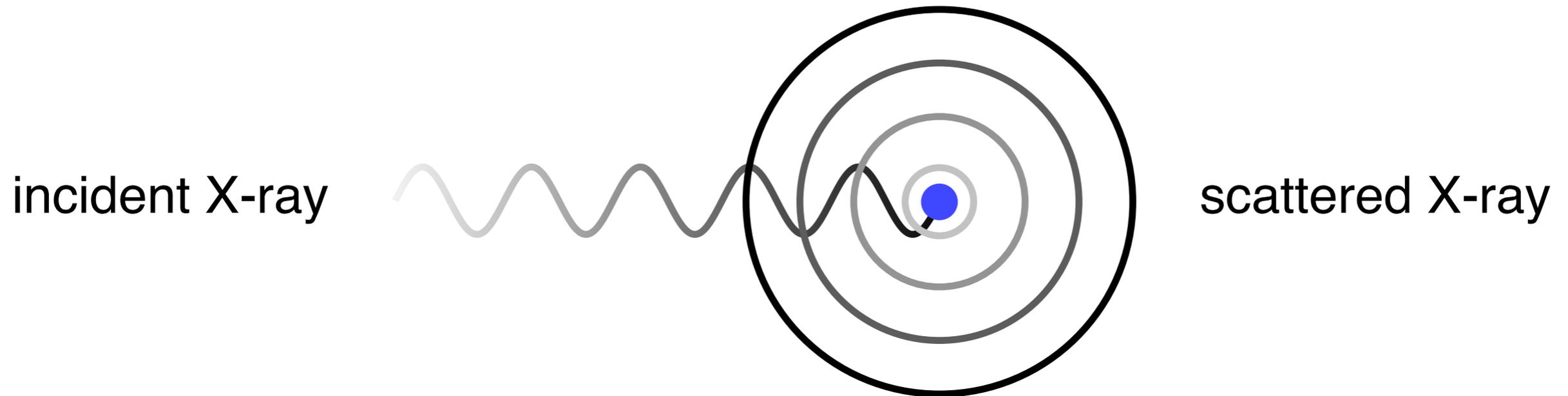
$$\vec{F} = q\vec{E}$$

Incident X-ray is a plane wave with wave vector,  $\vec{k}_0 = \frac{2\pi}{\lambda} \hat{k}$

Electric field component of plane wave at electron position,  $\mathbf{r}$ :

$$E(\vec{r}, t) = E_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

# Scattering from a single electron



Scattered X-ray is a spherical wave with wave vector,  $|\vec{k}| = |\vec{k}_0|$

**Thomson scattering:** Elastic scattering from free electron

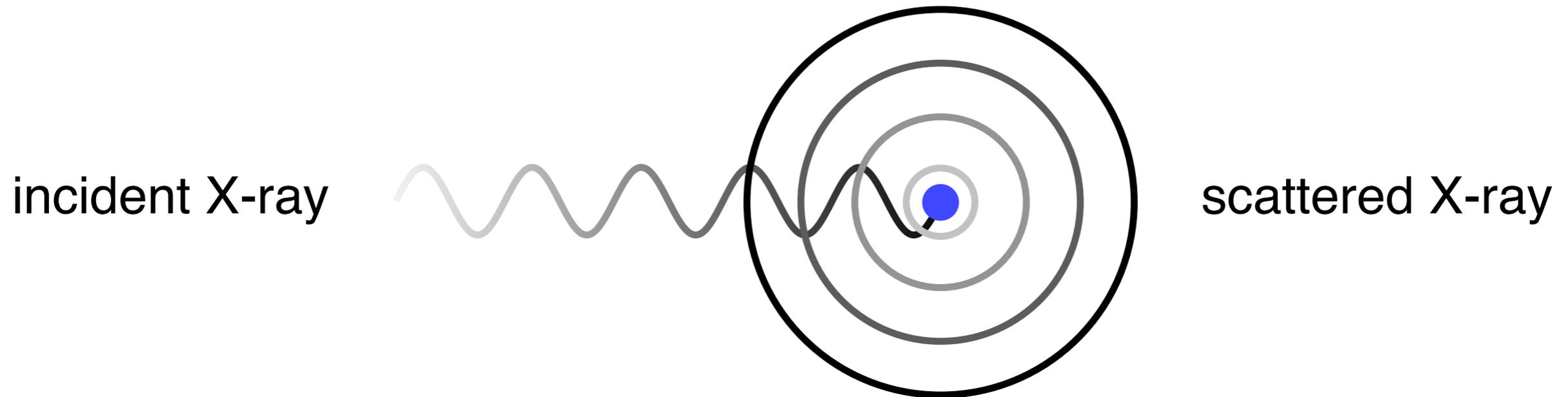
$$I_{elec} = |E_{elec}|^2 = E_0^2 \left( \frac{e^2}{4\pi\epsilon_0 c^2 m} \right)^2 \left( \frac{1 + \cos^2 2\theta}{2} \right) \frac{1}{R^2}$$

incident intensity

polarization factor

spherical wave

# Scattering from a single electron



Scattered X-ray is a spherical wave with wave vector,  $|\vec{k}| = |\vec{k}_0|$

**Thomson scattering:** Elastic scattering from free electron

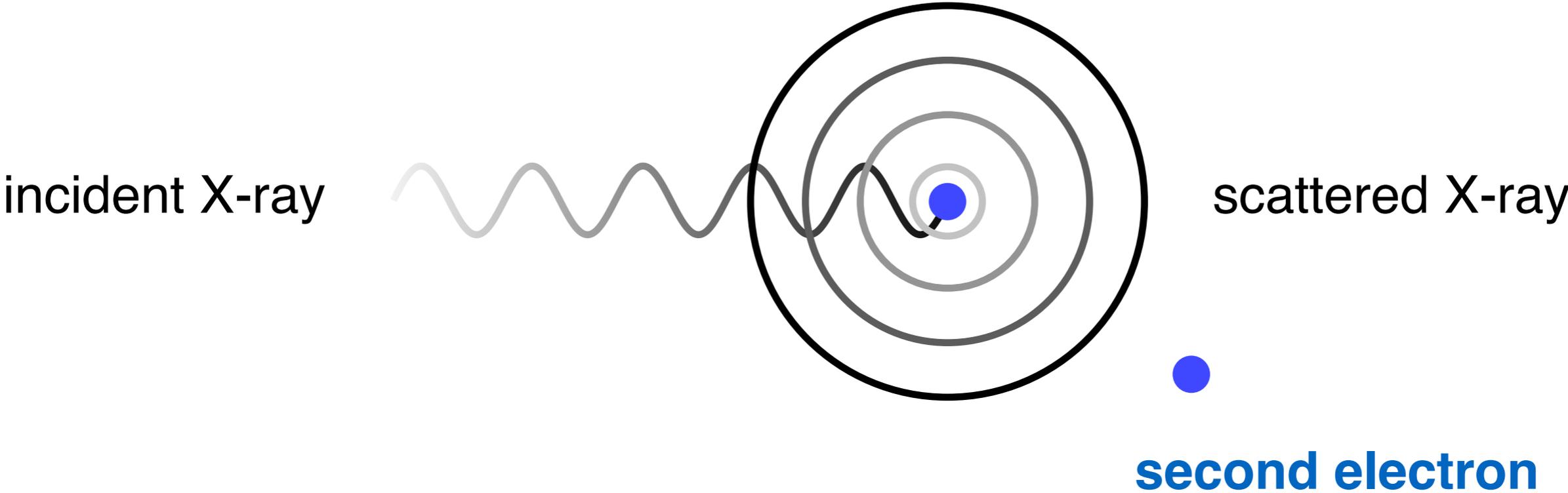
$$I_{elec} = |E_{elec}|^2 = E_0^2 \left( \frac{e^2}{4\pi\epsilon_0 c^2 m} \right)^2 \left( \frac{1 + \cos^2 2\theta}{2} \right) \frac{1}{R^2}$$

Scattering dominated by electrons:  
 $(m_e / m_p)^2 \sim 3 \times 10^{-7}$

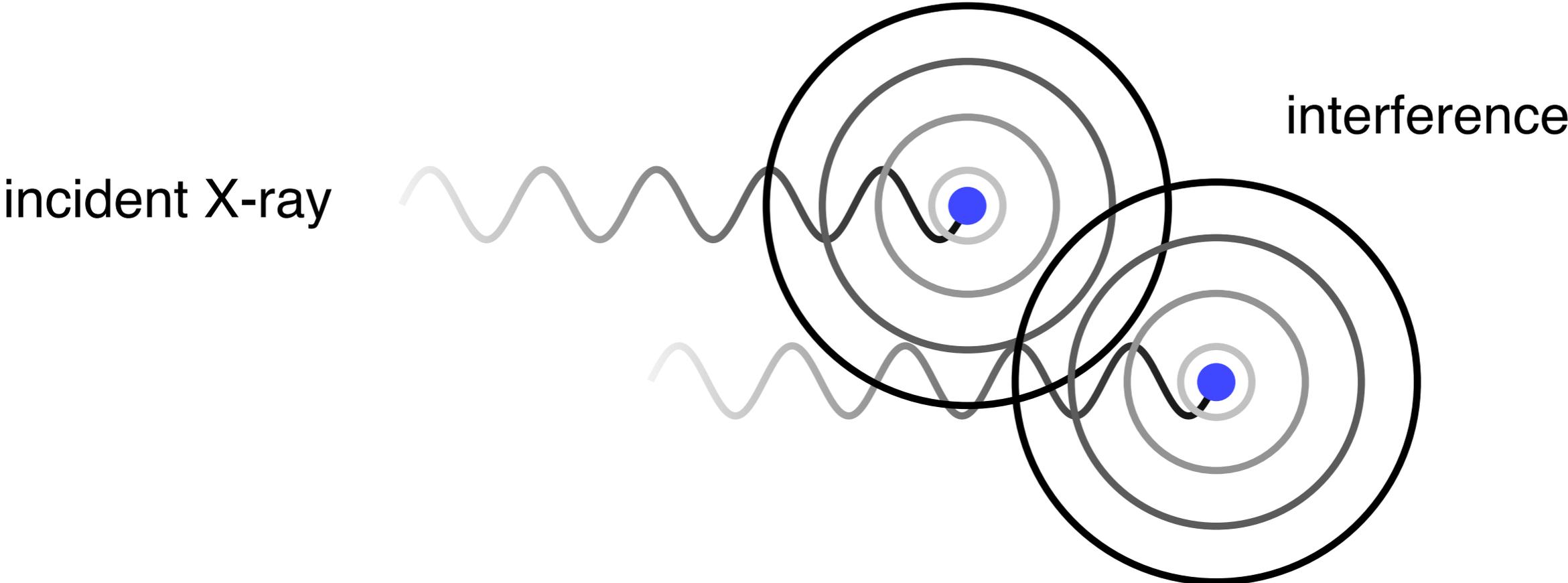
$\approx 1$  at small angles

# Scattering from a pair of electrons

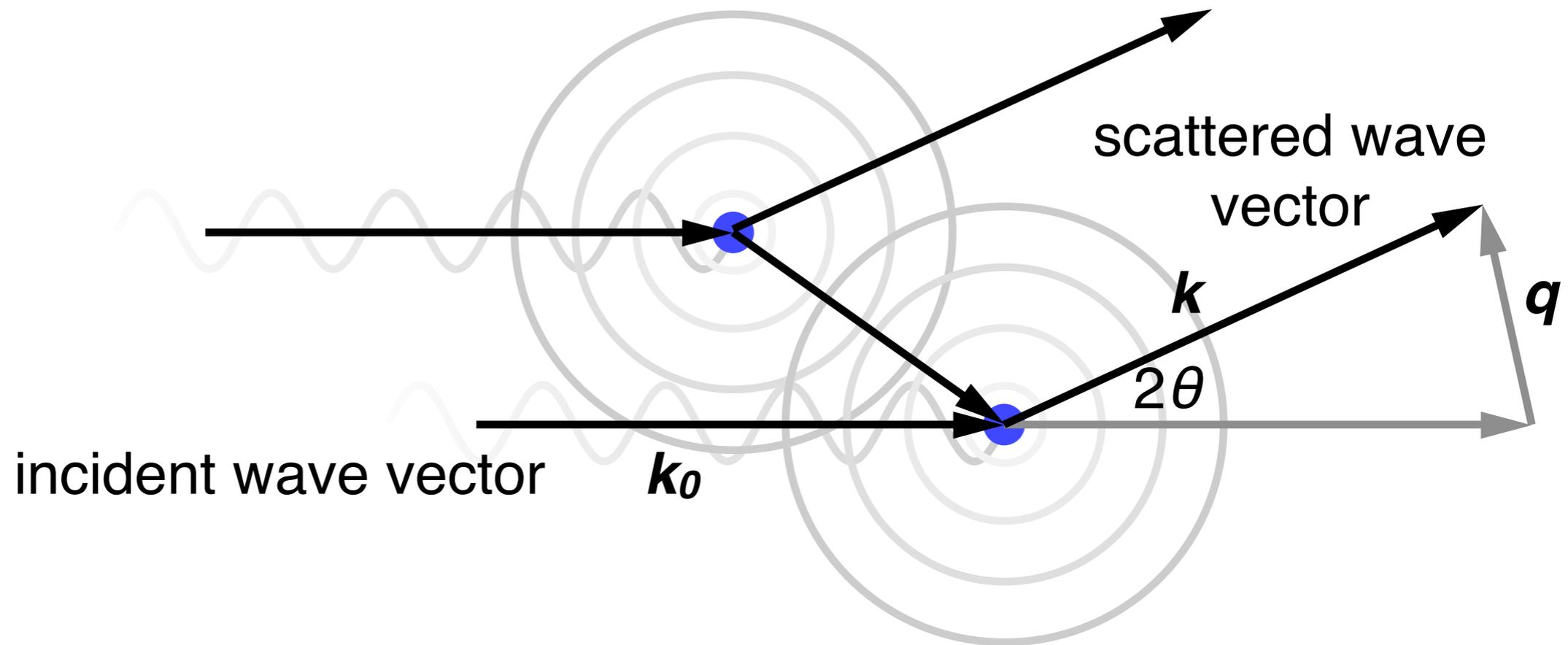
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# Scattering from a pair of electrons



# Scattering from a pair of electrons

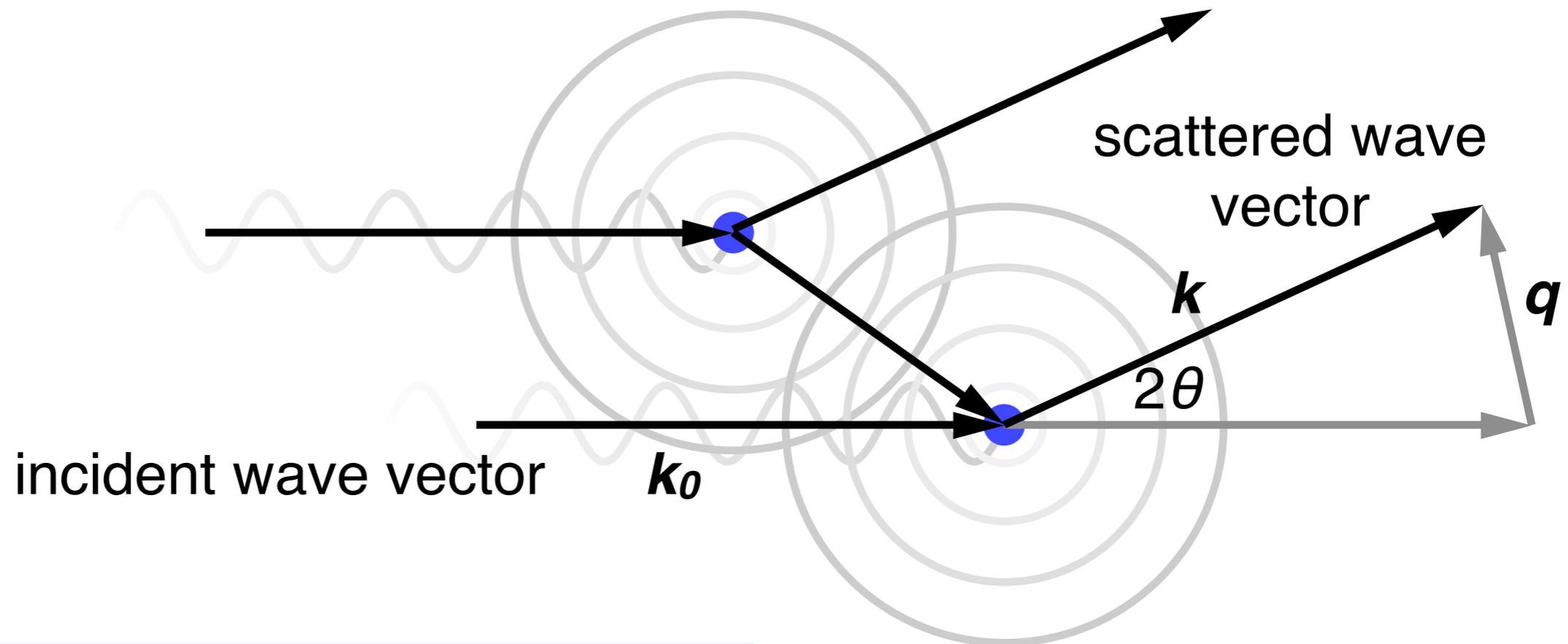


**Elastic scattering:**  $|\vec{k}| = |\vec{k}_0| = \frac{2\pi}{\lambda}$

**Derive:**  $q = \frac{4\pi}{\lambda} \sin \theta$

(homework)

# Scattering from a pair of electrons



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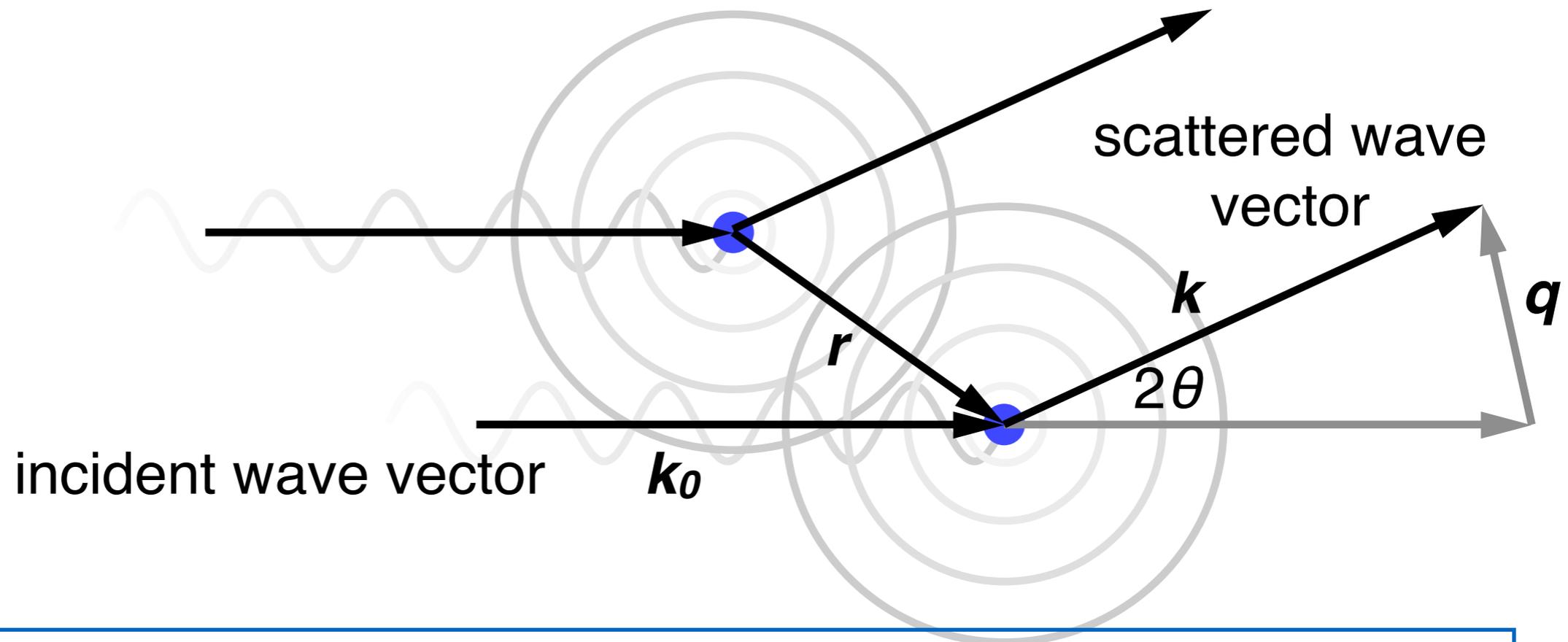
**Another notation:**

$$s = \frac{2}{\lambda} \sin \theta,$$

where  $q = 2\pi s$

units are  $1/\text{\AA}$  or  $1/\text{nm}$  for both  $q$  and  $s$ ; **define** notation in publications

# Scattering from a pair of electrons

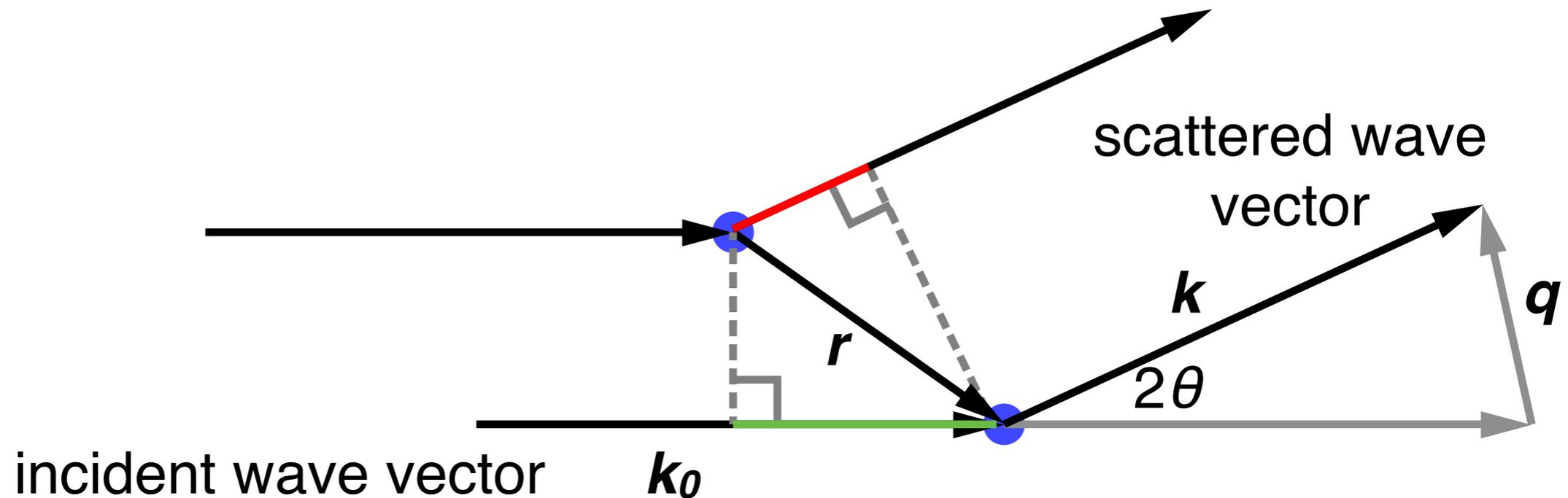


At small angles and far field ( $R \gg r$ ), individual scattered waves have **identical amplitudes**.

$$I_{elec} = |E_{elec}|^2 = E_0^2 \left( \frac{e^2}{4\pi\epsilon_0 c^2 m} \right)^2 \left( \frac{1 + \cos^2 2\theta}{2} \right) \frac{1}{R^2} \Rightarrow \text{constant}$$

↑  
amplitude  
squared

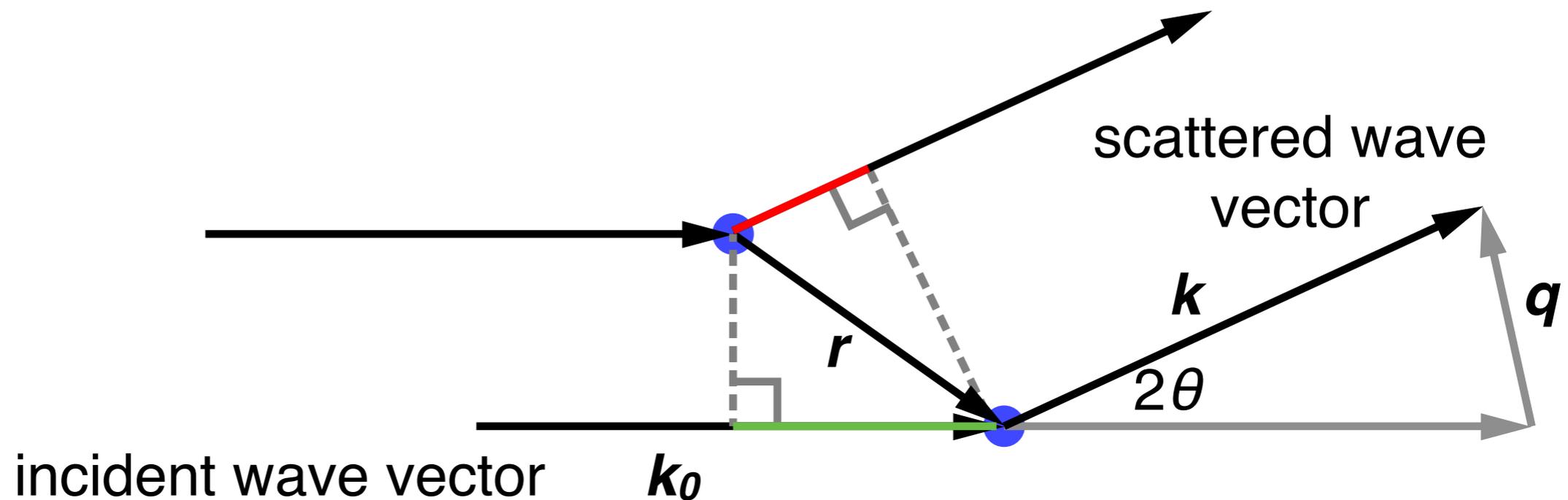
# Scattering from a pair of electrons



At small angles and far field ( $R \gg r$ ), individual scattered waves have **identical amplitudes**... but they differ by **phase shift**,  $\phi$ .

$$\phi = \vec{k}_0 \cdot \vec{r} - \vec{k} \cdot \vec{r} = -\vec{q} \cdot \vec{r}$$

# Scattering from a pair of electrons

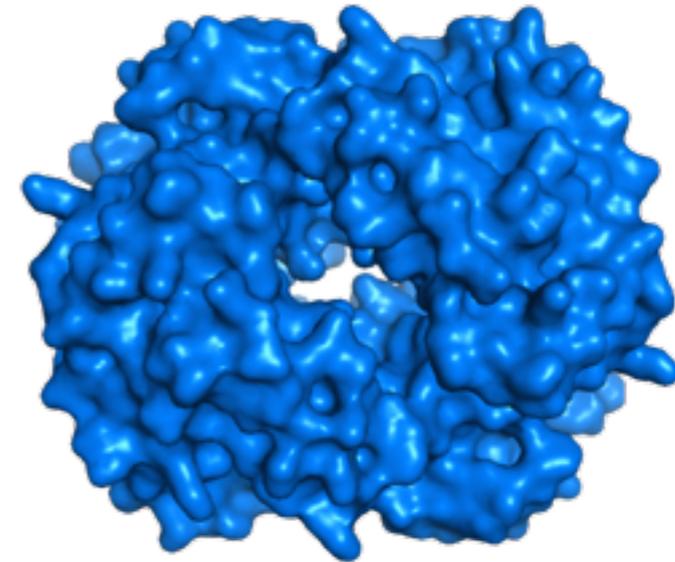


At small angles and far field ( $R \gg r$ ), individual scattered waves have **identical amplitudes**... but they differ by **phase shift**,  $\phi$ .

$$E_{pair}(q) = \text{wave 1} + \text{wave 2} = \sum_{j=1}^2 |E_{elec}| e^{i\phi_j} = |E_{elec}| e^0 + |E_{elec}| e^{-i\vec{q} \cdot \vec{r}}$$

# Scattering from a protein (with $n$ electrons)

incident X-ray

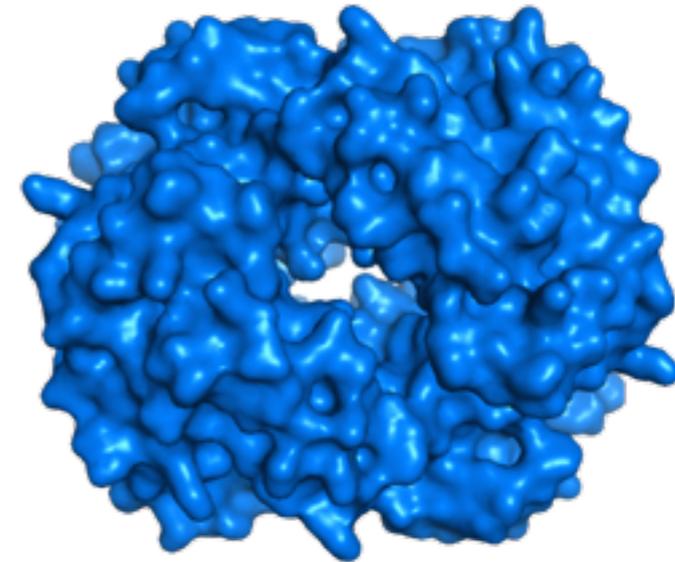


At X-ray energies far from binding energies (absorption edges), all electrons can be treated as if free.

$$E(q) = |E_{elec}| \sum_j^n e^{-i\vec{q}\cdot\vec{r}_j} \propto \int_V \rho(\vec{r}) e^{-i\vec{q}\cdot\vec{r}} d\vec{r} = F[\rho(\vec{r})]$$

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Scattering amplitude from a protein is the Fourier Transform of its electron density distribution,  $\rho(r)$ , ***in other words structure!!!***

# Scattering from a uniform material (e.g. buffer)

incident X-ray



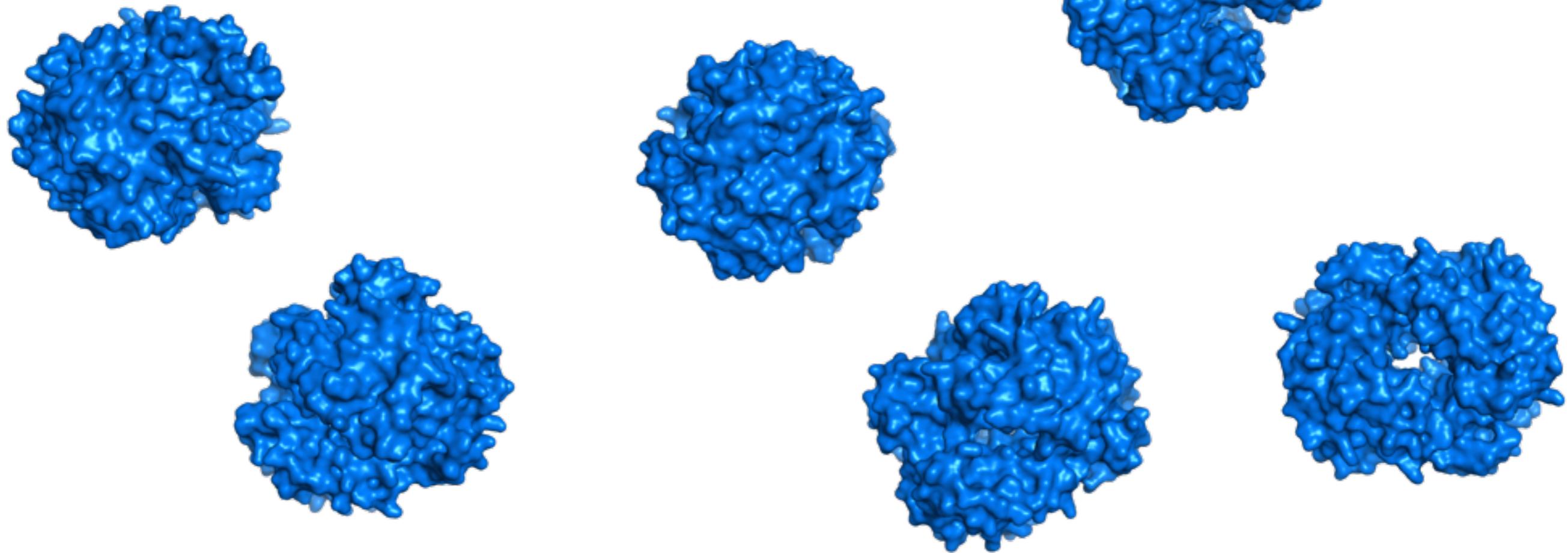
Scattering from a uniform material (constant  $\rho$ ):

$$E(q) \propto \rho \int e^{-i\vec{q}\cdot\vec{r}} d\vec{r} = \rho \cdot \delta(\vec{q} - 0)$$

→ zero everywhere except at zero-angle:  $\mathbf{q} = 0$

In other words, a uniform sample does not deflect X-rays!

# Scattering from proteins in solution



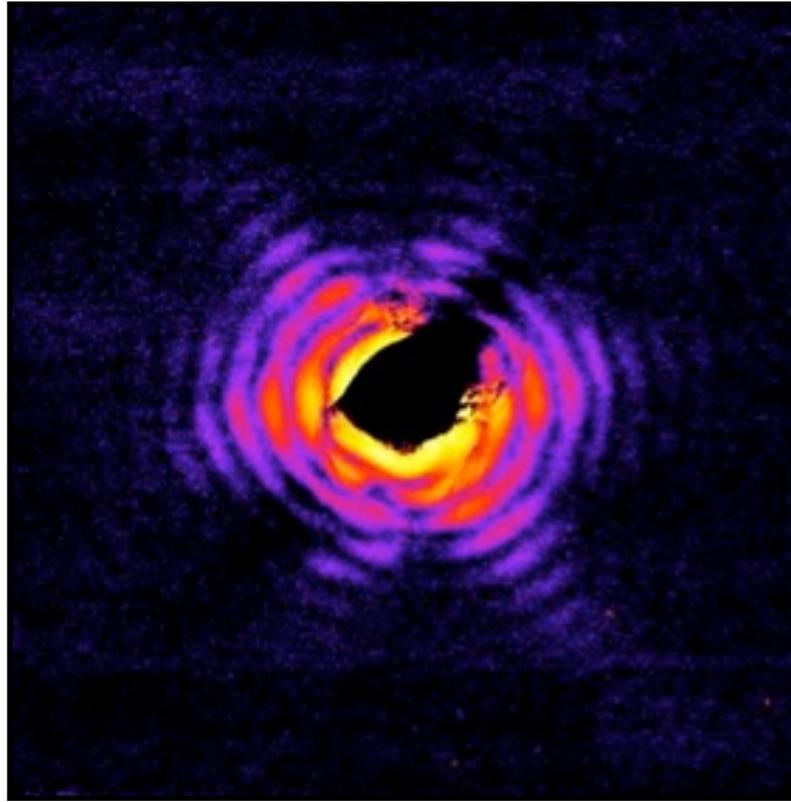
Scattering from  $N$  identical proteins under dilute conditions:

$$I(q) \propto N \left\langle \left| F[\rho(\vec{r})] \right|^2 \right\rangle_{\Omega}$$

Appears like the rotational average of a single protein.

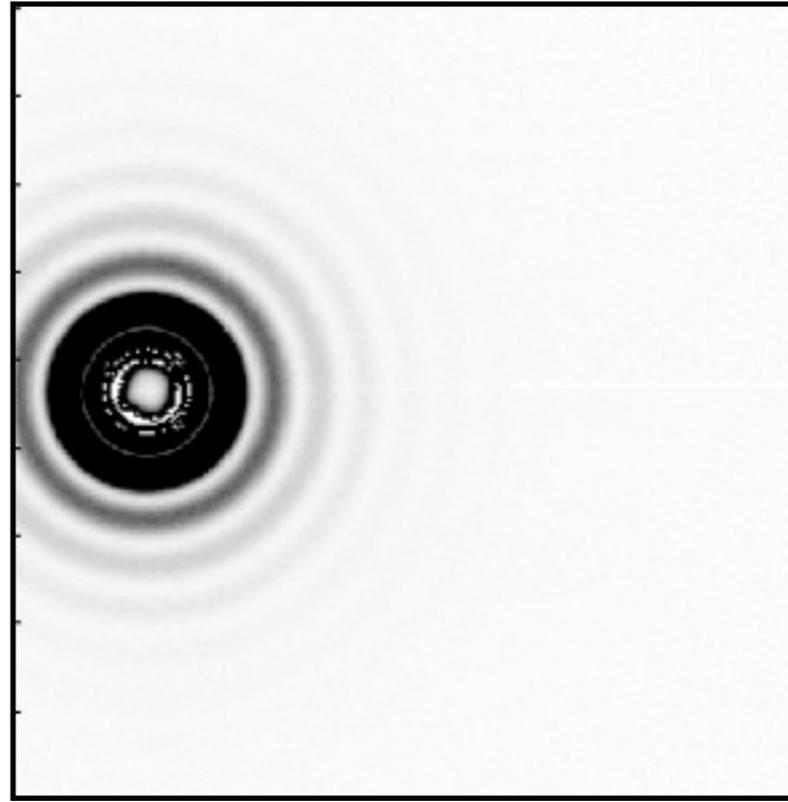
# Scattering overview

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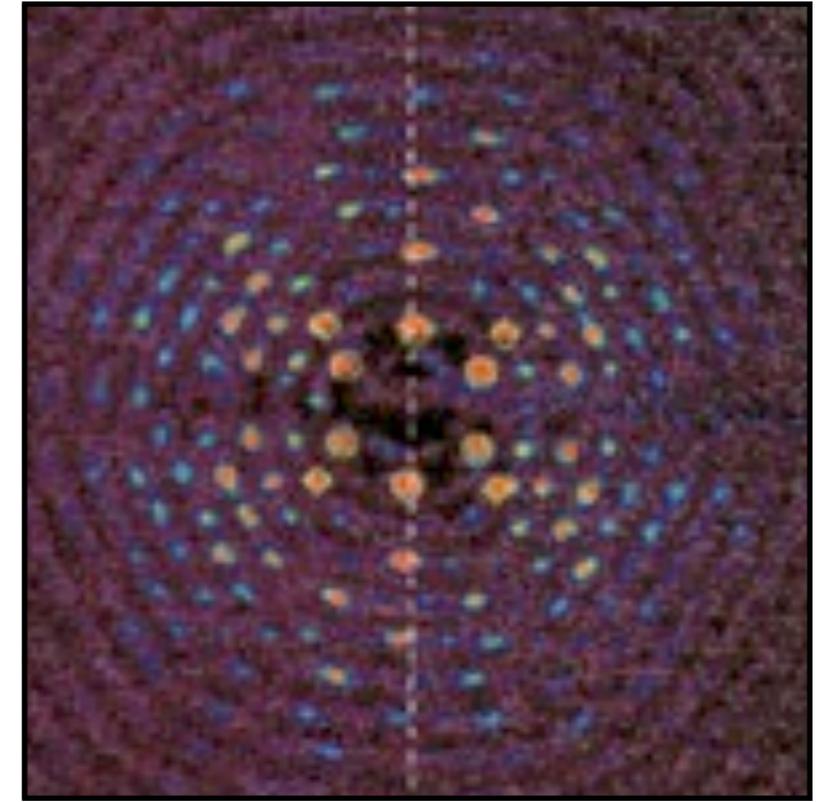
single-sphere scattering

[ESRF Sept. 2008 newsletter]



solution SAXS

[A. Finnefrock]

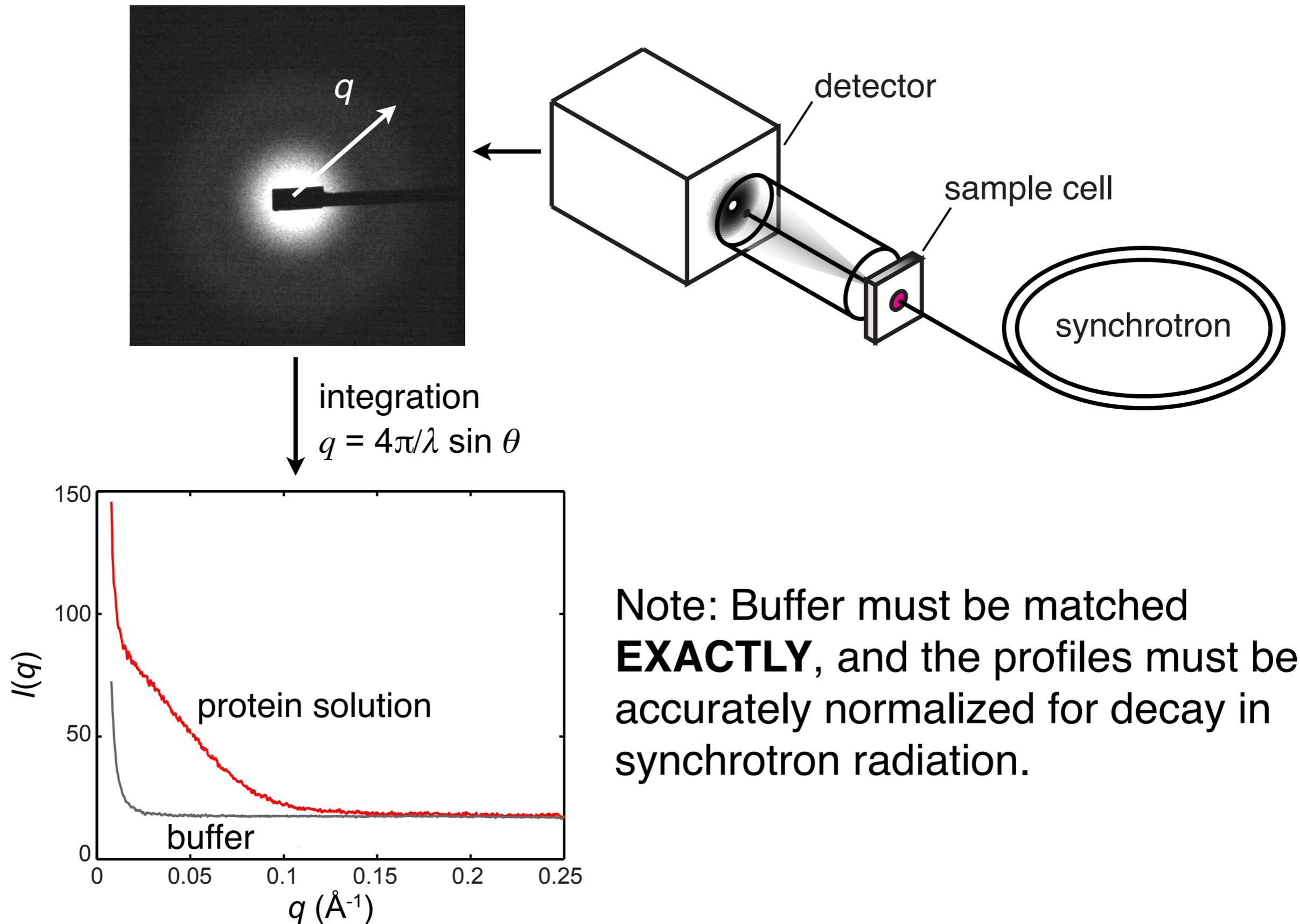


crystal diffraction

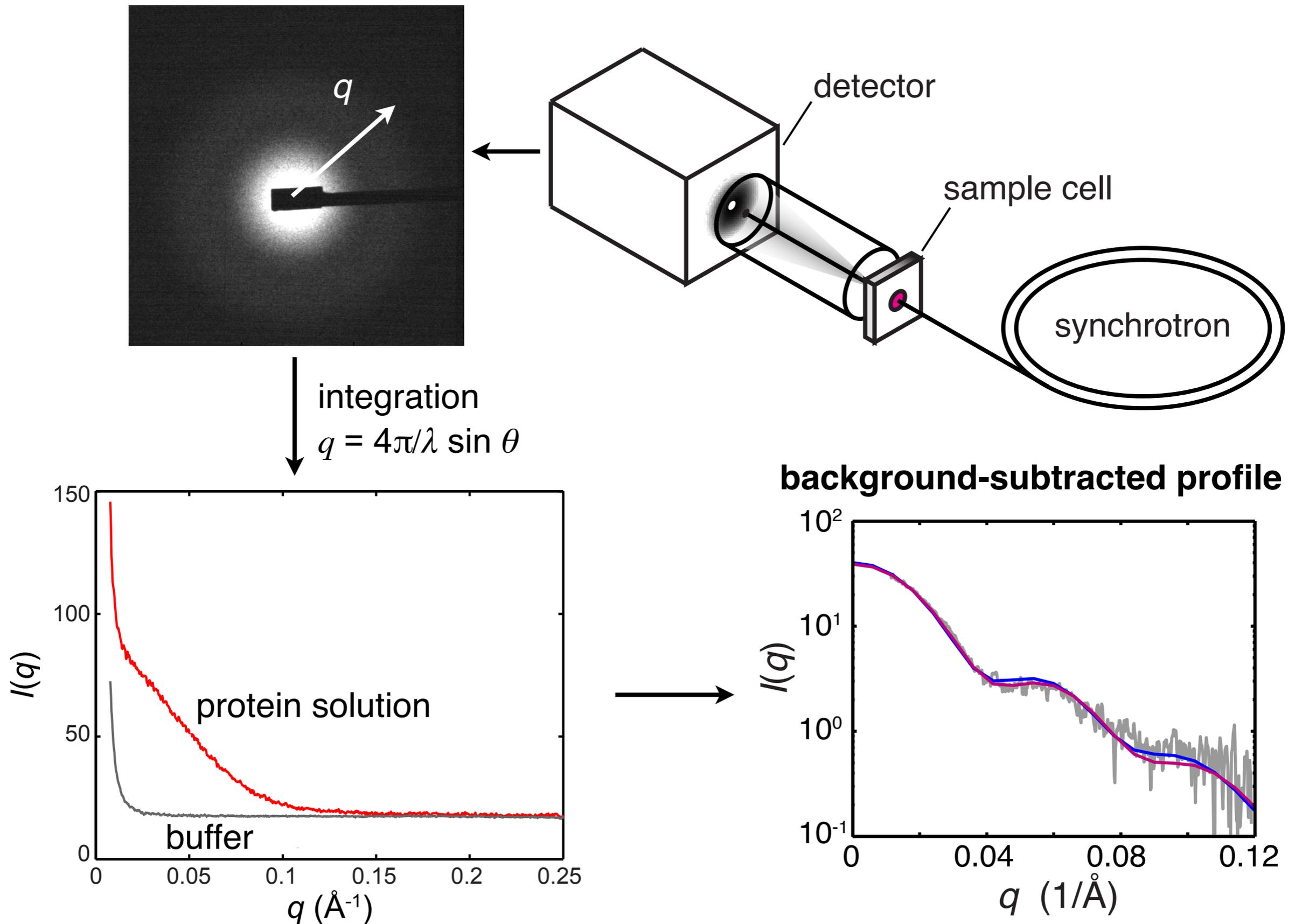
[W. L. Vos, et al. Langmuir 1997]

## **Part 2: Basic Interpretation of SAXS Data**

# SAXS begins with measurement of two solutions



# Analysis is done on background-subtracted profiles



# Radius of gyration and the zero-angle intensity

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From rotational averaging, we can re-write:

$$I(q) = 4\pi \int_0^{D_{\max}} P(r) \frac{\sin qr}{qr} dr$$

$P(r)$  is the pair-distance distribution, a histogram of all distances in a protein.

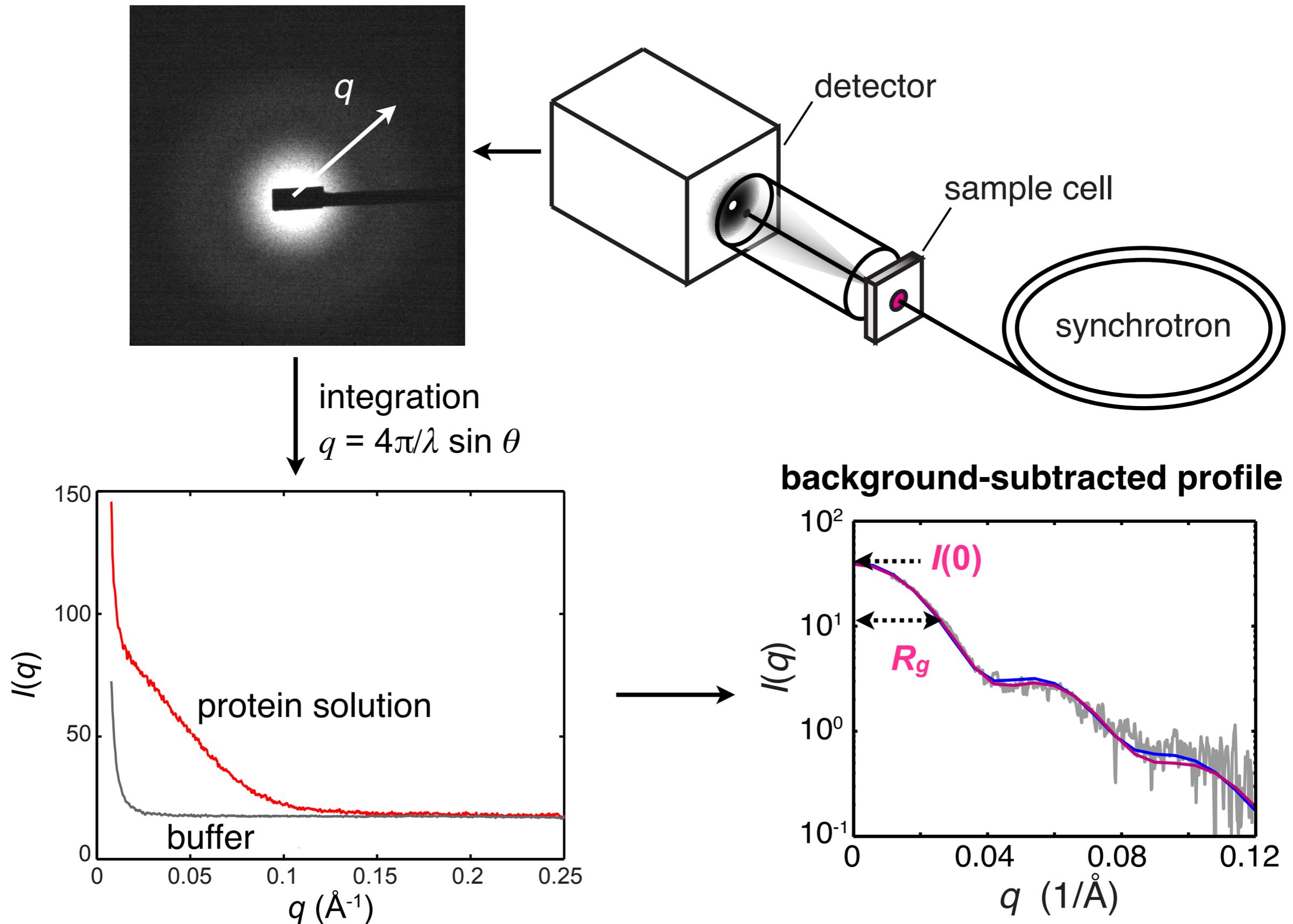
Radius of gyration = root mean square distance of all electrons:

$$R_g^2 = \frac{\int_0^{D_{\max}} r^2 P(r) dr}{2 \int_0^{D_{\max}} P(r) dr}$$

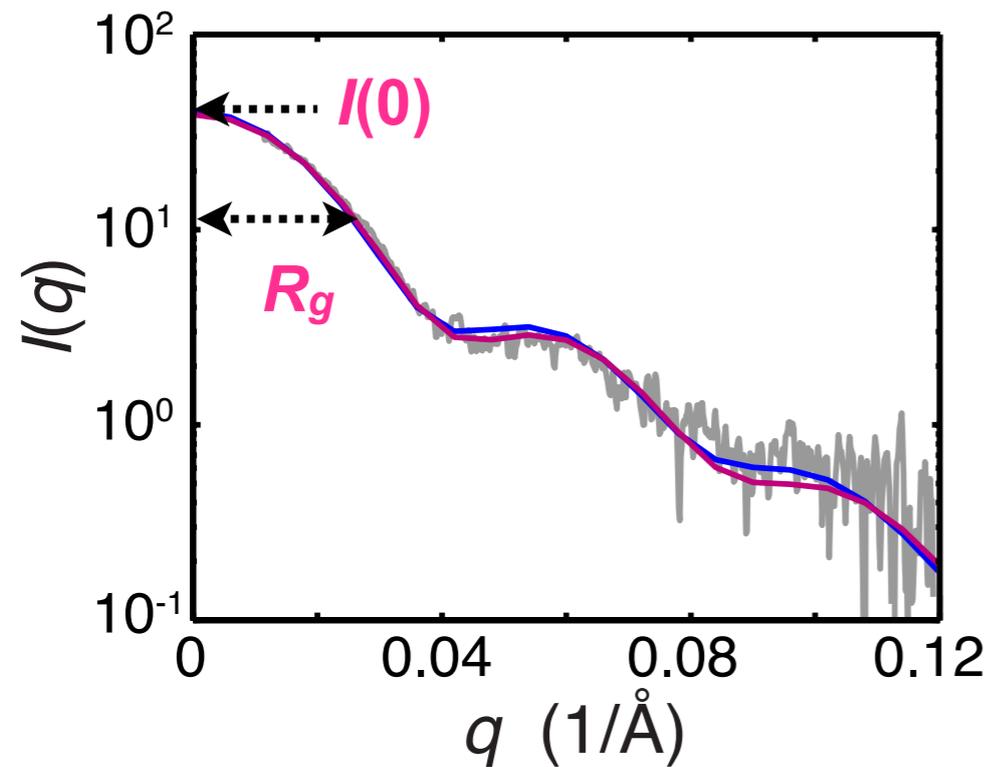
The zero-angle scattering intensity (or forward scattering intensity) is a function of the number of electrons ( $n$ ) and the contrast,  $\Delta\rho$ :

$$I(0) = n / V (\Delta\rho \cdot V_p)^2 \approx f(\text{mass})$$

# Interpretation of background-subtracted profiles



# Obtaining $R_g$ and $I(0)$ from Guinier analysis



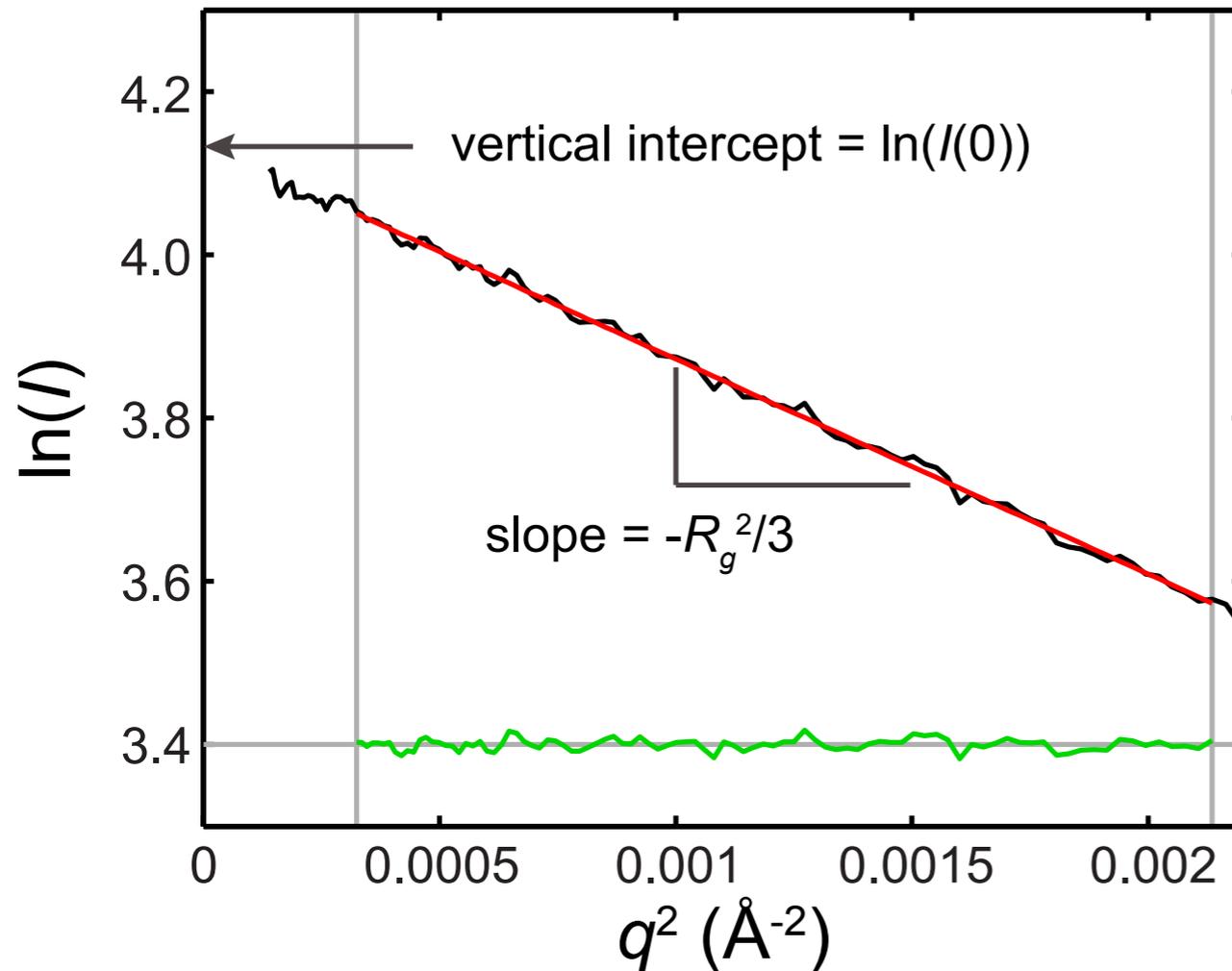
Guinier approximation at small angles ( $qR_g < 1.3$ ):

$$I(q) = I(0)e^{-R_g^2 q^2 / 3}$$

$$\ln[I(q)] = \ln[I(0)] - \frac{R_g^2}{3} q^2$$

Skou, *et al.* (2014) Nature Procols, in press.

# Obtaining $R_g$ and $I(0)$ from Guinier analysis



“Guinier plot”

Guinier approximation at small angles ( $qR_g < 1.3$ ):

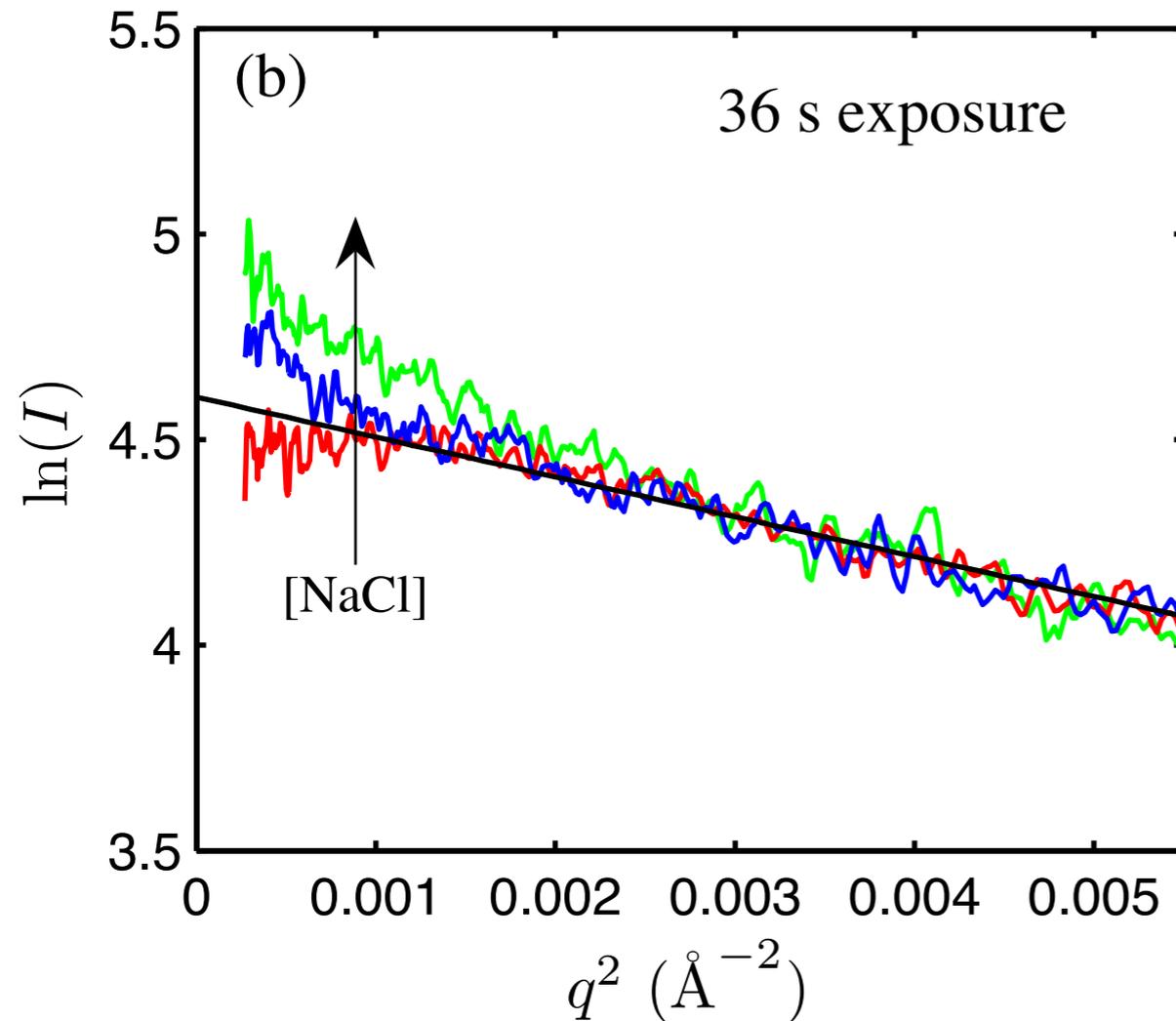
$$I(q) = I(0)e^{-R_g^2 q^2 / 3}$$

$$\ln[I(q)] = \ln[I(0)] - \frac{R_g^2}{3} q^2$$

Reviewers want to see a linear Guinier plot, which is an indication of high sample/data quality.

Skou, *et al.* (2014) Nature Procols, in press.

# Guinier plots are sensitive to aggregation



Guinier approximation at small angles ( $qR_g < 1.3$ ):

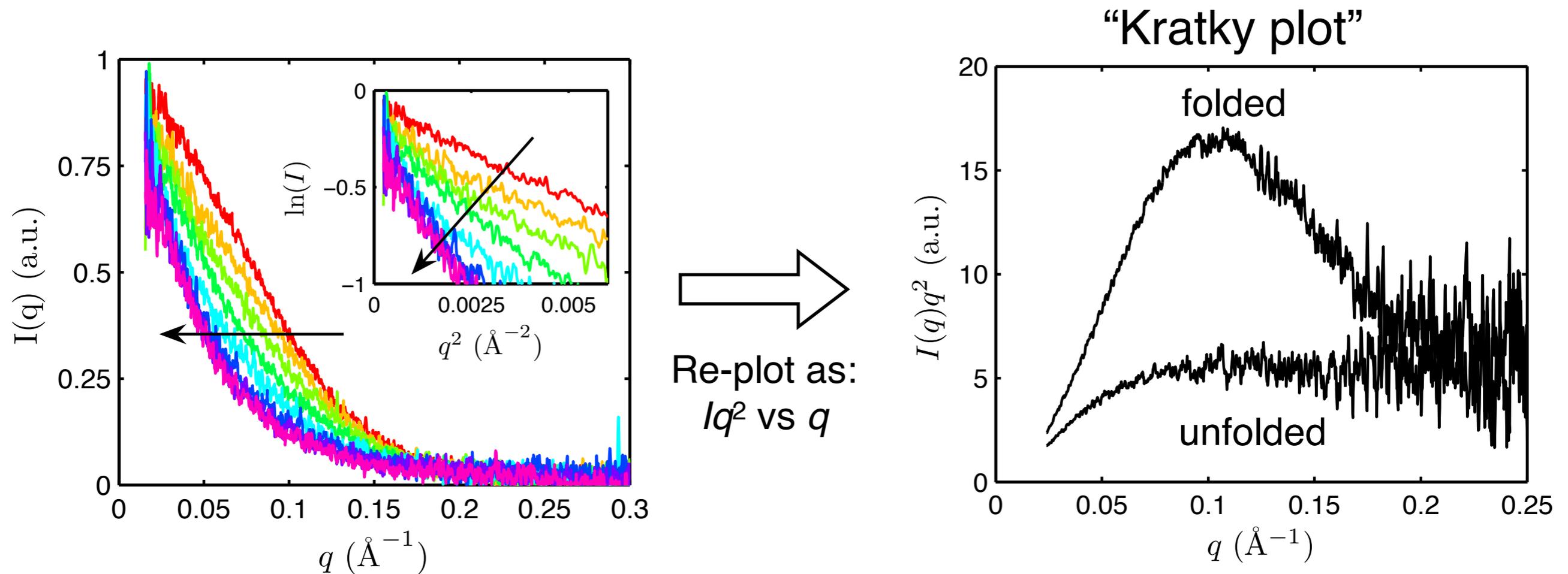
$$I(q) = I(0)e^{-R_g^2 q^2 / 3}$$

$$\ln[I(q)] = \ln[I(0)] - \frac{R_g^2}{3} q^2$$

Radiation damage manifests as aggregation, which leads to an upturn at low  $q$ .

Skou, *et al.* (2014) Nature Procols, in press.

# Shape information from Kratky analysis



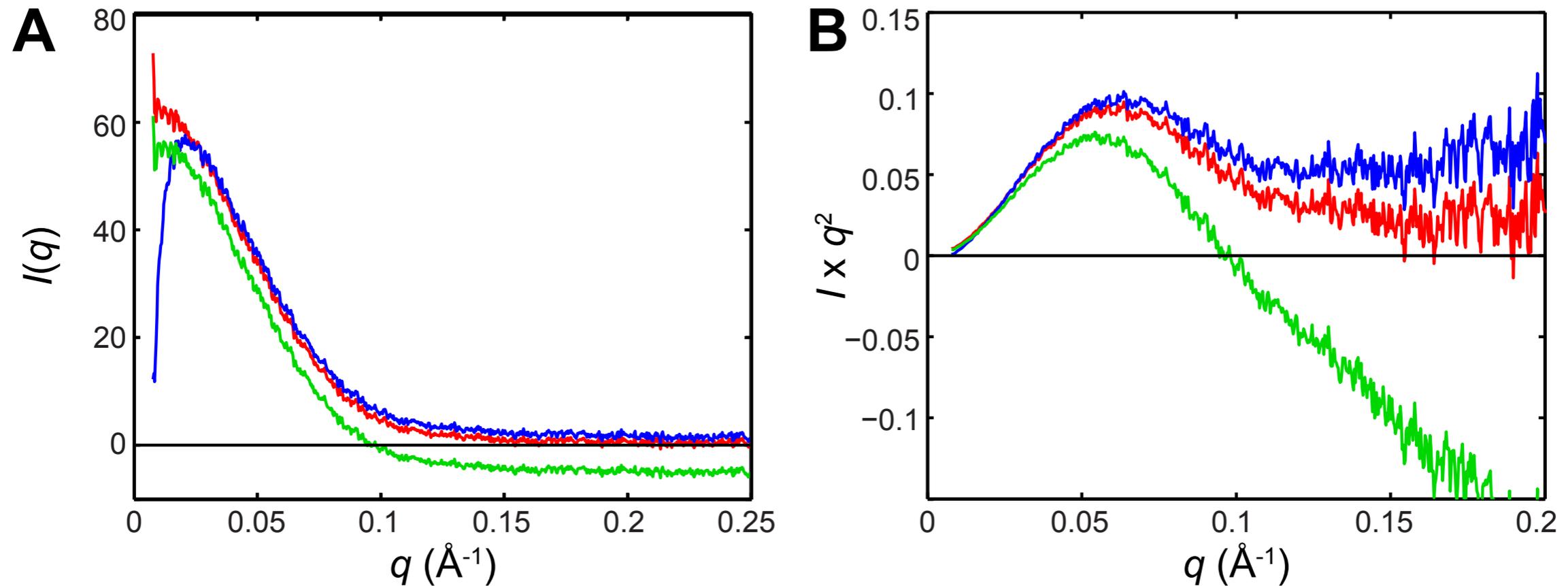
Emphasizes the power-law dependence in mid- $q$  region ( $\sim q^{-n}$ ) that contains information about foldedness, compactness, flexibility.

folded, compact:  $n \sim -4 \Rightarrow$  peak in  $P(r)$   
random polymer chain:  $n \sim -2 \Rightarrow$  plateau in  $P(r)$   
extended polymer chain:  $n \sim -1 \Rightarrow$  rise in  $P(r)$

Ando, *et al.* (2008). *Biochemistry*, 47(42), 11097–11109.

# Kratky plots are sensitive to background subtractions

Extreme examples of buffer mismatch:

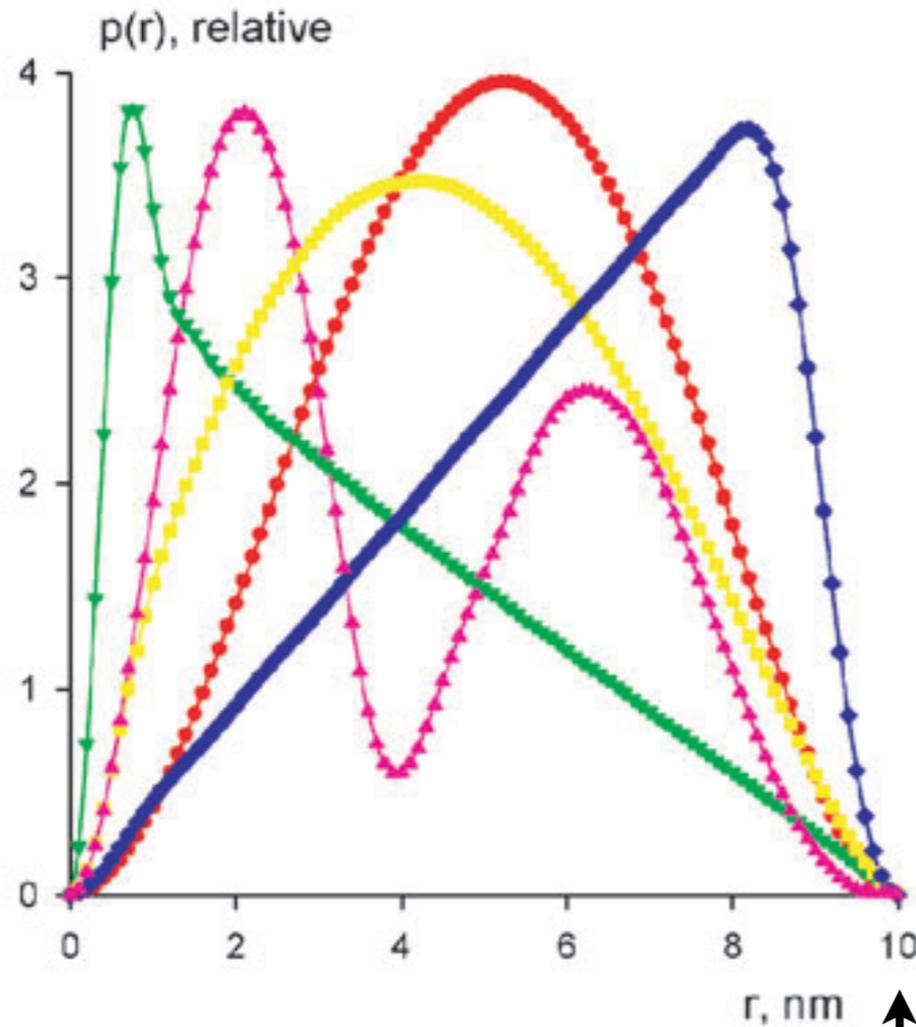
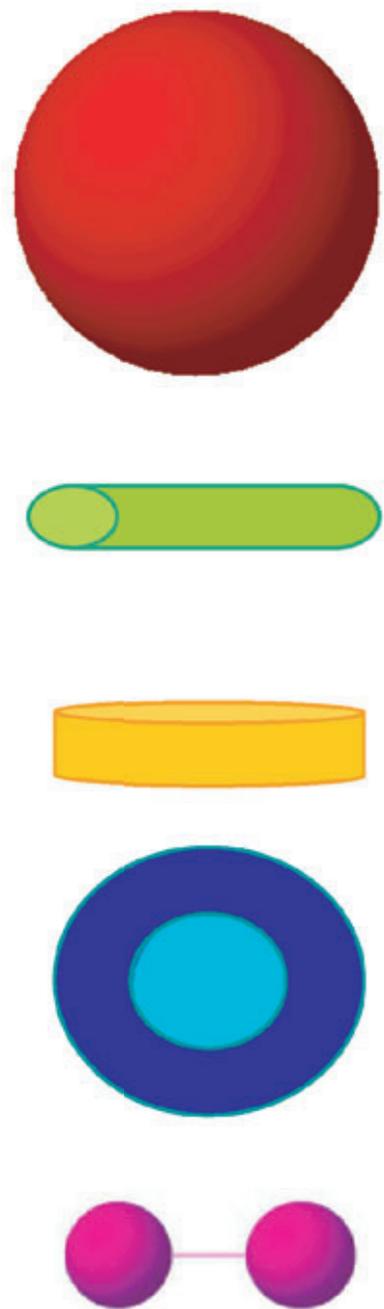


properly matched buffer  
similar buffer with glycerol added  
different buffer

Skou, *et al.* (2014) Nature Procols, in press.

# Pair-distance distribution function

$P(r)$  is the inverse Fourier Transform of  $I(q)$  and represents a histogram of electron-pair distances in a protein.



$D_{max}$  = maximum dimension

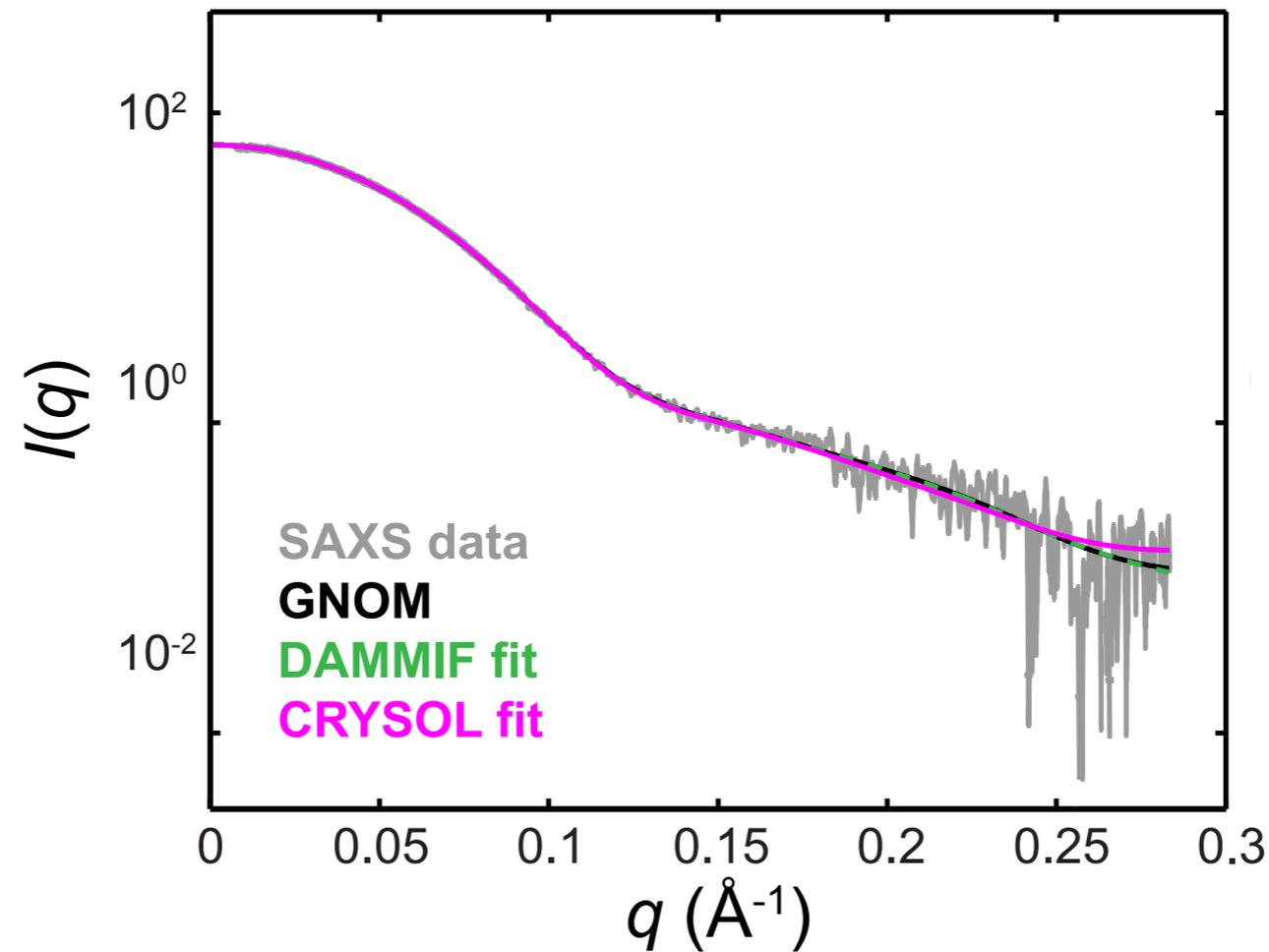
When  $D_{max}$  is well-defined, gives:  $R_g$ ,  $I(0)$ , shape info

$$R_g^2 = \frac{\int_0^{D_{max}} r^2 P(r) dr}{2 \int_0^{D_{max}} P(r) dr}$$

Svergun & Koch (2003). Reports on Progress in Physics, 66, 1735–1782.

# Fitting data to theoretical scattering

- CRY SOL from ATSAS package (Svergun group, EMBL)
- FoXS server (Sali group, UCSF)



Skou, *et al.* (2014) Nature Procols,in press.

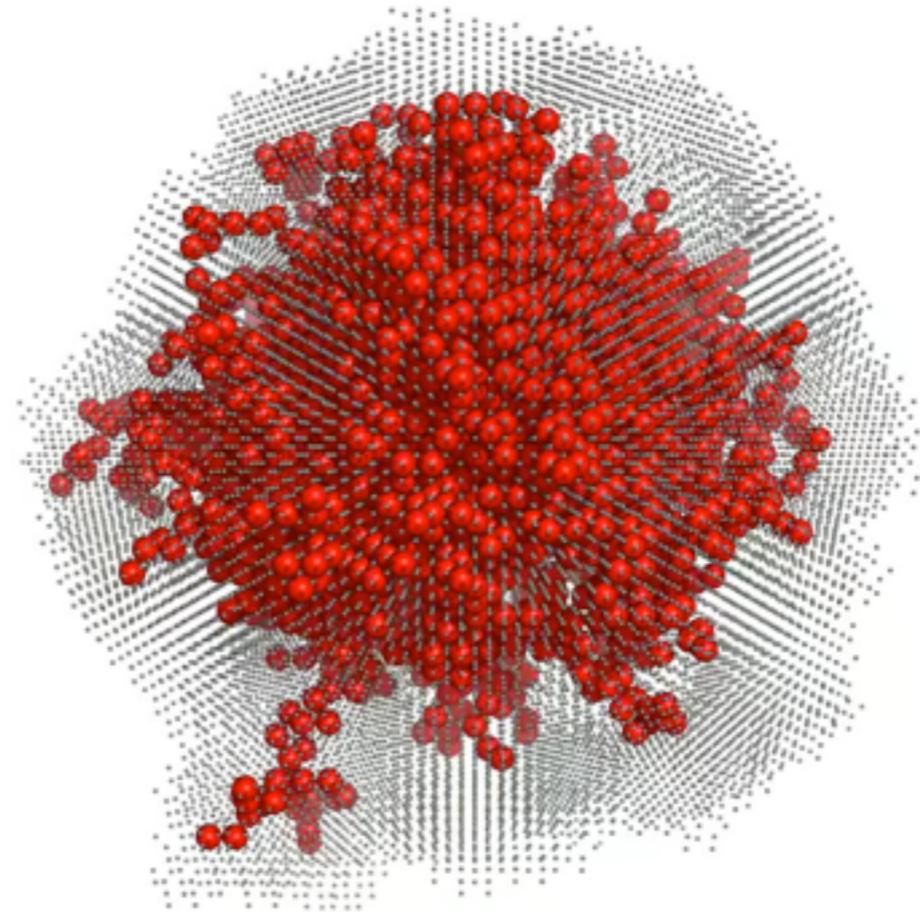
# *Ab initio* shape reconstructions

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Following  $P(r)$  analysis, a low-resolution 3D model can be reconstructed from SAXS data.

From ATSAS package:

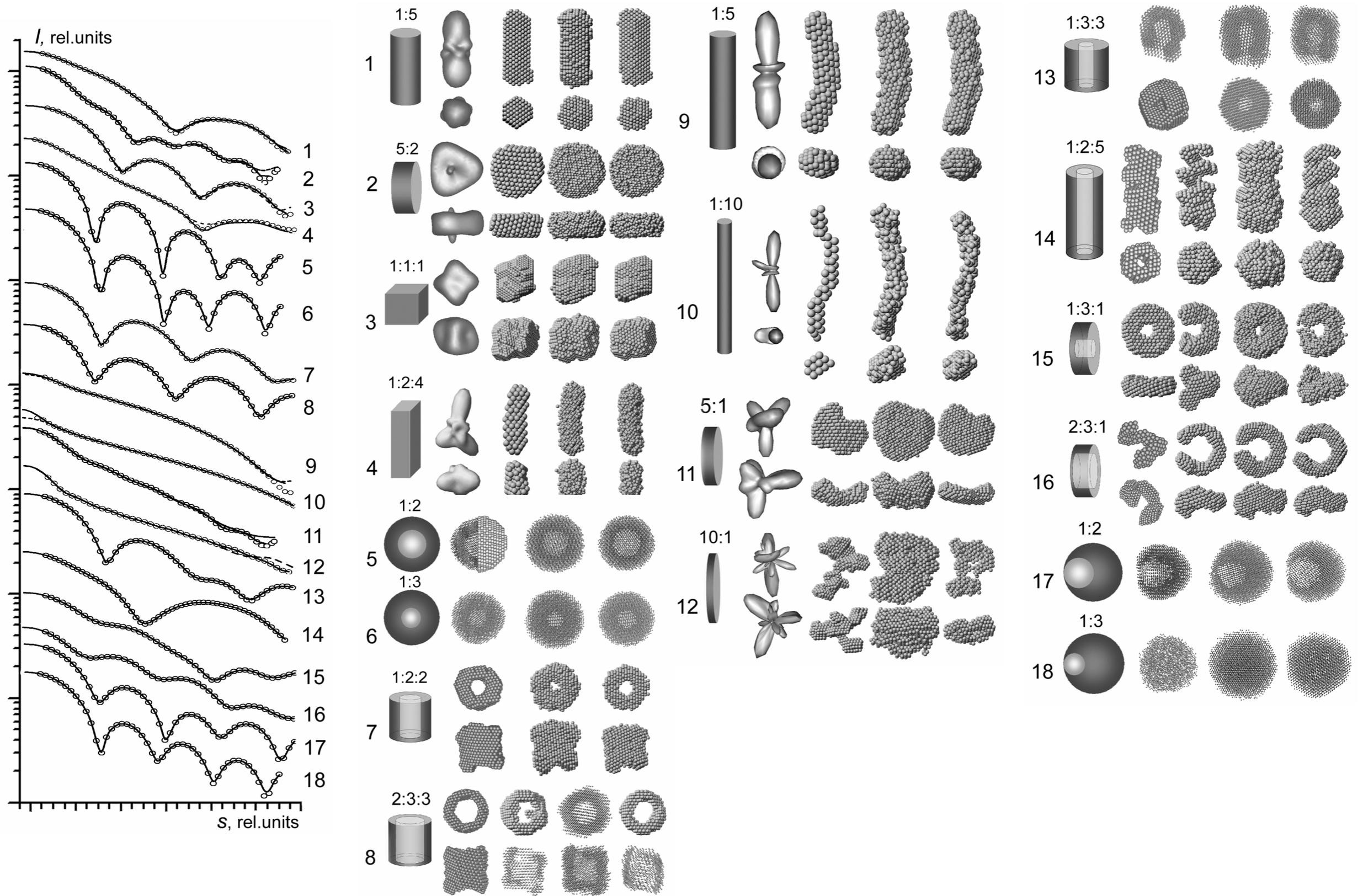
- `dammif`
- `dammin`
- `gasbor`
- `damaver`
- `supcomb`



Repeat at least 10 times, align and average in *damaver*.

Skou, *et al.* (2014) Nature Procols,in press.

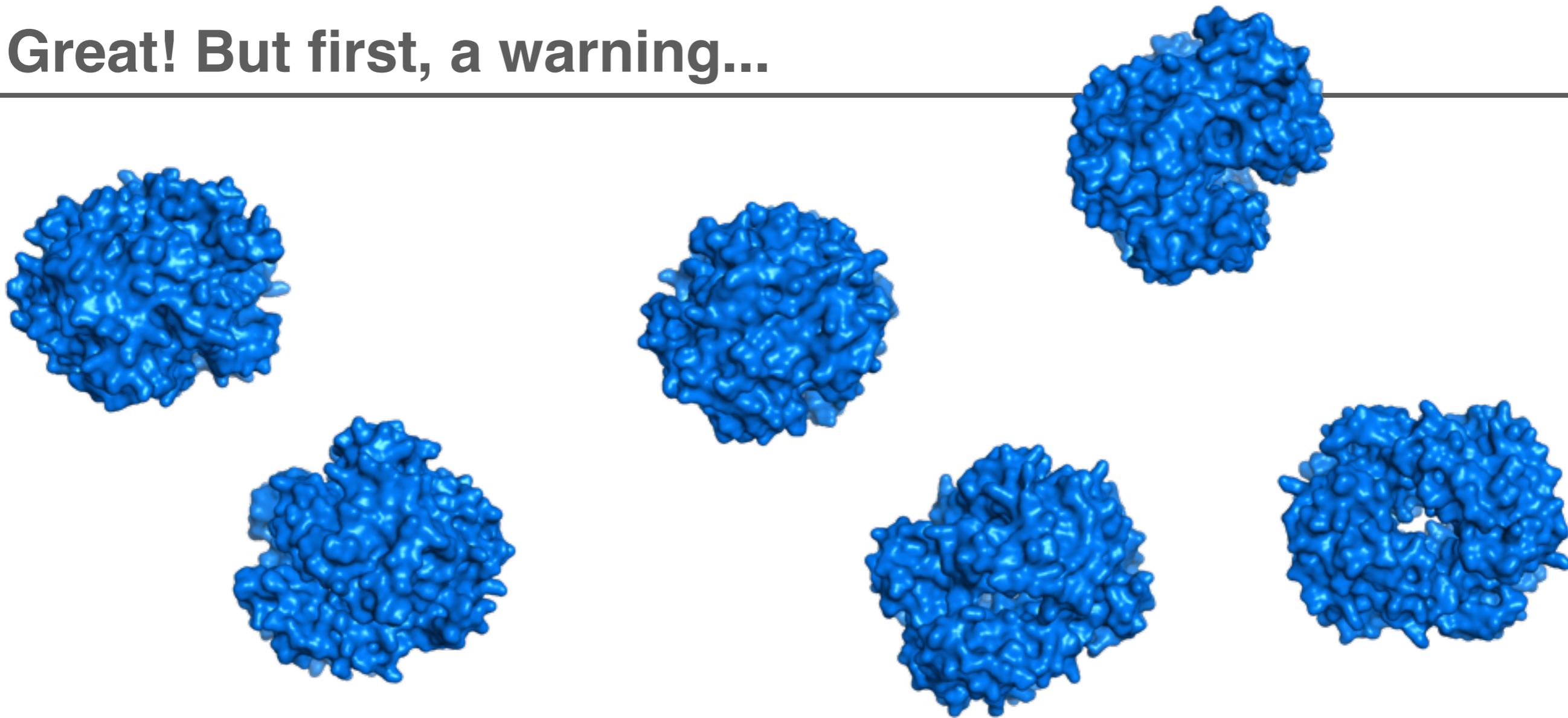
# Ab initio shape reconstructions are models



Volkov & Svergun (2003). J. Appl. Cryst. 36(3), 860–864.

# Great! But first, a warning...

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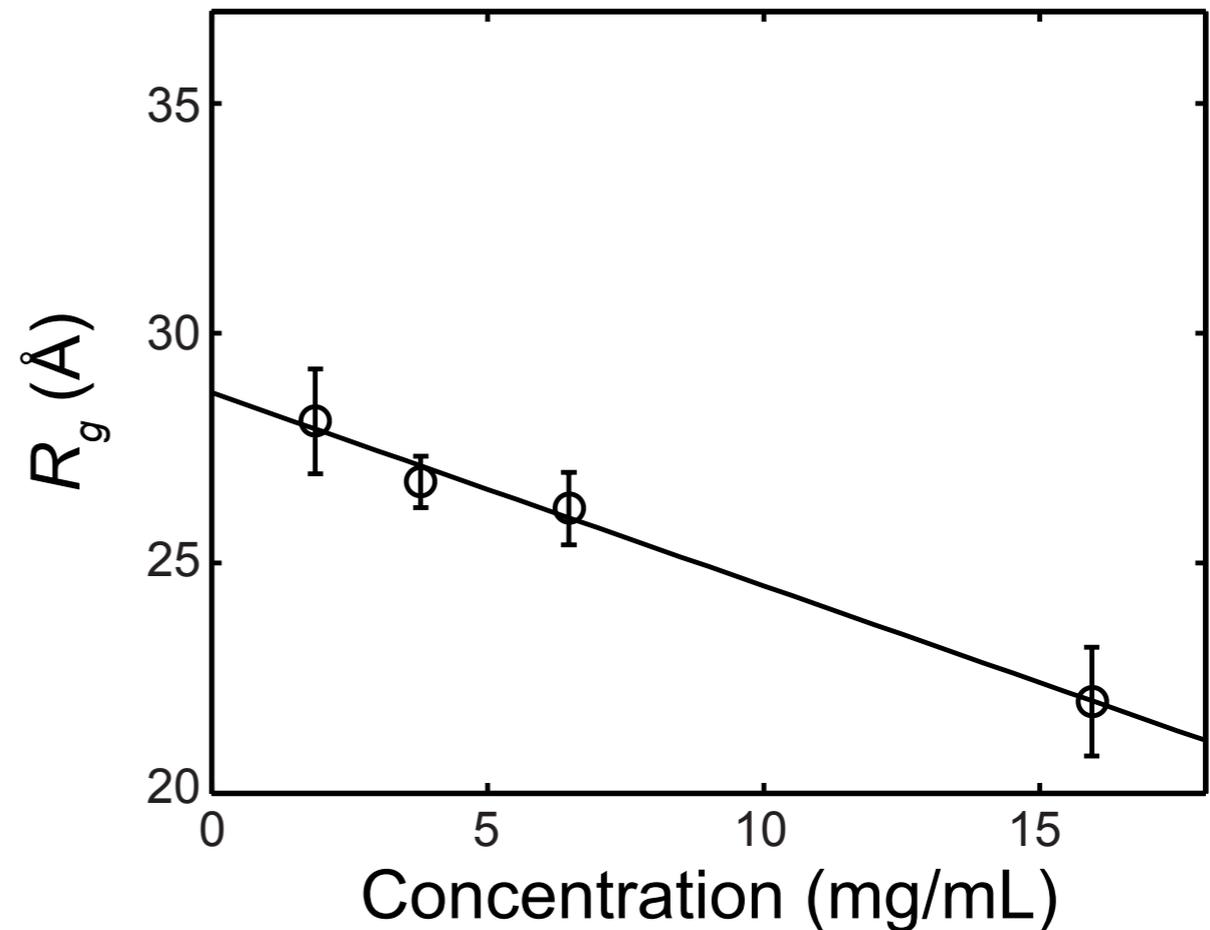
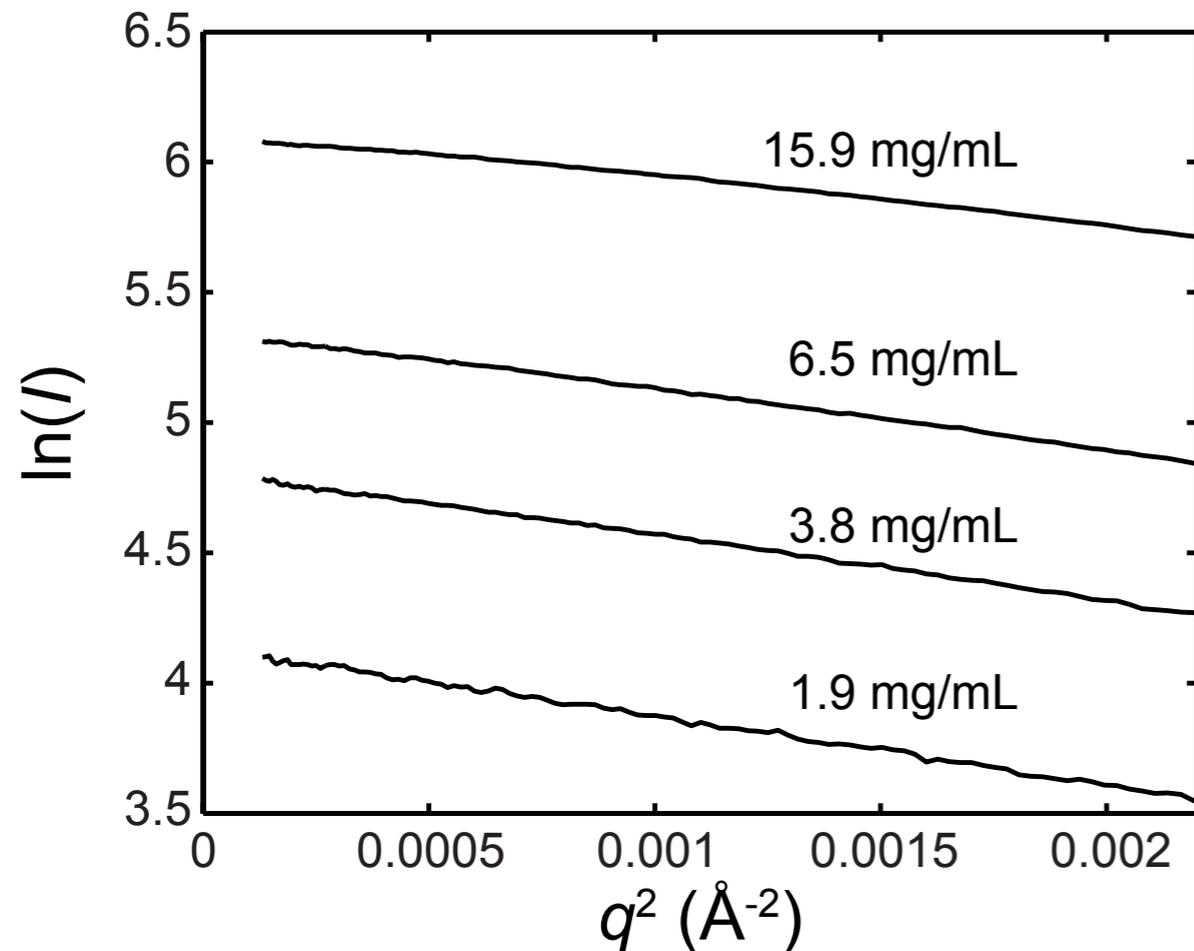


Recall that scattering from  $N$  proteins under **dilute** conditions:

$$I(q) \propto N \left\langle \left| F[\rho(\vec{r})] \right|^2 \right\rangle_{\Omega}$$

As long as  $N > 1$ , we must be aware of **inter-particle interference**.

# Concentration effects must be examined

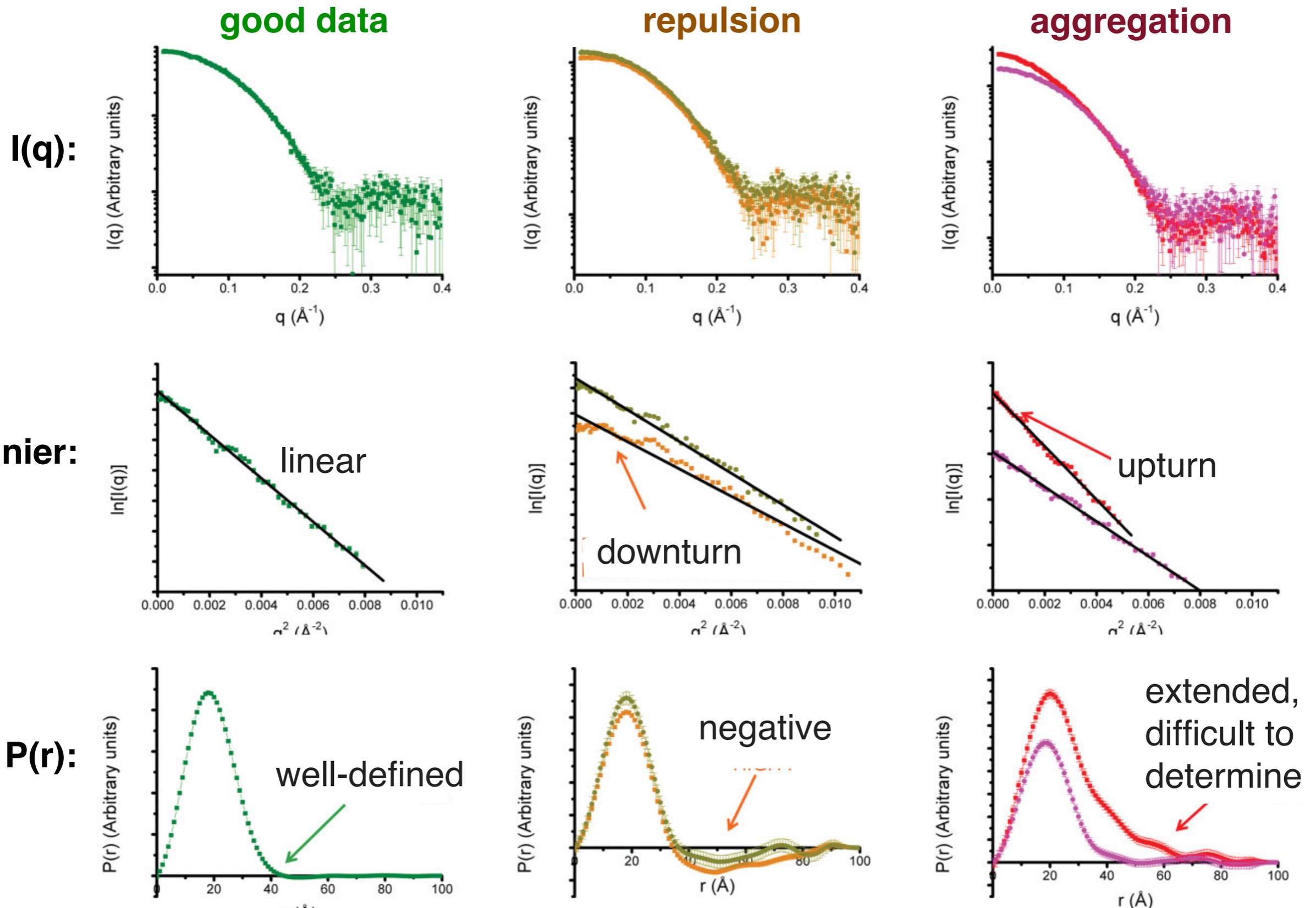


**Always,** examine sample at multiple concentrations.

Report the  $R_g$  value extrapolated to infinite dilution (vertical intercept), where we can approximate “ideal-gas” like conditions.

Ando, Dissertation (2008)

# $P(r)$ is extremely sensitive to inter-particle effects



Jacques & Trewhella (2010). *Protein Science*, 19(4), 642–657.

# Part 3: Experimental Requirements

# What do you need for SAXS?

---

## **A protein stock solution:**

- Must be well characterized by other methods for high purity, conformational homogeneity, no aggregates.
- Must have accurately measured concentration (typically 1-10 mg/ml required for good signal, but depends on MW).
- Typical sample volumes are only 10-40  $\mu\text{L}$ , but you will do many measurements, so bring as much as you can.

# What do you need for SAXS?

---

## **Buffer solution:**

- Almost all buffers useful for biochemistry are compatible.
- Should contain all ingredients needed for protein stability.
- Unless necessary, avoid adding excess of ingredients that can reduce contrast (e.g. >15% glycerol, excess metallocofactors).
- Most importantly, must be exactly matched to protein solution (use dialysate or elution buffer).

# What do you need for SAXS?

---

## Equipment requirements:

- X-ray source.
- Good PIN-diode beamstop to accurately measure transmission intensity.
- Good detector.
- Beamline and sample cell with low background scatter.
- Computers and software to process raw images and analyze processed data.

# What do you need for SAXS?

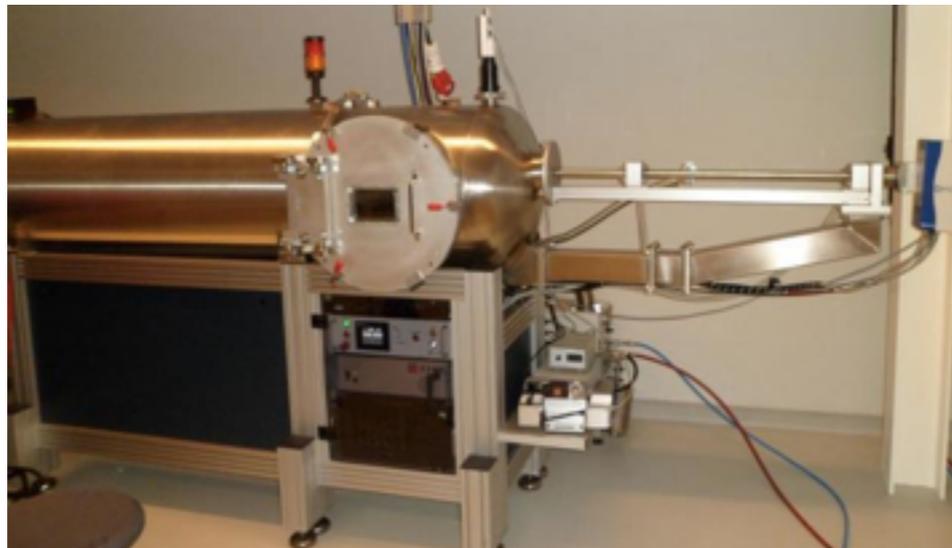
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## **Good experimental technique:**

- Document everything.
- Accurate pipetting.
- Be observant (watch detector images as they are generated, monitor flux, etc).
- Back up data, and don't discard raw images.

# Laboratory anode-based X-ray generators

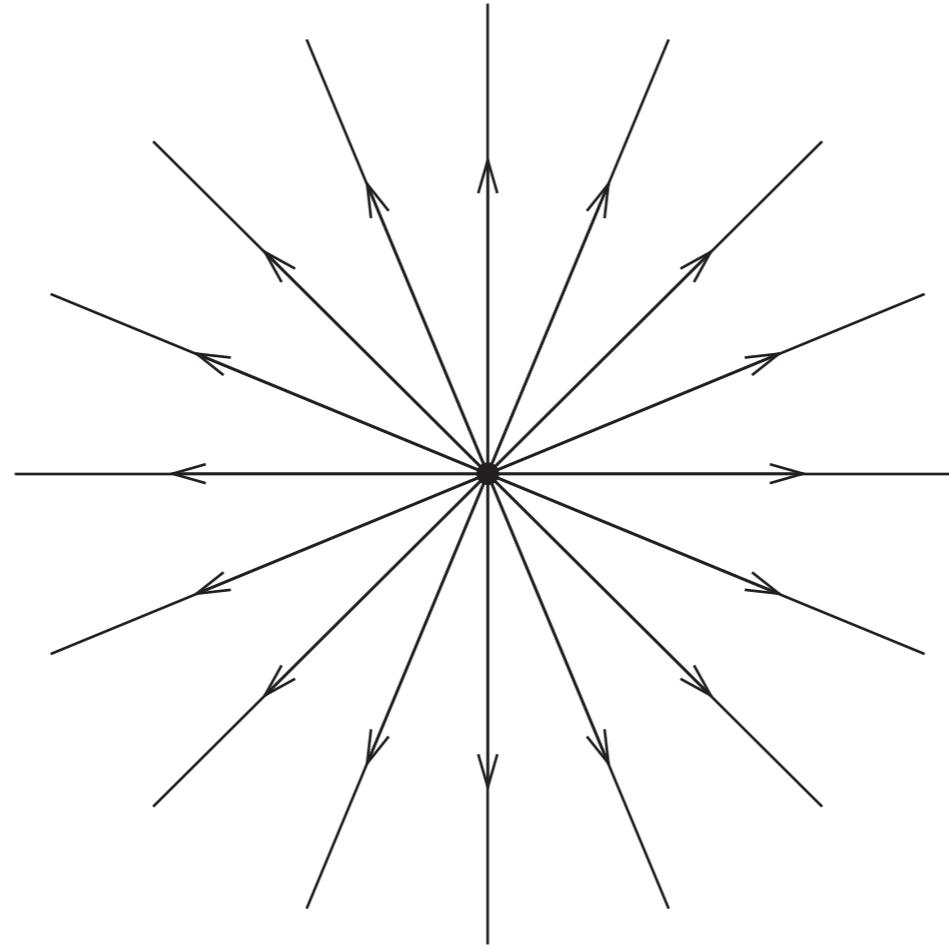
- Bruker NanoStar
- Anton Paar SAXSess
- Rigaku BioSAXS
- SAXSLab Ganesha



# Synchrotron radiation

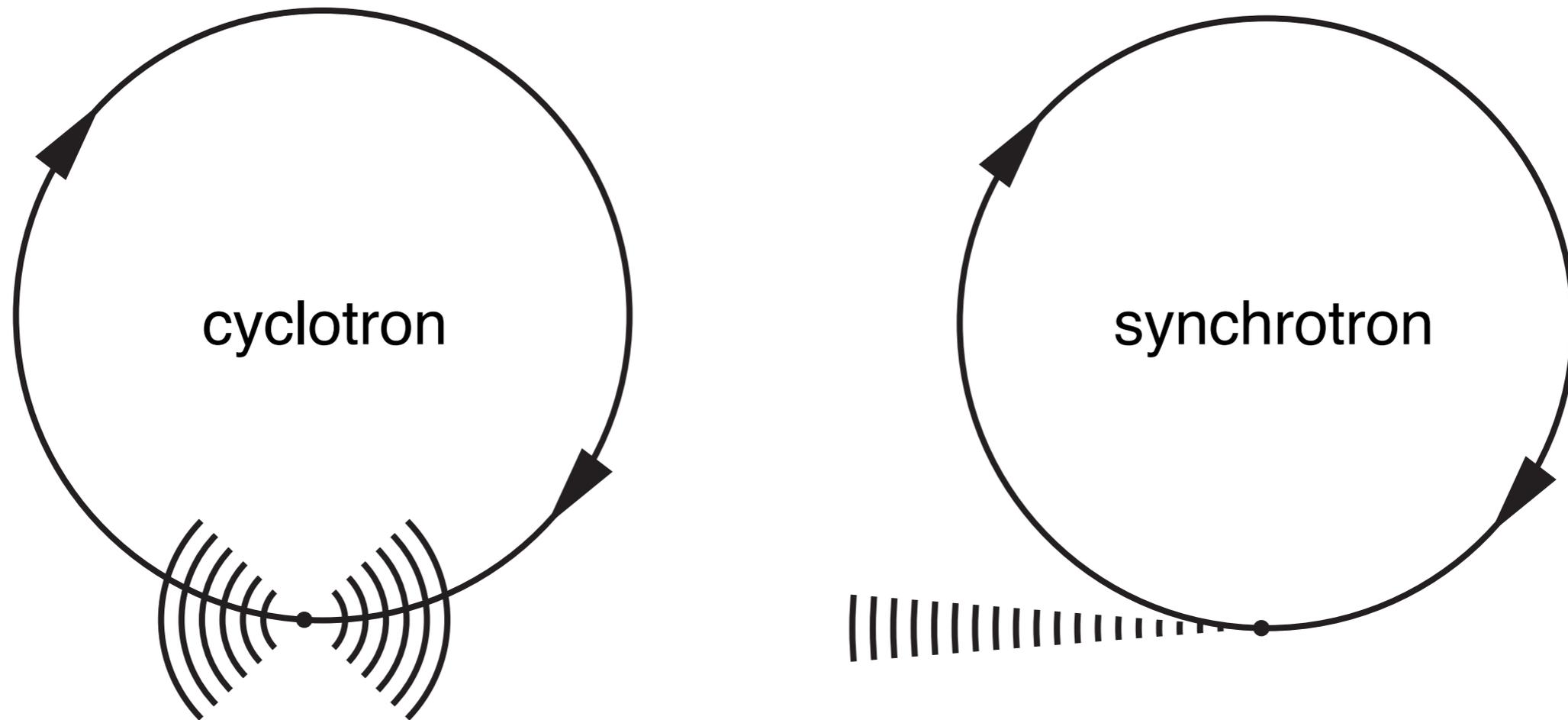
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Recipe for EM waves: charged particle supplies electric field



**Accelerate** the charged particle in order to create a wave front in the electric field and induce a magnetic field.

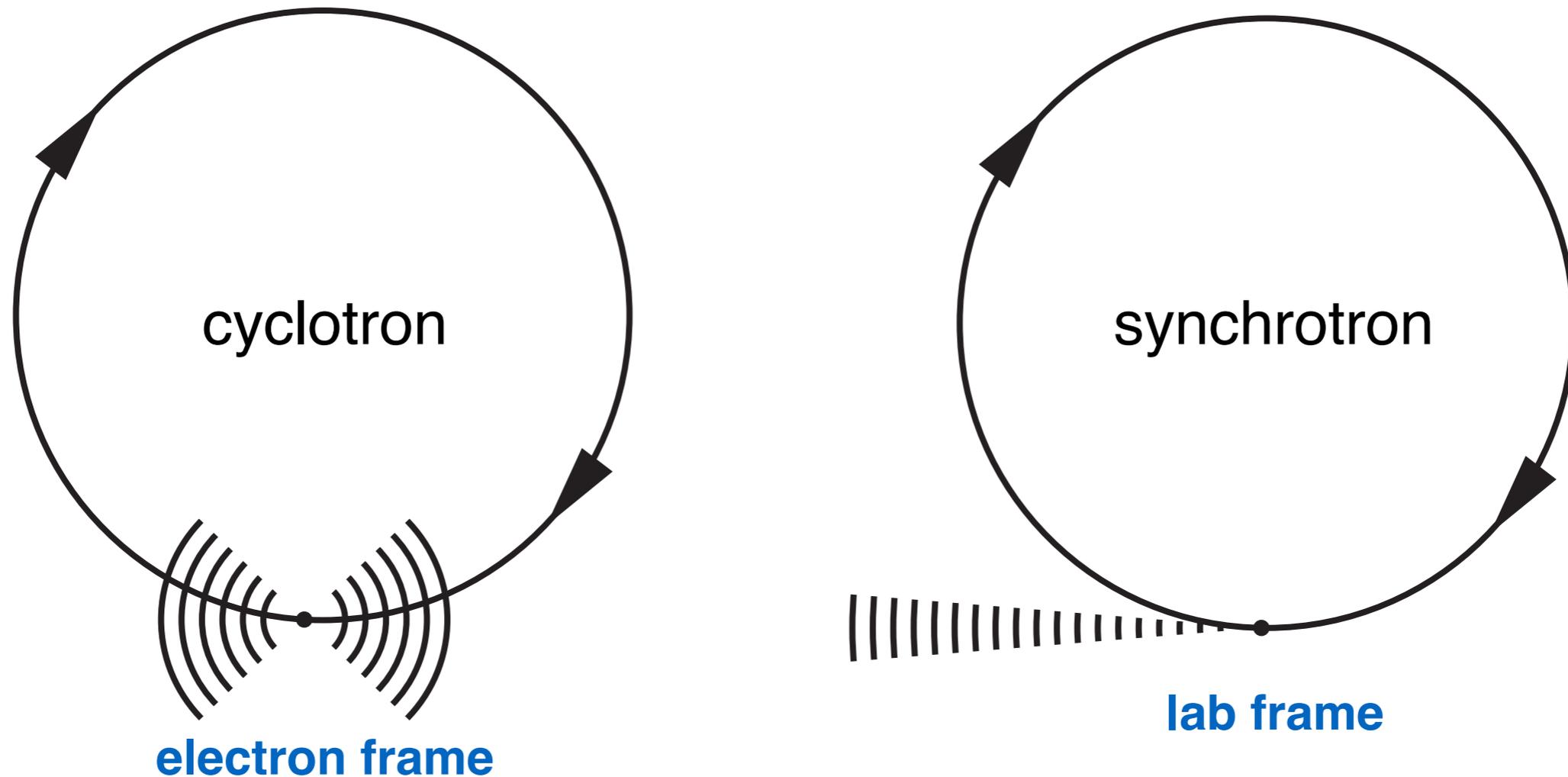
# Synchrotron radiation comes from relativistic charge



**Synchrotron:** 5 GeV electrons move at  $.99999999$  c.

- Relativistic beaming = highly intense, collimated beams.
- Spectrum of radiation = tunable wavelength source.

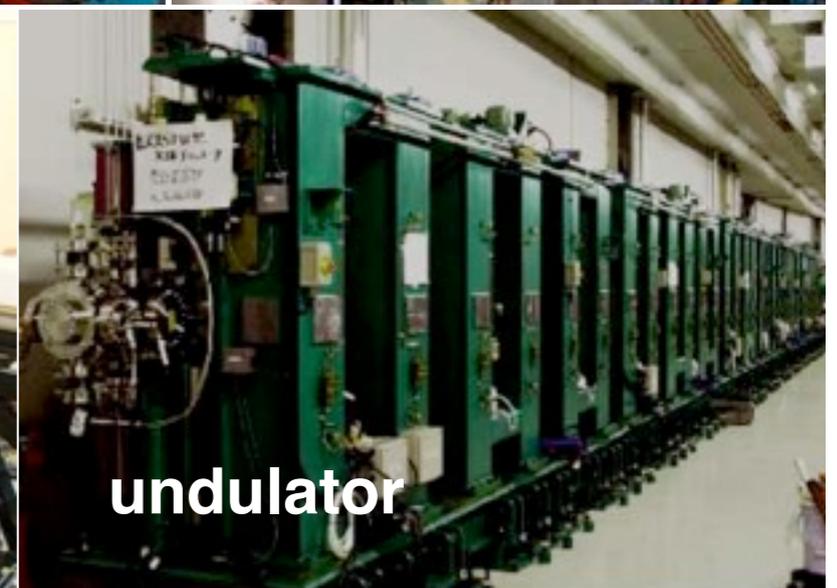
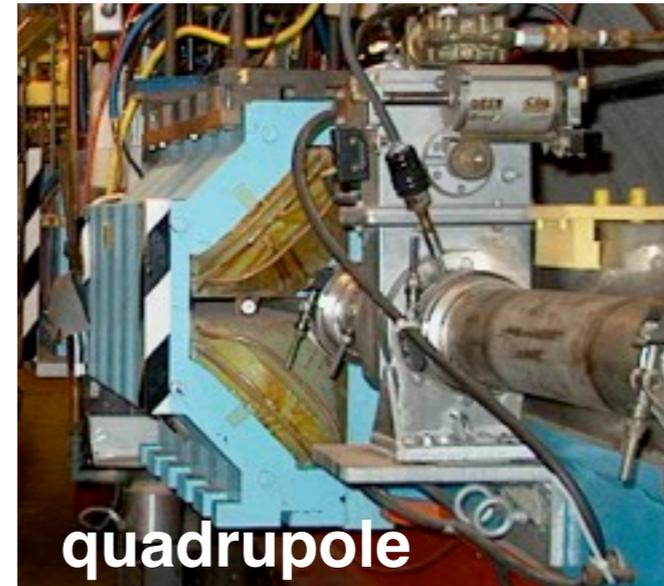
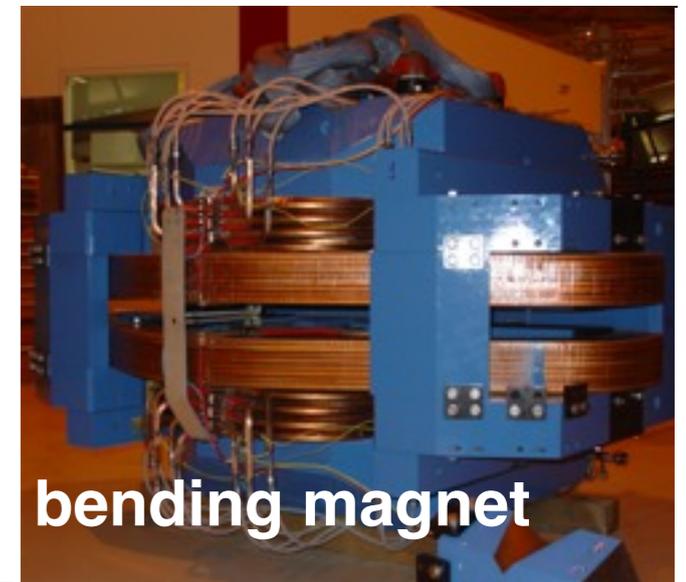
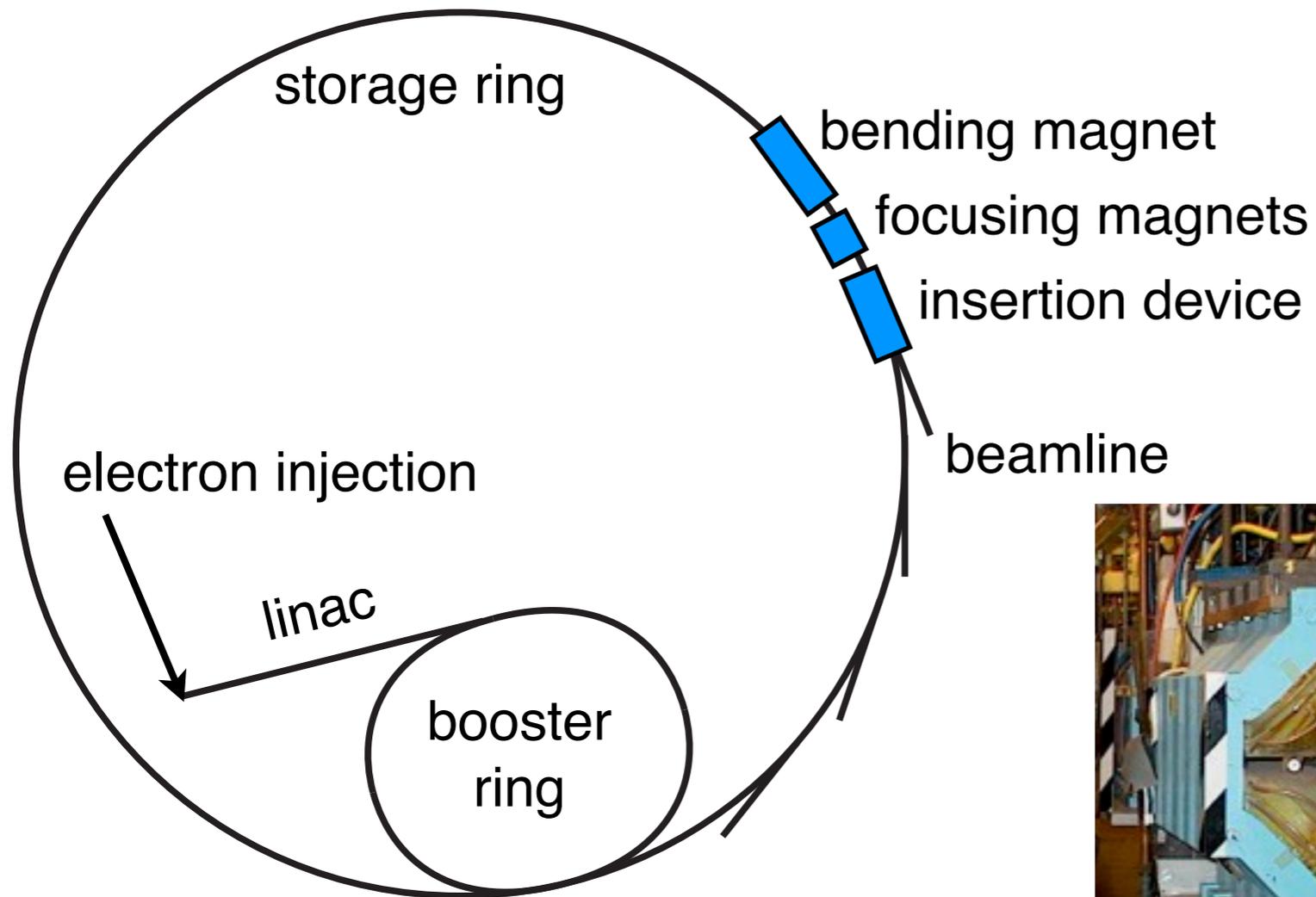
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# Elements of storage ring synchrotrons



# Access to synchrotron facilities

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Most operate on a rapid-access proposal mechanism.

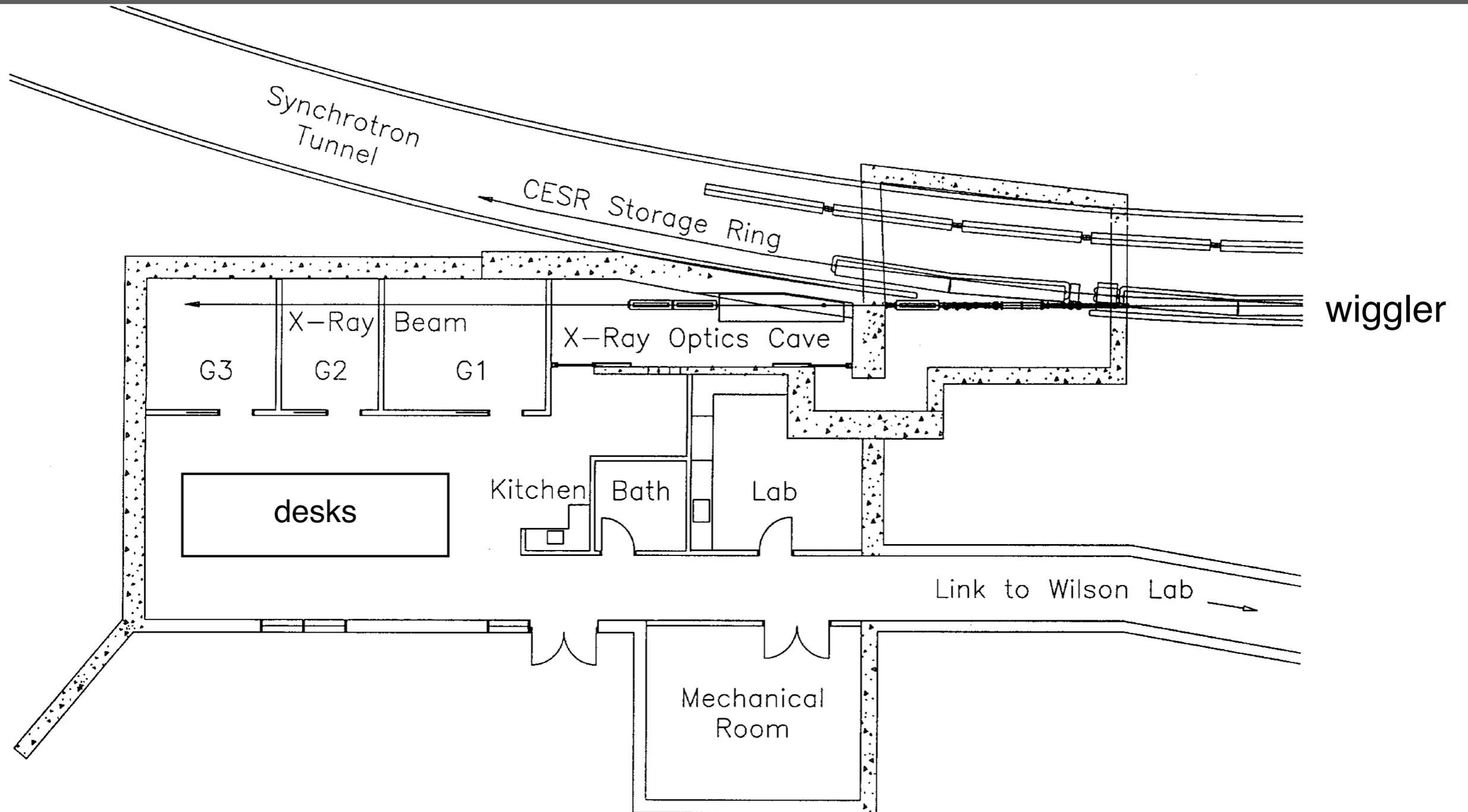
Many host SAXS workshops.

Get in touch with a beamline scientist.

Beam time:

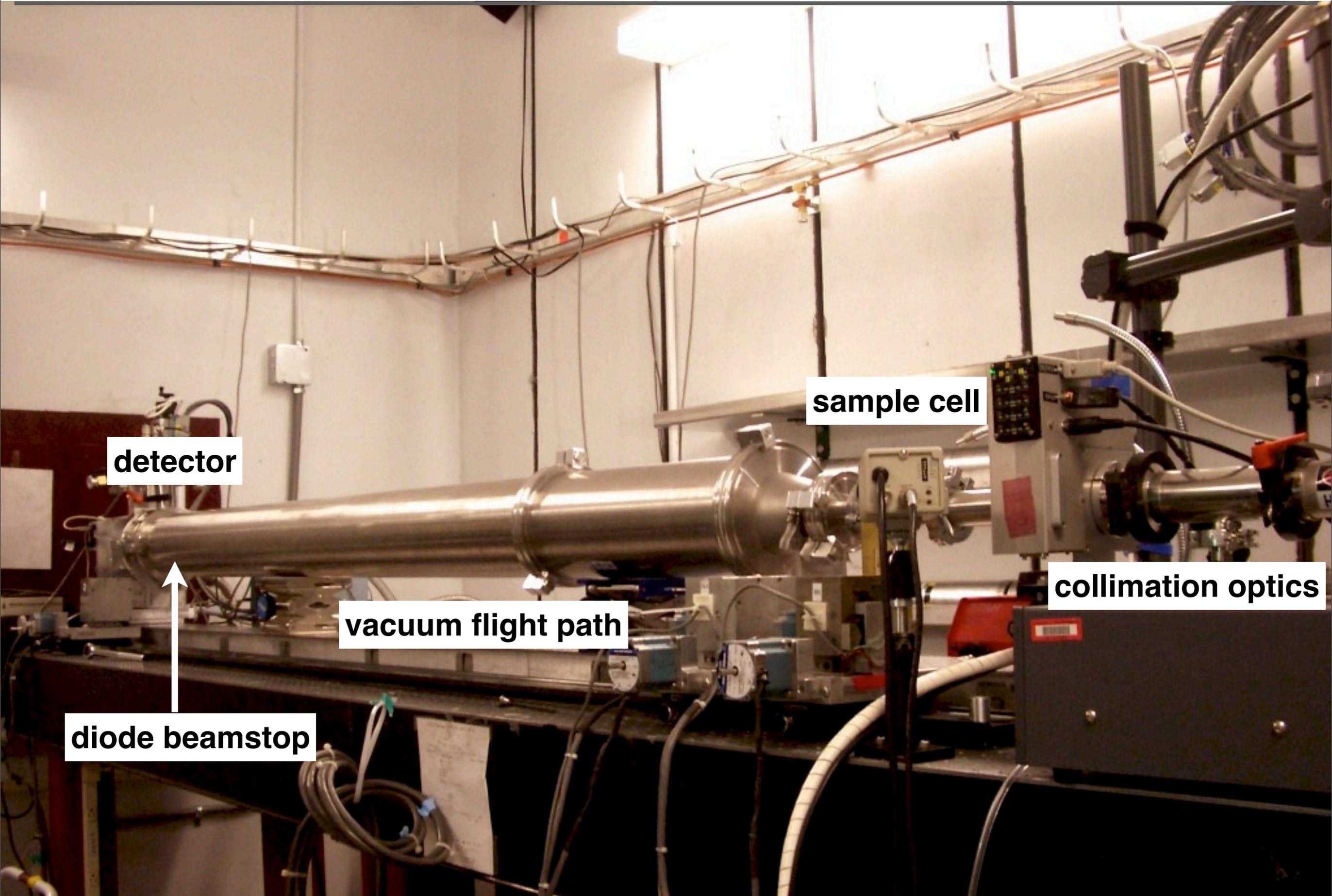
- 1-2 day (for routine SAXS, beamline is set up by staff)
- ~1 week or more (for non-conventional SAXS)

# SAXS beamline at synchrotron



Cornell High Energy Synchrotron Source (CHESS)

# Experimental hutch



**detector**

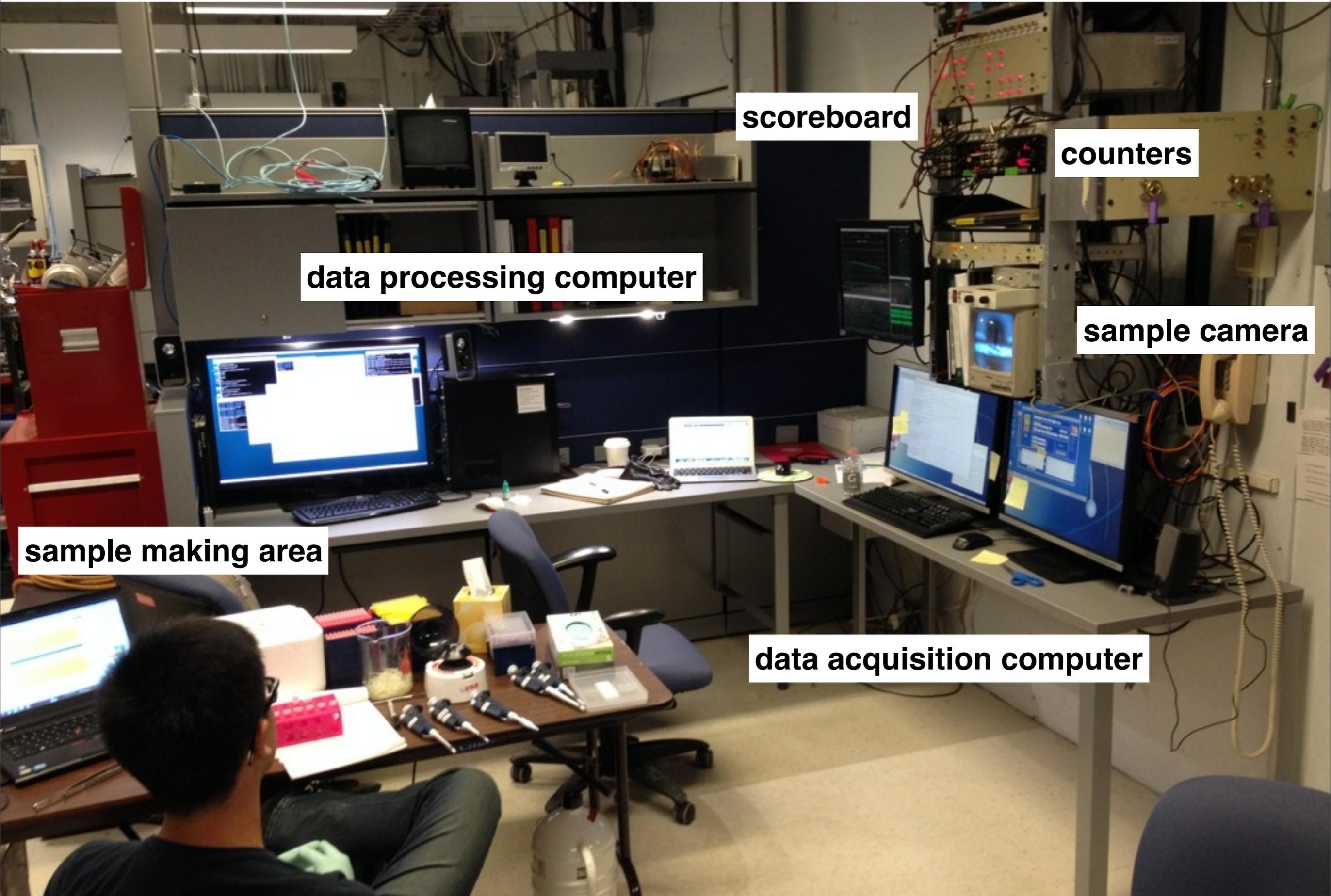
**sample cell**

**collimation optics**

**vacuum flight path**

**diode beamstop**

# Outside of experimental hutch



scoreboard

counters

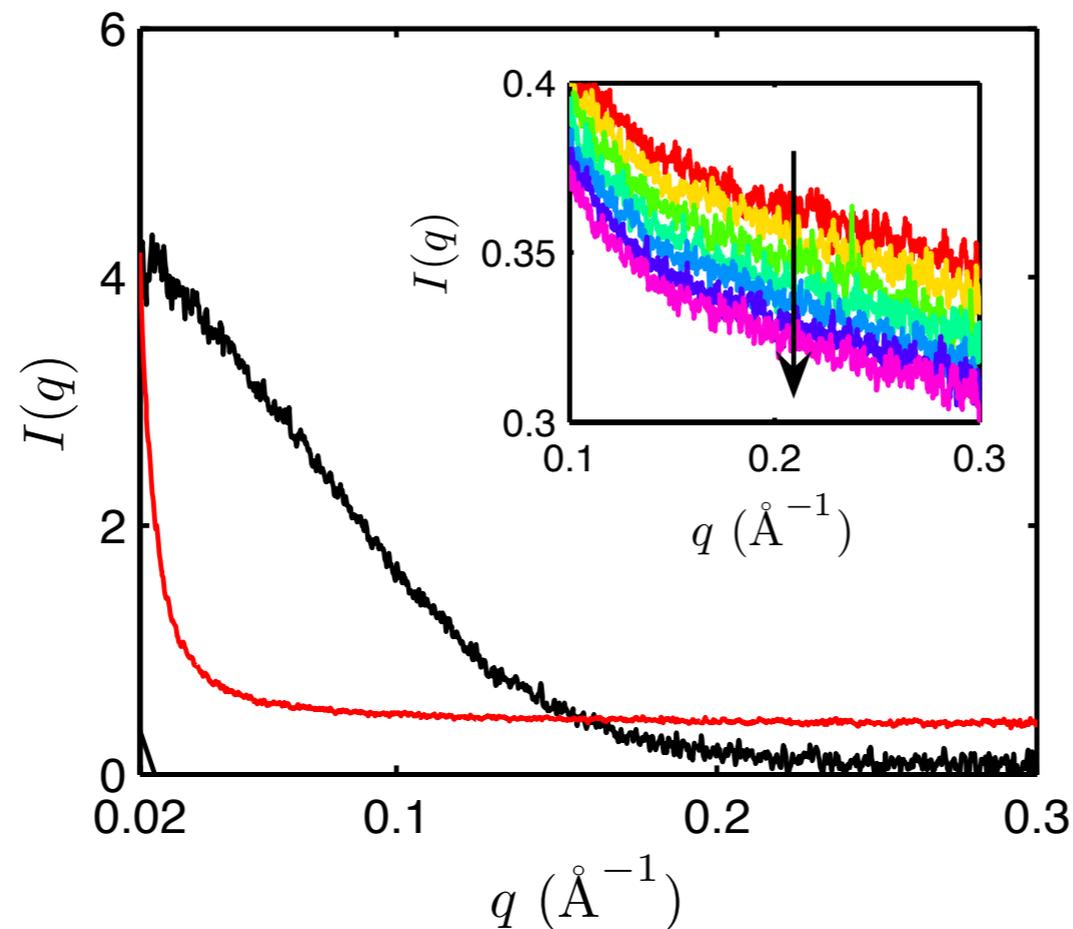
data processing computer

sample camera

sample making area

data acquisition computer

# Potential pitfall: Background subtraction



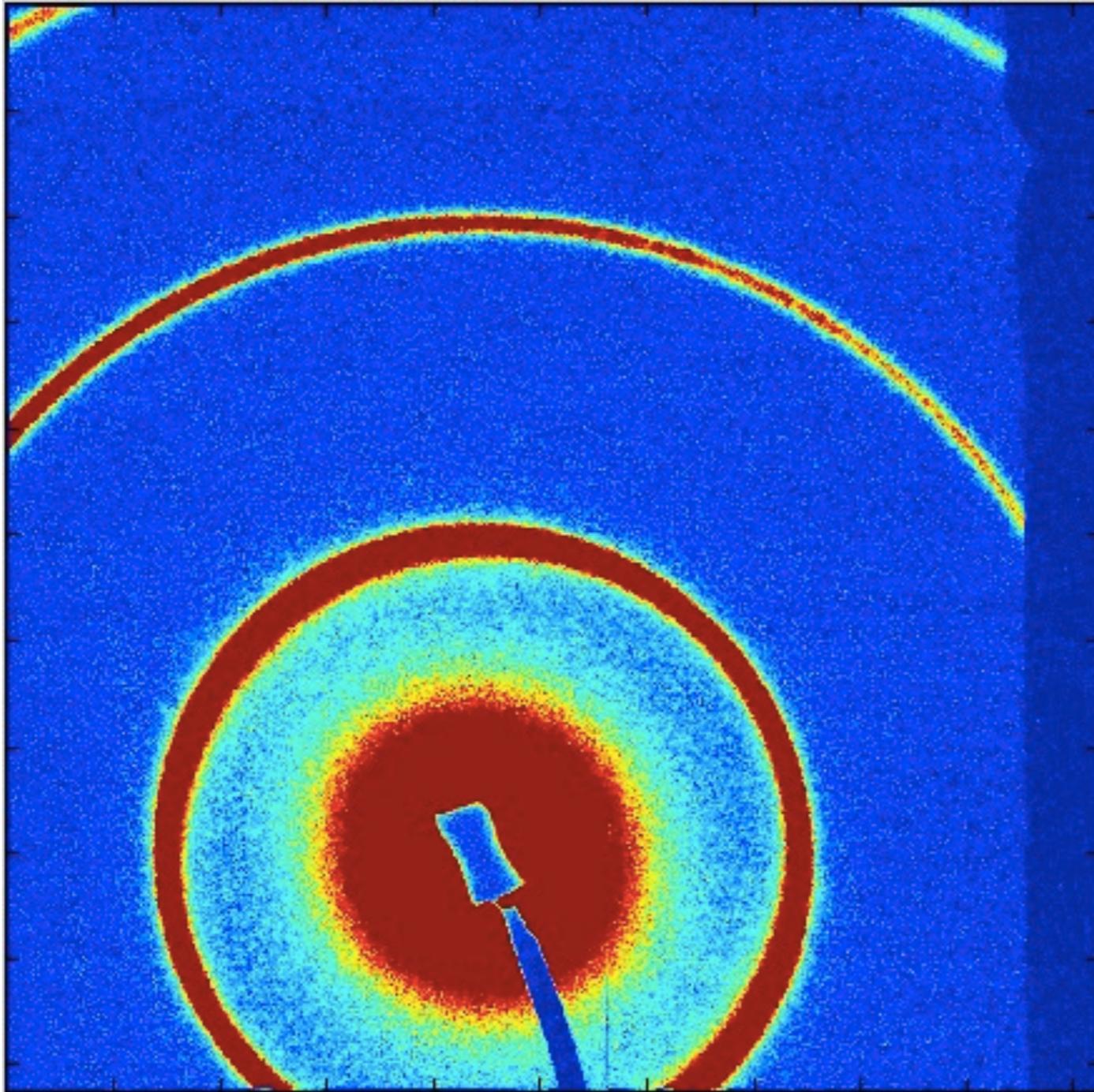
Protein signals at high  $q$  will be  $<0.5\%$  of background level.

## Need:

- exactly matched background solution
- very good detector and understanding of its limitations
- very good PIN-diode beamstop for accurate intensity normalization

Ando, *et al.* (2008) *J. Appl. Cryst.*, 41, 167–175.

# Detectors

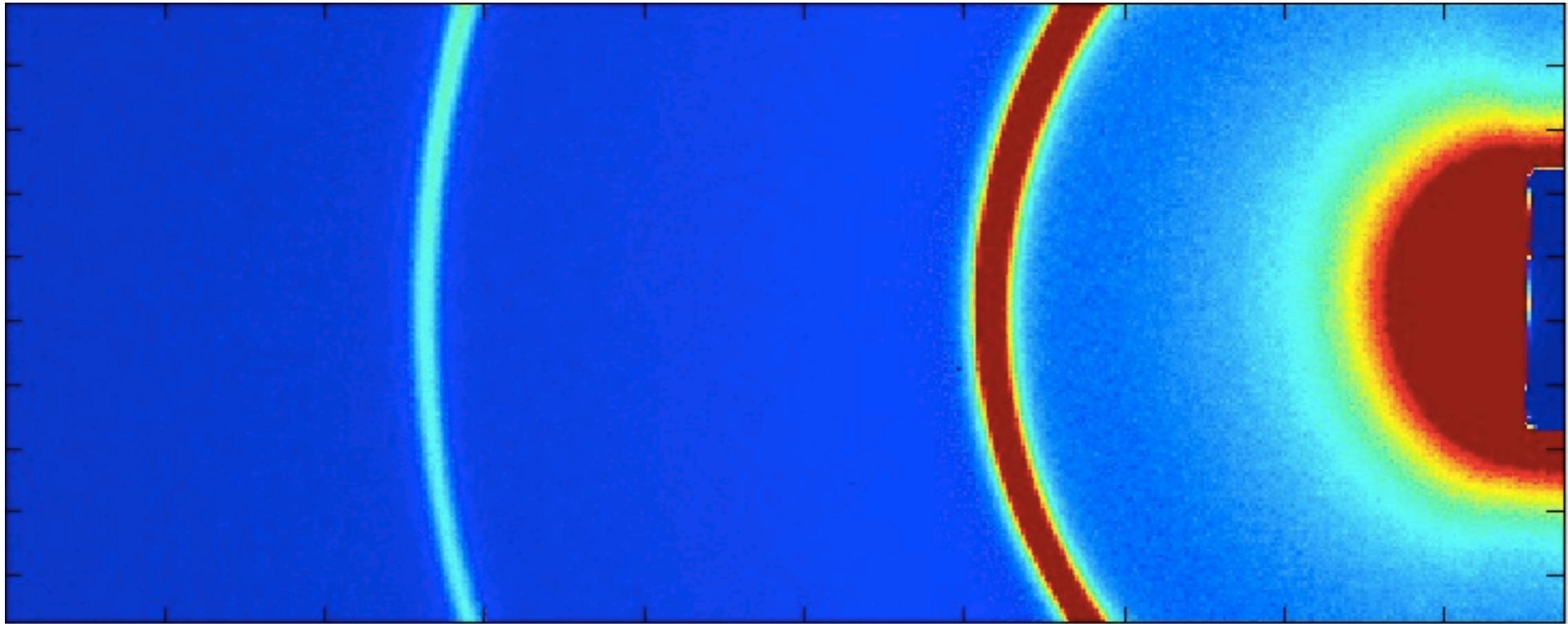


FLIcam detector @ CHESS

## CCD Detectors:

- Images should be corrected for intensity and distortion.
- Detector offset should be measured.
- Dark current images should be measured frequently.

**Note: If you can see a pattern in the detector image that is not from SAXS (e.g. faint shadows, stripes, sharp scattering, bright spots), then it will affect data.**



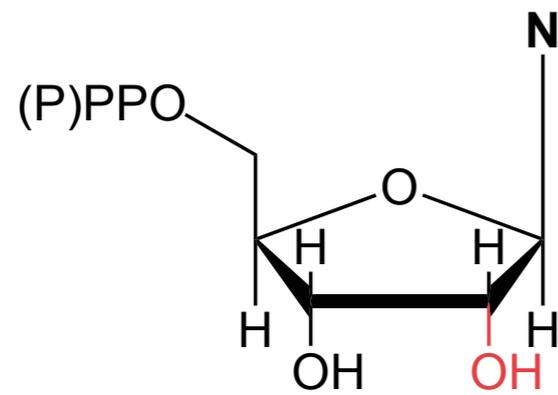
Pilatus 100K-S @ CHESS

## **Photon-counting Detectors:**

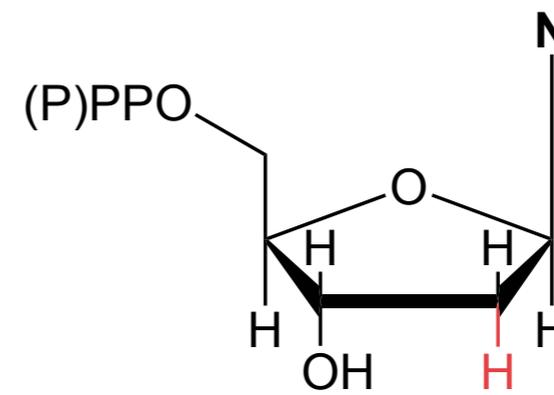
- No image corrections, low read-out noise.
- Fast read-out.

# Part 4: An Advanced Example

# Example: Ribonucleotide reductase (RNR)



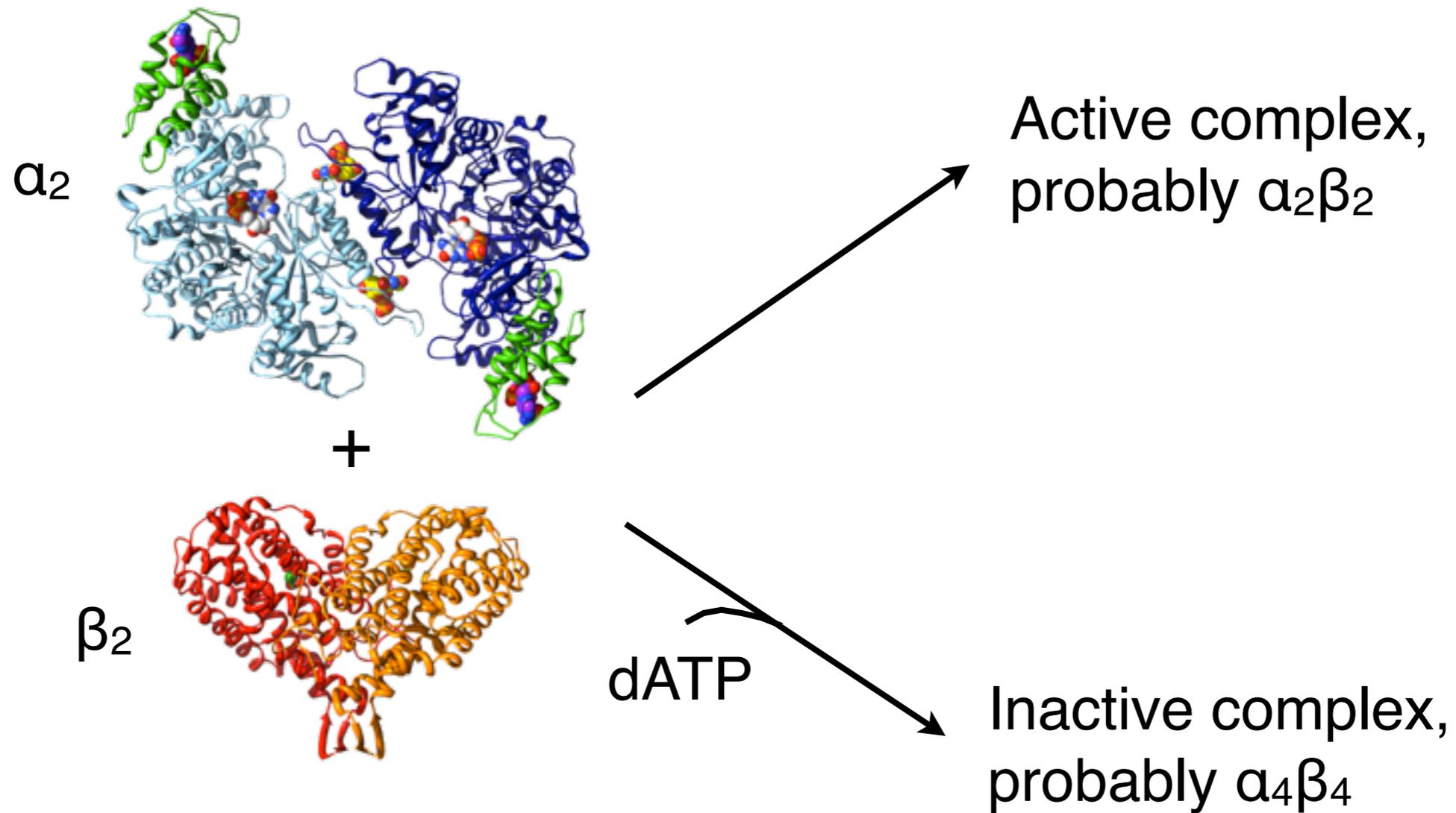
ribonucleotides  
(RNA precursors)



deoxyribonucleotides  
(DNA precursors)

Ando, et al. (2011). PNAS, 108(52), 21046–21051.

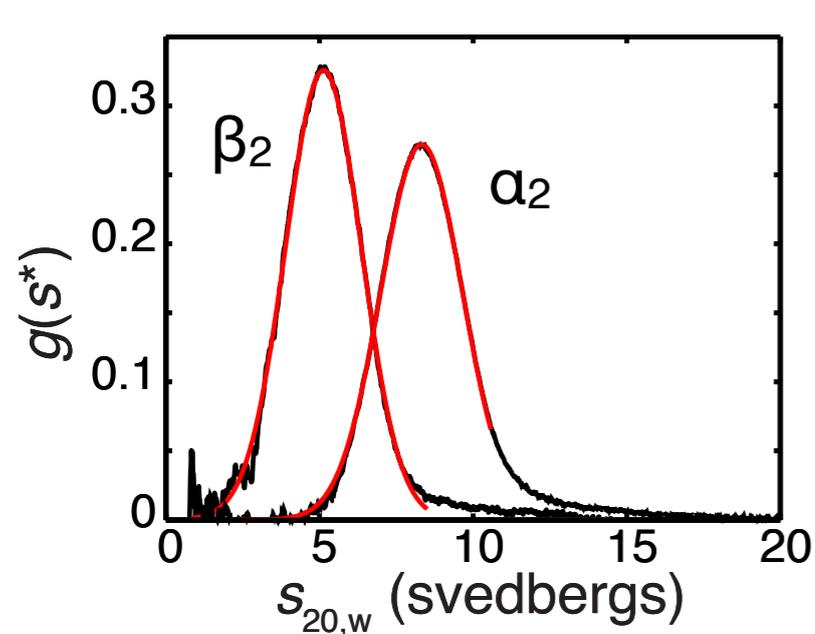
# Two subunits form at least two unknown complexes.



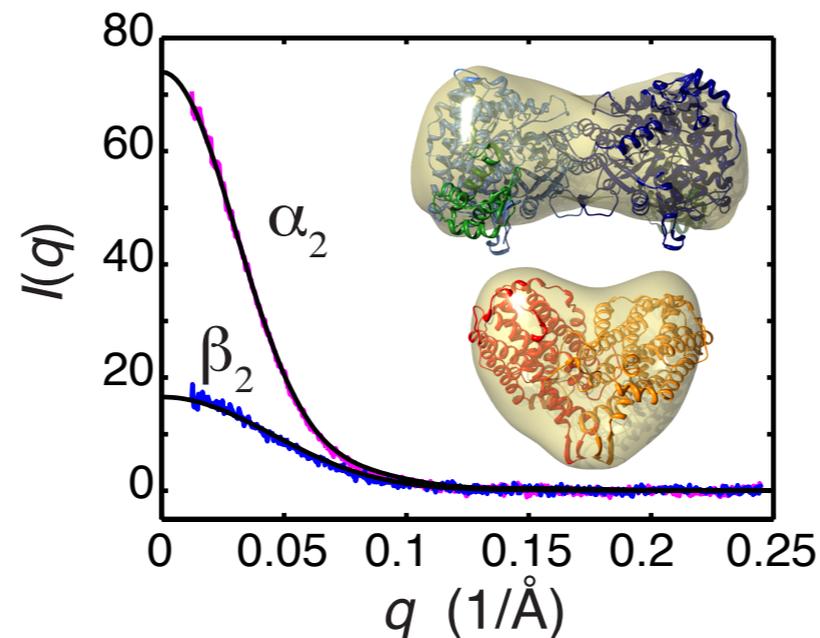
Catalytic mechanism requires long-range radical transfer between  $\beta_2$  and  $\alpha_2$ . Allosterically inhibited by dATP.

# 1. Check solution behavior of free subunits

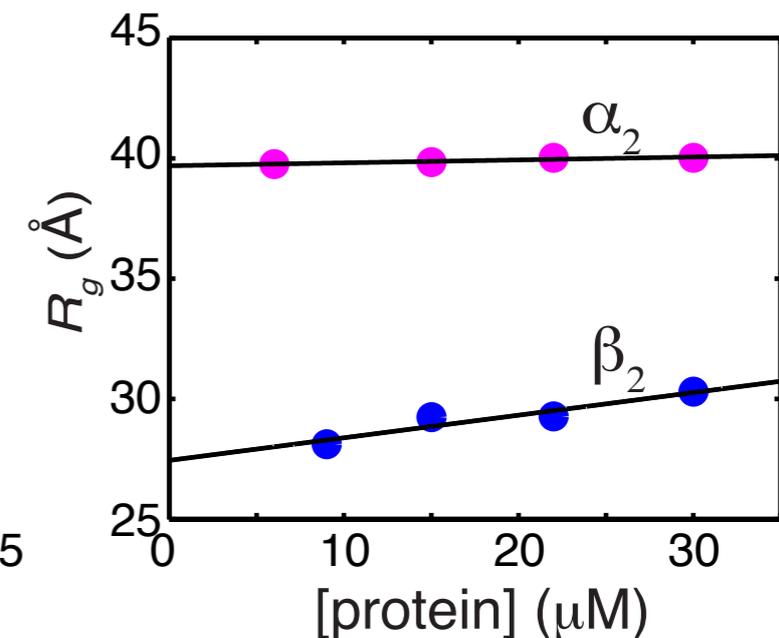
- Confirm purity, oligomerization state (MW), and concentration dependence.
- Compare with any known structures.



A. Purity, molecular weight, hydrodynamic quantities



B. Protein shape in real and reciprocal space



C. Concentration effects (aggregation, dissociation?)

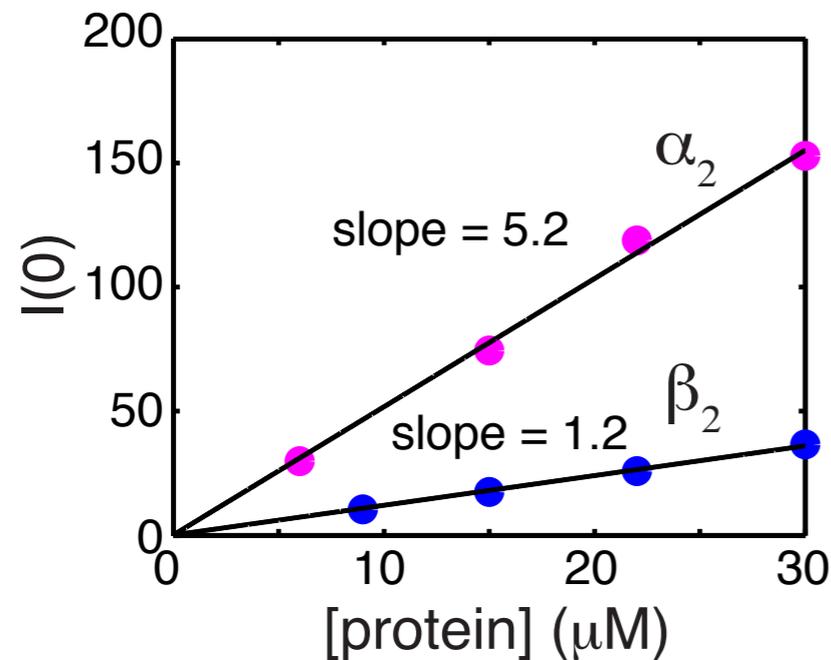
Compare with theoretical values calculated from crystal structures.

- AUC: hydropro
- SAXS: crysol

	$s_{20,w}$ (expt)	$s_{20,w}$ (theor)	$R_g$ (expt)	$R_g$ (theor)
$\beta_2$	5.2 S	5.8 S	$27.4 \pm 1.9 \text{ \AA}$	27.0 $\text{\AA}$
$\alpha_2$	8.4 S	8.4 S	$39.7 \pm 0.3 \text{ \AA}$	39.3 $\text{\AA}$

## 2. Check molar ratio of protein stock solutions

- Confirm protein concentrations with SAXS.



$$I(0) \propto c \times MW^2 \propto c' \times MW$$

$c$  = molar concentration

$c'$  = concentration in mg/ml

1. Make sure the proteins are in the same buffer.
2. Measure SAXS data at several different protein concentrations.
3. Determine  $I(0)$  by Guinier analysis or by  $P(r)$  analysis.
4. Make sure  $I(0)$  vs. concentration is linear. Then determine slope through origin.

Example using molar concentrations:

MW of  $\alpha_2$  = 172 kDa; MW of  $\beta_2$  = 87 kDa

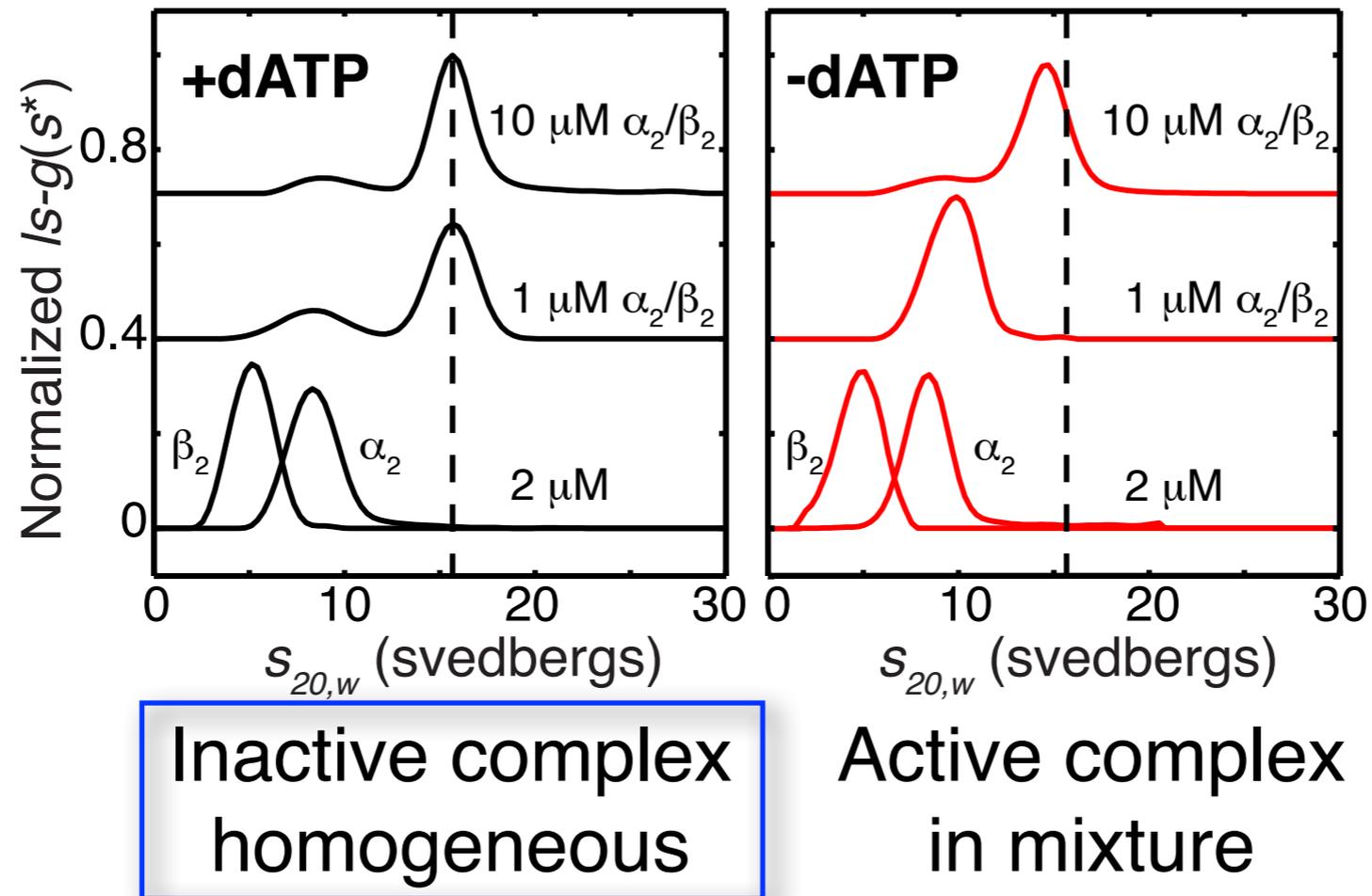
→ Actual MW ratio ~ 2.0

Slope of  $I(0)$  vs  $[\alpha_2]$  = 5.2; Slope of  $I(0)$  vs  $[\beta_2]$  = 1.2

→ Calculated MW ratio =  $(5.2/1.2)^{1/2} = 2.1$

### 3. Check stability of complex

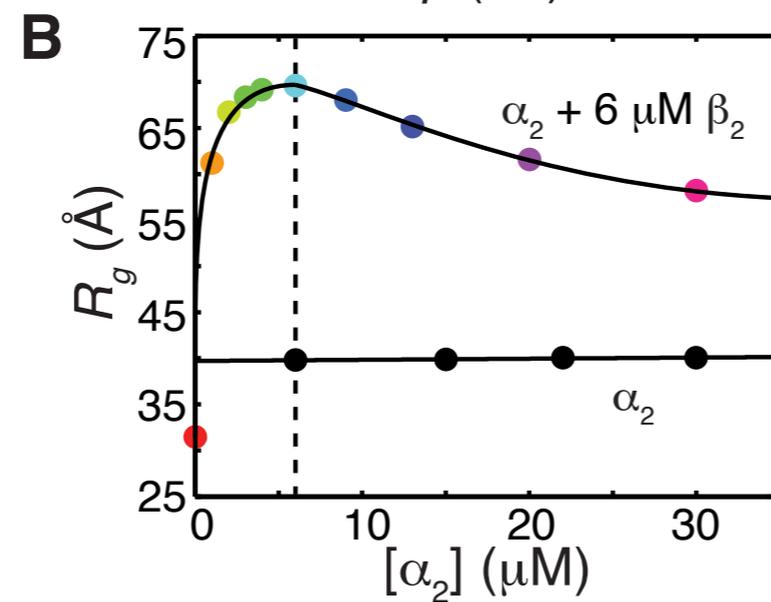
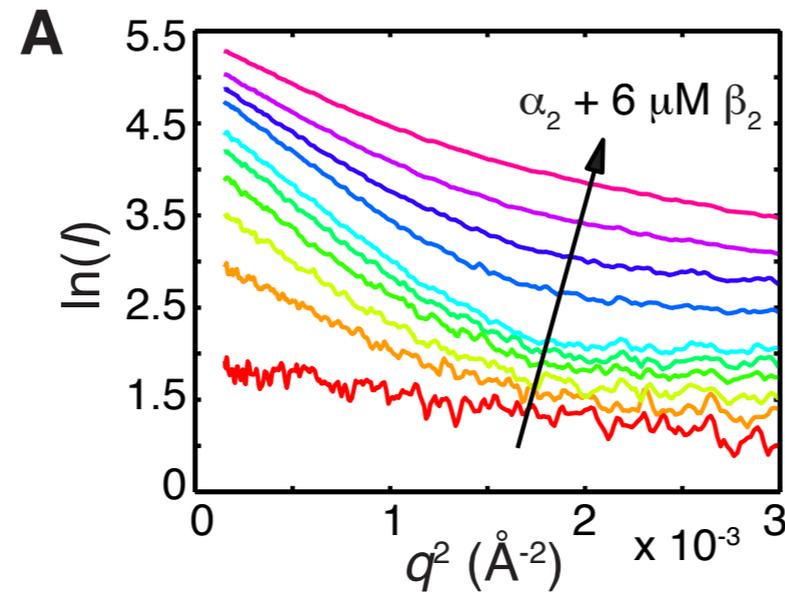
- One way: examine concentration dependence with sedimentation velocity analytical ultracentrifugation (AUC)



- Concentration-independent sedimentation is a sign that the species is non-interacting (slowly dissociating). MW and sedimentation coefficient are tractable by peak fitting.
- Concentration-dependent sedimentation is a sign that there are multiple species rapidly exchanging (mass action).
- This is also true of size exclusion chromatography, but controlling/varying concentration is much more difficult.

# 4. Determine subunit stoichiometry of complex

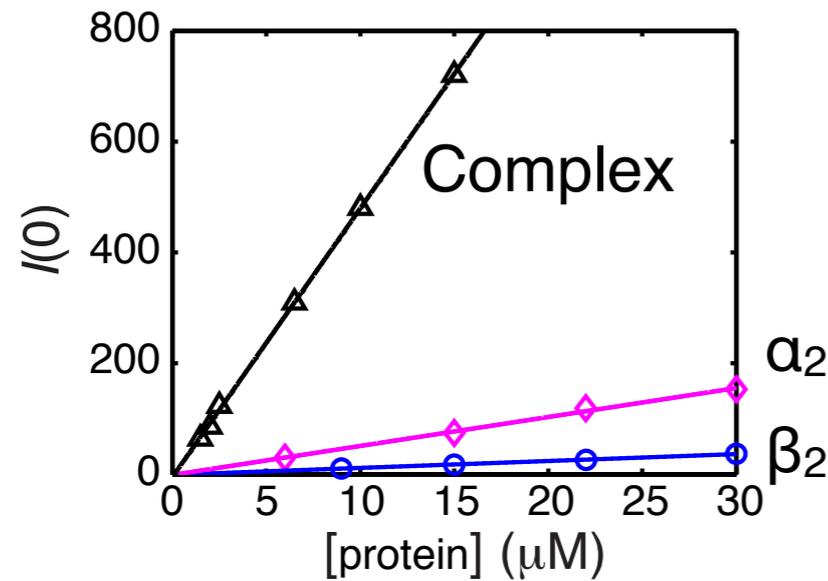
- Subunit titration with SAXS.



→ 1:1 complex

# 5. Oligomerization state of inactive complex

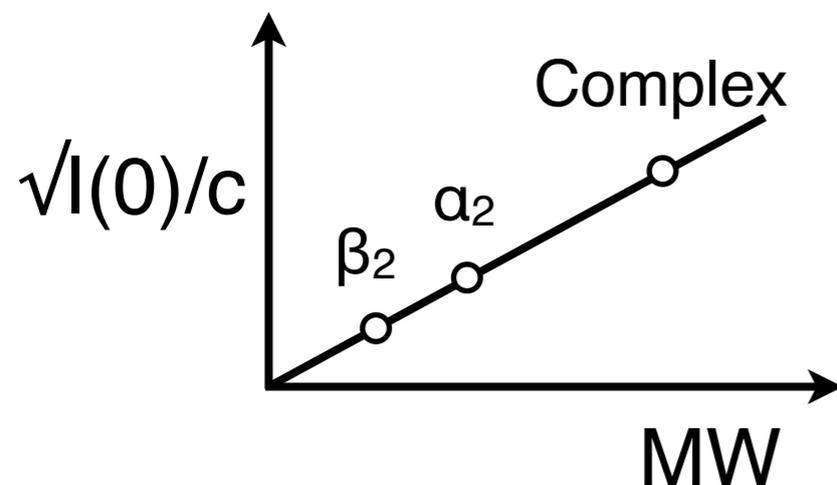
- Use free subunits as standards and check molecular weight of complex.
- Measure  $I(0)$  of complex at multiple concentrations.



$$I(0) \propto c \times MW^2 \propto c' \times MW$$

$c$  = molar concentration

$c'$  = concentration in mg/ml



Gives predicted MW of 512 kDa.

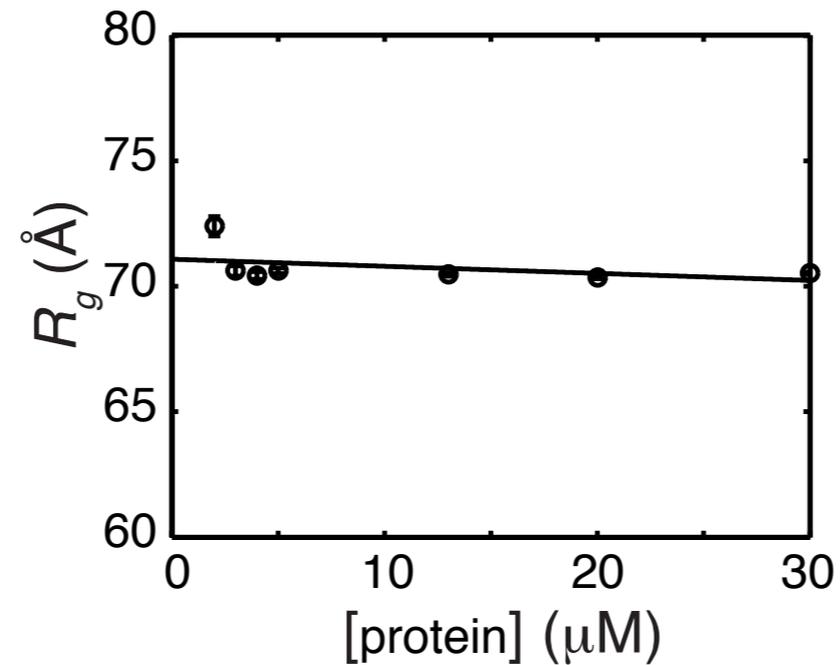
We know complex has 1:1 subunit stoichiometry.

→ Consistent with  $\alpha_4\beta_4$  (actual MW = 517 kDa).

## 6. Concentration dependence of inactive complex

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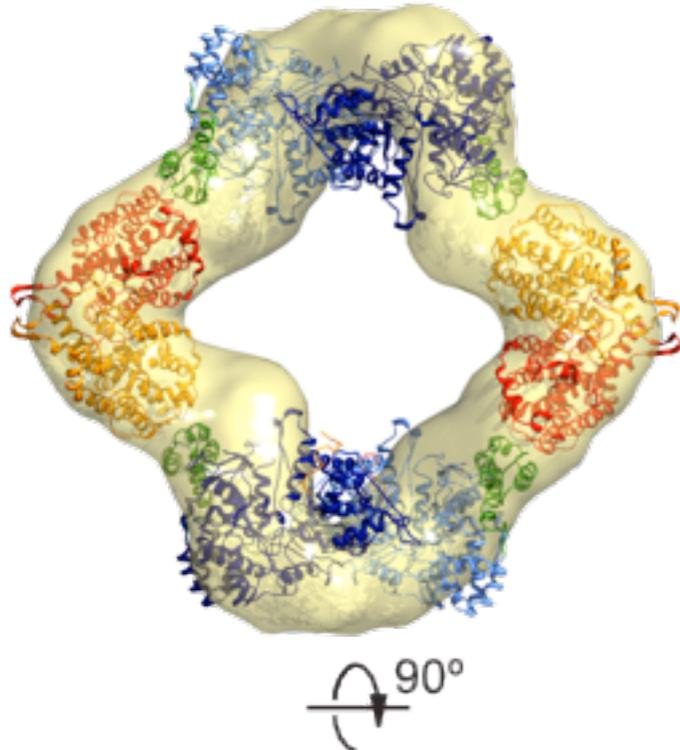
- Measure  $R_g$  of complex at multiple concentrations.



→ Complex is stable over a wide range of protein concentrations.

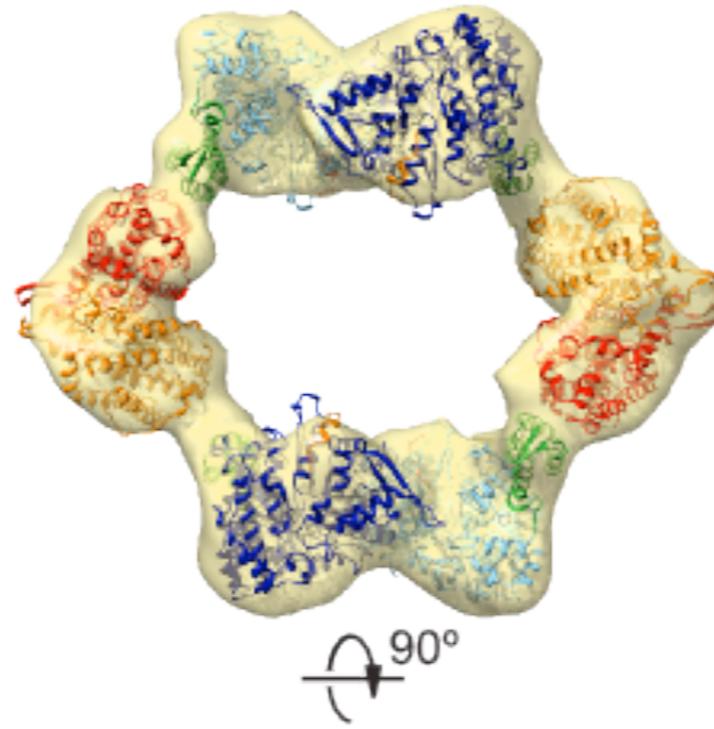
# 7. Structure determination of inactive complex

SAXS (2  $\mu\text{M}$  RNR)



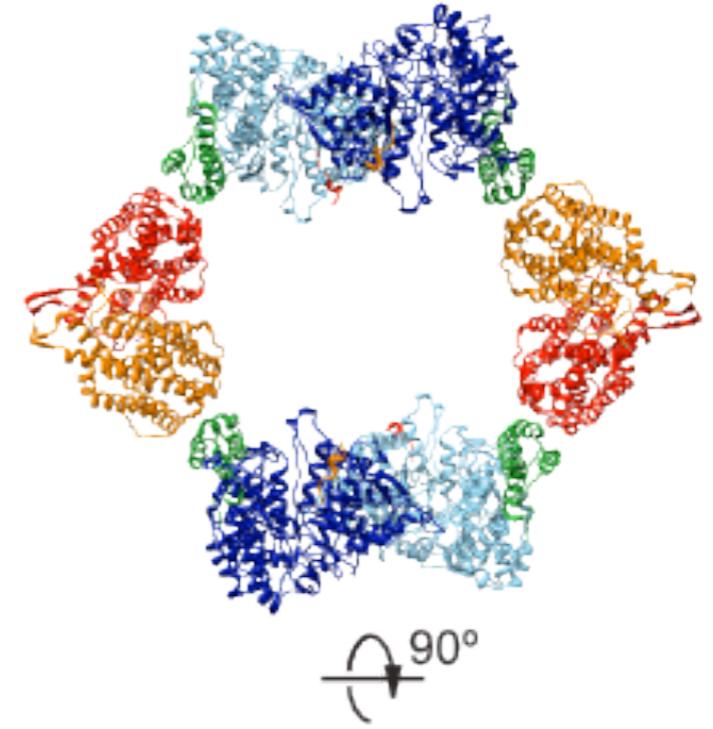
57 Å

EM (0.15  $\mu\text{M}$  RNR)



23 Å

XTAL (25  $\mu\text{M}$  RNR)

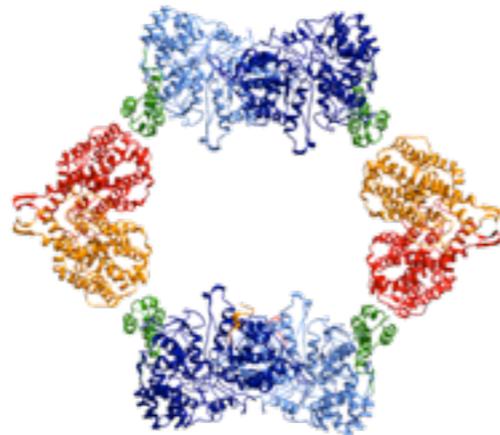
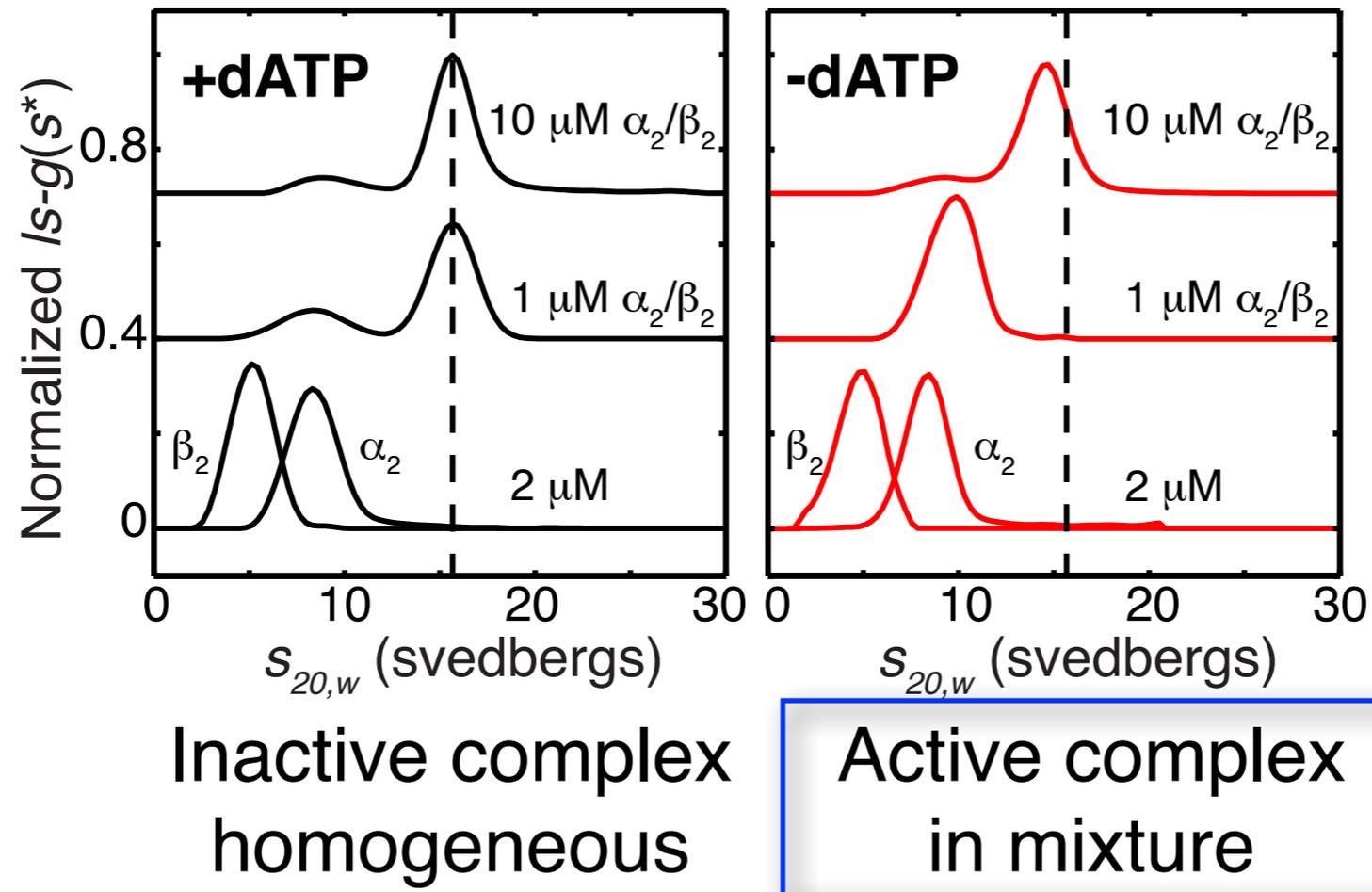


5.65 Å

Inactive complex is an  $\alpha_4\beta_4$  ring.

# 8. Extracting the active complex from mixture

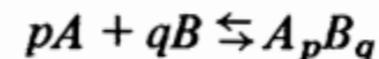
- Active complex is embedded in equilibrium mixture, likely exchanging with inactive complex.



# 8. Extracting the active complex from mixture

## C. Equilibrium Data

In special cases it is possible also to obtain equilibrium data from SAXS. The accuracy of such results obtained from more or less polydisperse solutions is, of course, rather limited; but it can be high enough, as shown by Österberg *et al.* (1975), to decide which type of complex is formed between two macromolecules  $A$  and  $B$ , when the equilibrium of the system is described by the general reaction



which has the associated equilibrium constant  $\beta_{pq}$ . Österberg *et al.* (1975) showed by analysis of SAXS data that the predominant complex ( $AB$ ) formed in solution between lysine:  $tRNA$  ligase ( $A$ ) from yeast and  $tRNA$  ( $B$ ) consists of two ligase molecules and one molecule of  $tRNA$  ( $A_2B$ -type).

For a sufficiently diluted system the scattered intensity  $I$  at any angle can be written as the sum of intensities scattered from all types of particles in the solution, i.e.

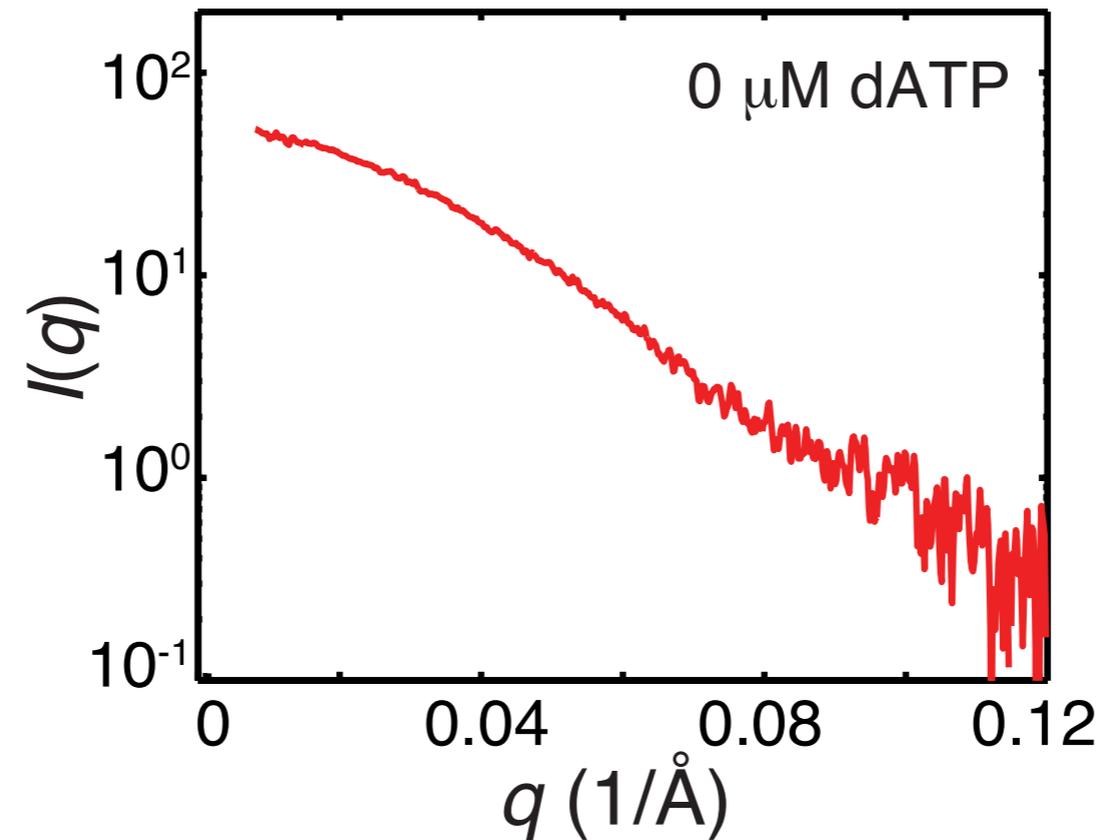
$$I = I_A + I_B + \sum \sum I_{pq}$$

where  $I_A$ ,  $I_B$  and  $I_{pq}$  are the scattered intensities from the molecules  $A$ ,  $B$  and the complexes  $A_pB_q$ , respectively. The intensity contributed by each type of particle at a particular small angle depends on its concentration, its molecular weight and the excess electrons  $\Delta z$  (see Chapter 4). The scattered intensity increases with an increase in any of these parameters.

Glatter & Kratky Ch. 8, p. 286 (1982)

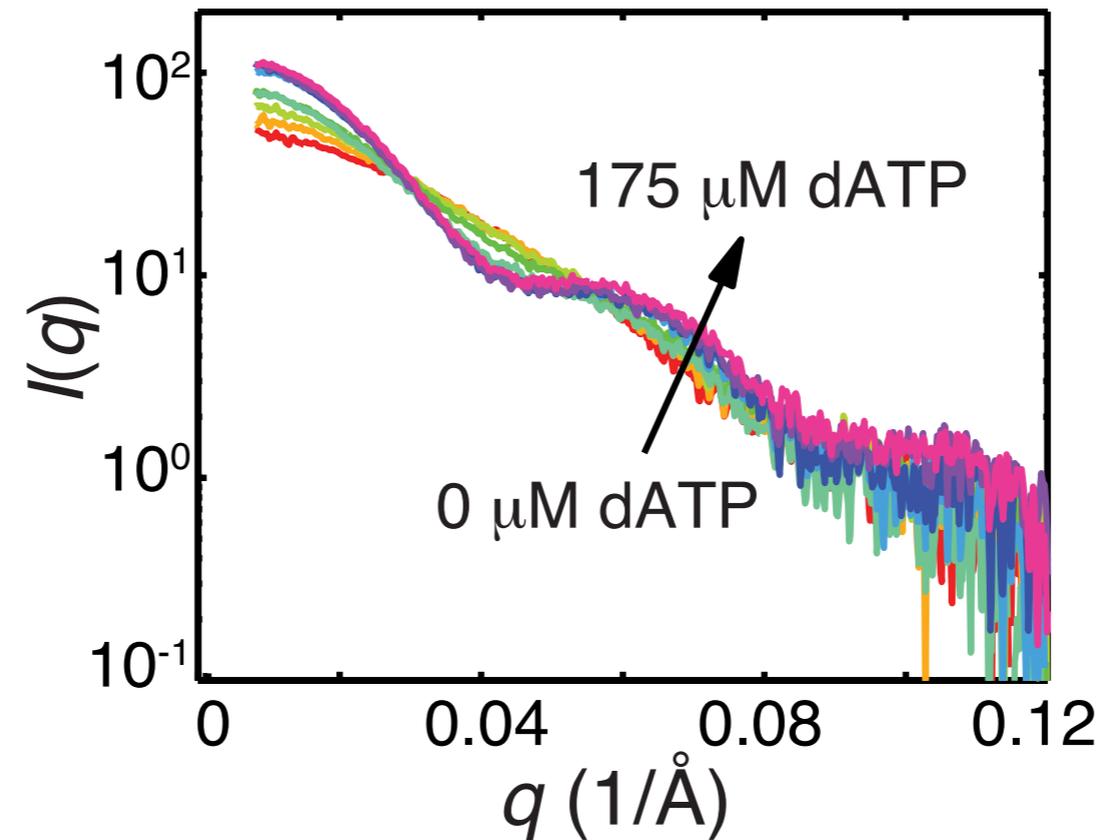
# 9. Shifting the equilibrium from active to inactive

- Titration of dATP (negative activity effector) with SAXS.



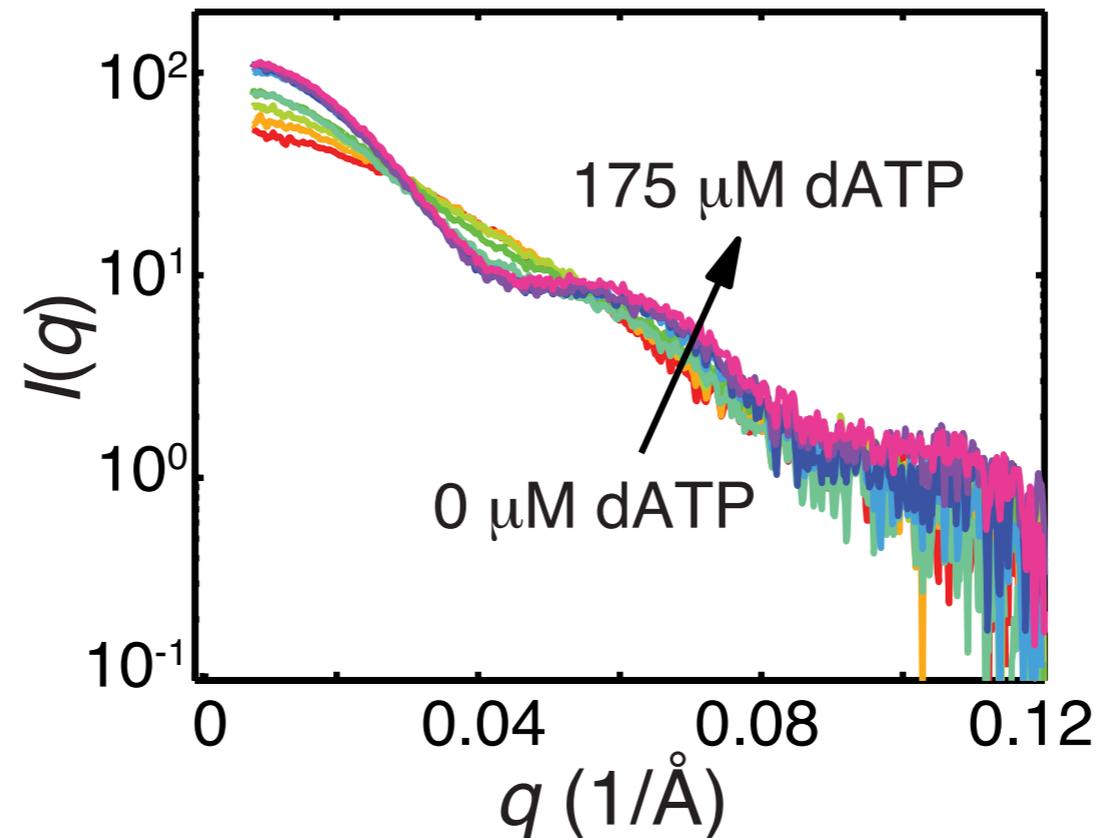
# 9. Shifting the equilibrium from active to inactive

- Titration of dATP (negative activity effector) with SAXS.



# 9. Shifting the equilibrium from active to inactive

- Titration of dATP (negative activity effector) with SAXS.



How many states in this transition?

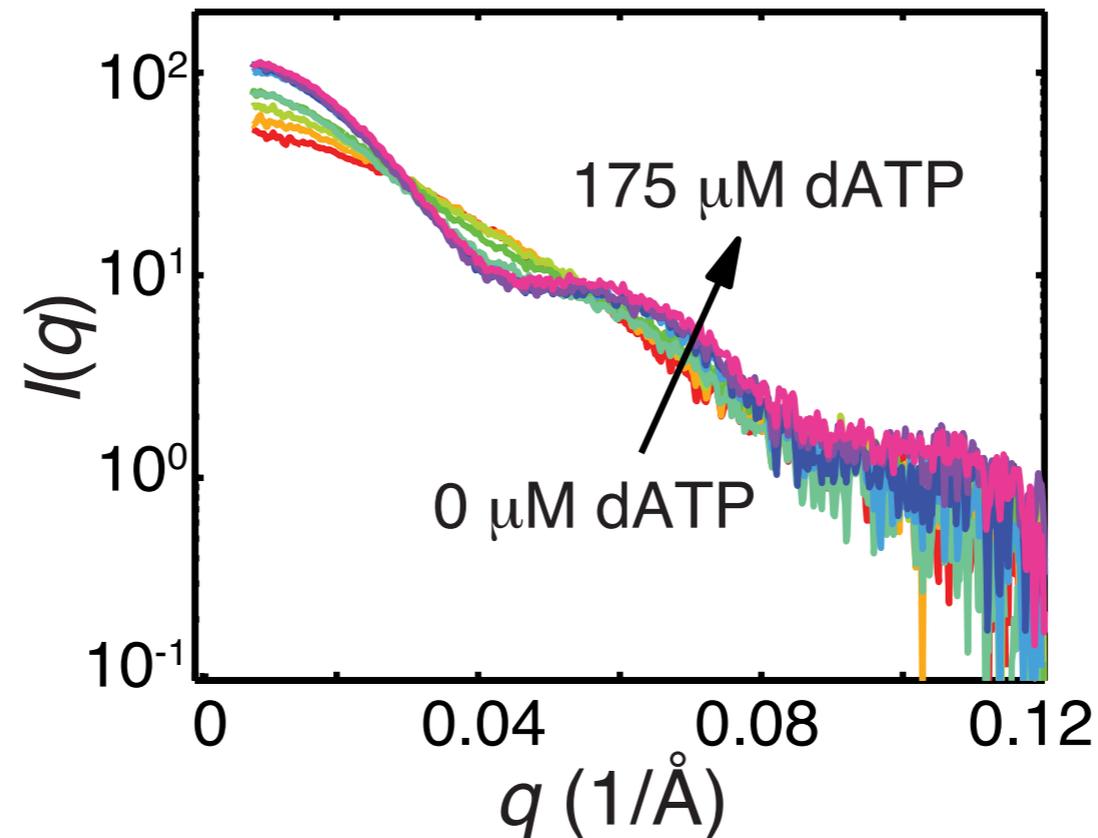
A) observation of iso-scattering points

B) singular value decomposition  $\rightarrow$  2 “eigenstates”

**dATP drives a two-state transition.**

# 9. Shifting the equilibrium from active to inactive

- Titration of dATP (negative activity effector) with SAXS.



A) Increase in scattering signal:  $I(0) \sim c \times MW^2$       223 → 512 kDa

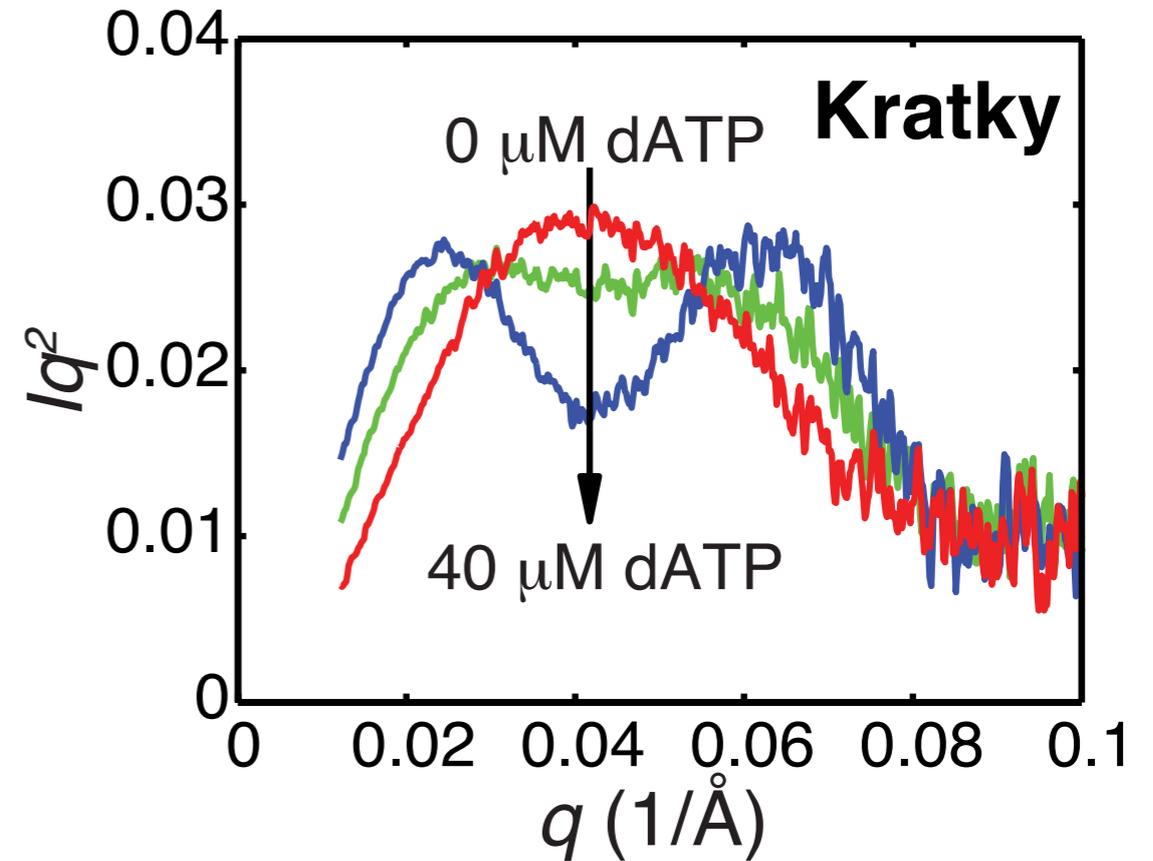
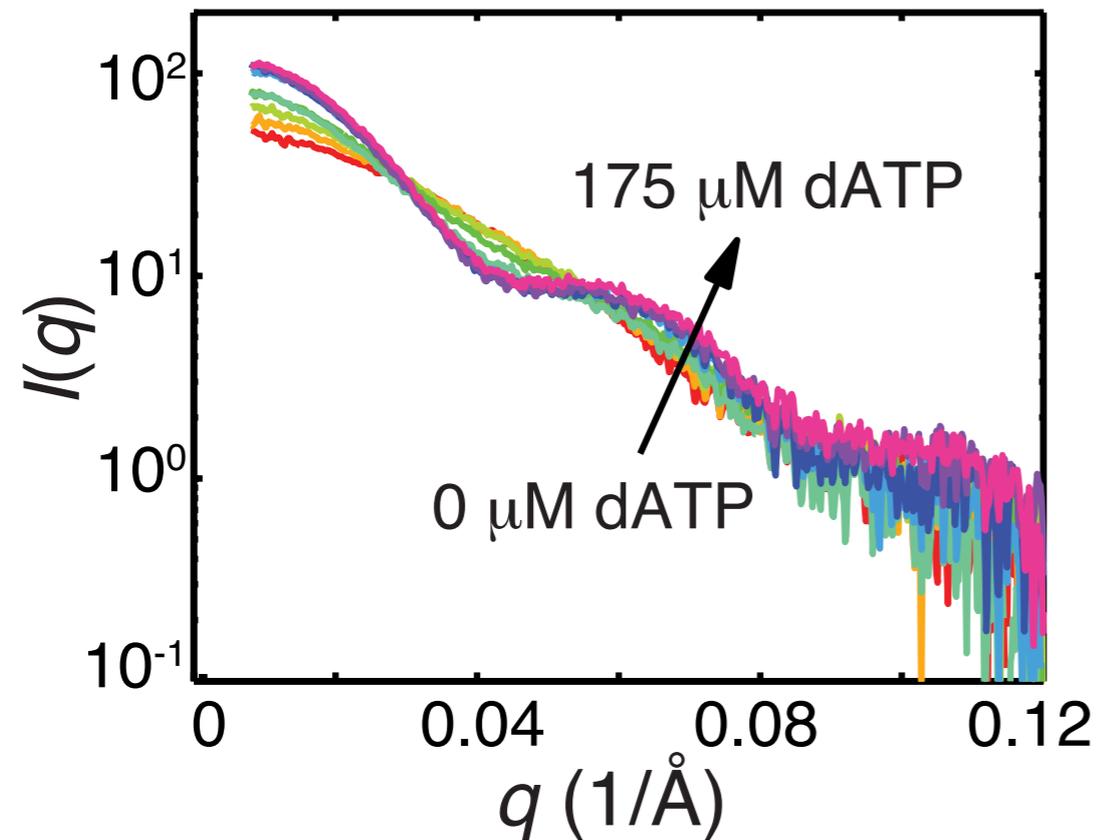
B) Porod invariant method\* (area under the curve)      269 → 549 kDa

$\alpha_2\beta_2$  (259 kDa) →  $\alpha_4\beta_4$  (517 kDa)

\*<http://www.ifsc.usp.br/~saxs/saxsmow.html>

# 9. Shifting the equilibrium from active to inactive

- Titration of dATP (negative activity effector) with SAXS.

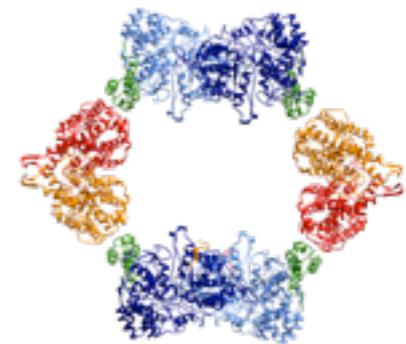


Kratky analysis ( $Iq^2$  vs.  $q$ )

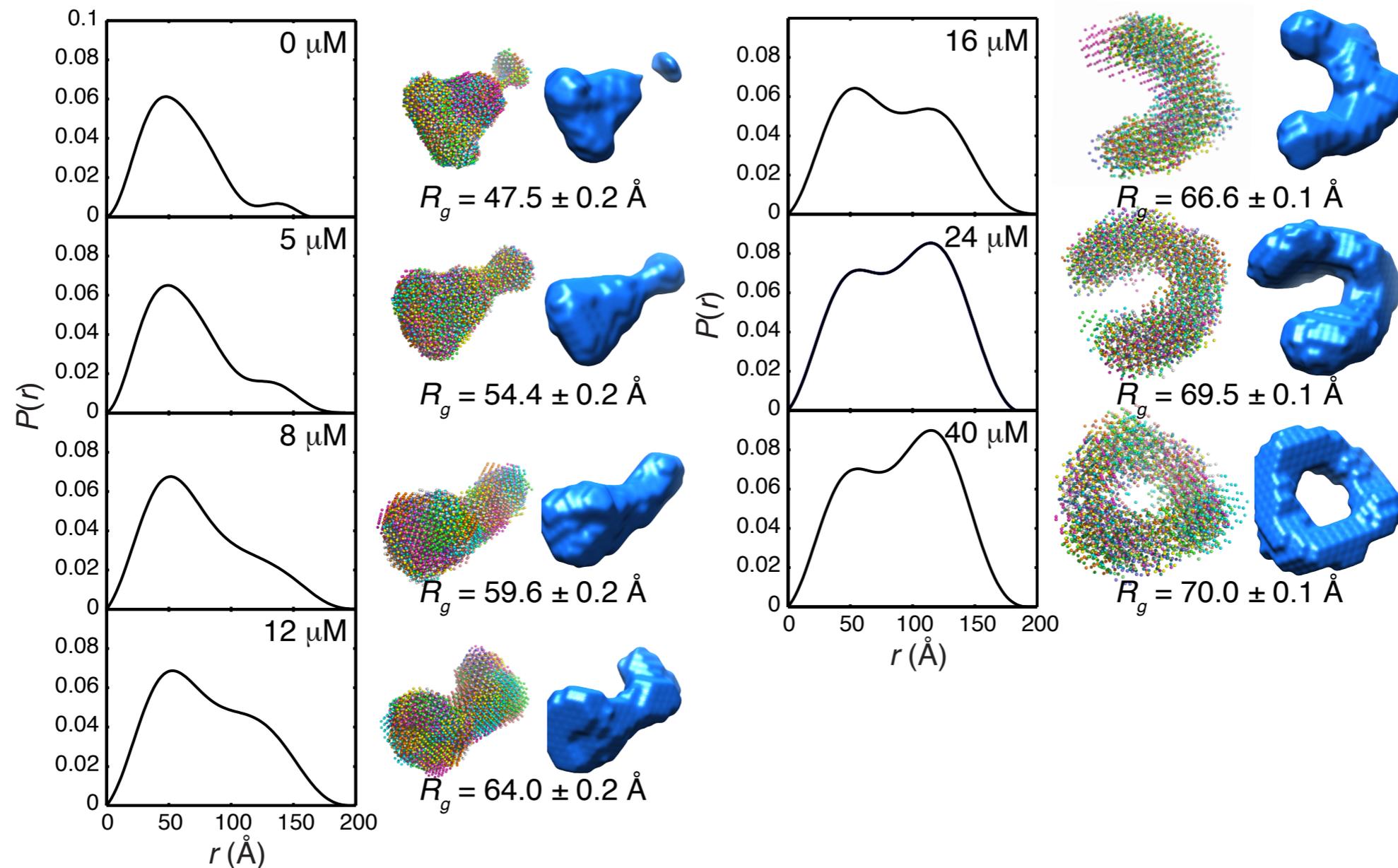
single peak = globular, compact

multiple peaks = non-globular arrangement of multi-domains

globular  $\alpha_2\beta_2 \rightarrow$  non-globular  $\alpha_4\beta_4$

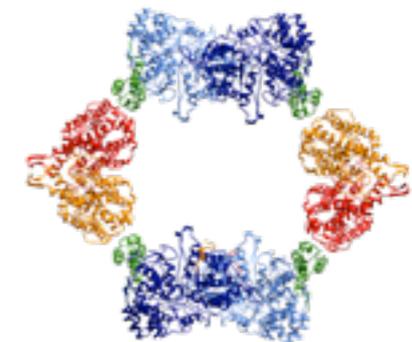


# 10. Visualizing the two-state transition



- Shapes are “average” of two species (active and inactive complexes)

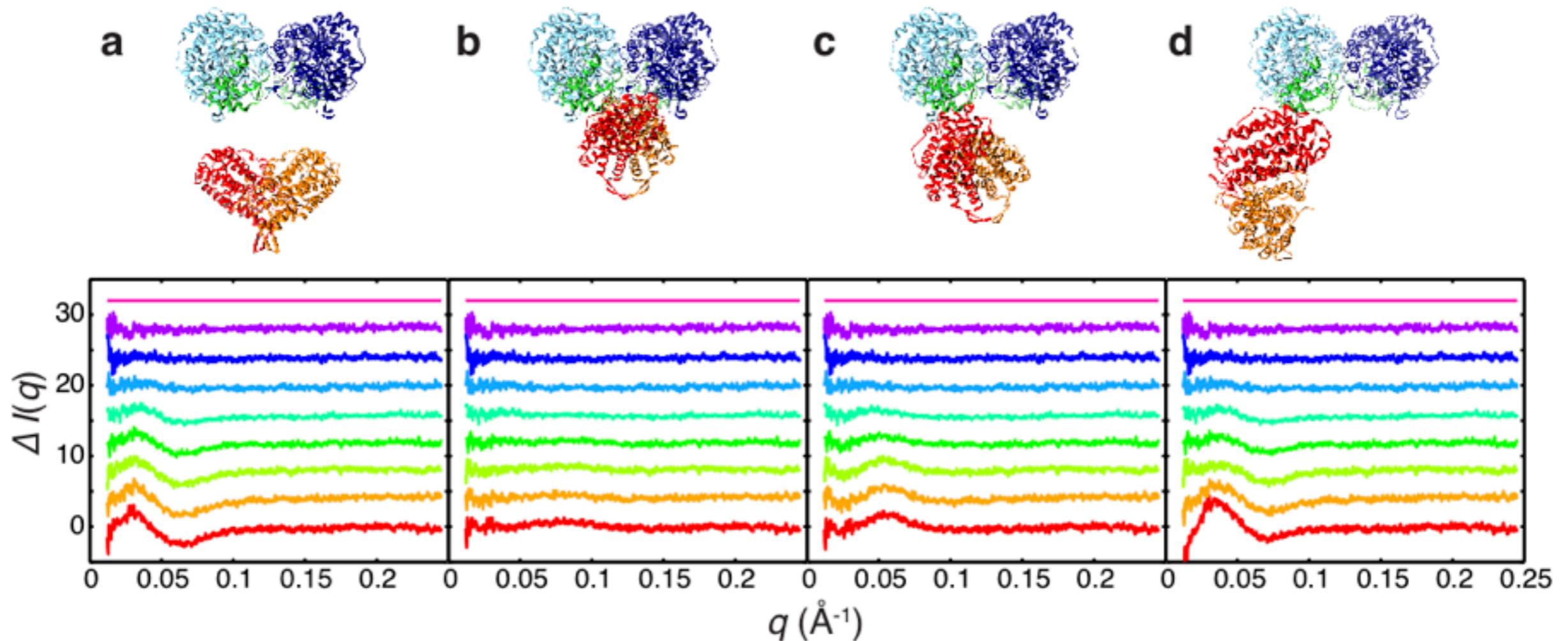
globular  $\alpha_2\beta_2 \rightarrow$  non-globular  $\alpha_4\beta_4$



# 11. Rigid body modeling of active complex

- Two-component fitting of dATP titration data with several candidate models for the active complex in equilibrium with inactive complex. Compare residuals.

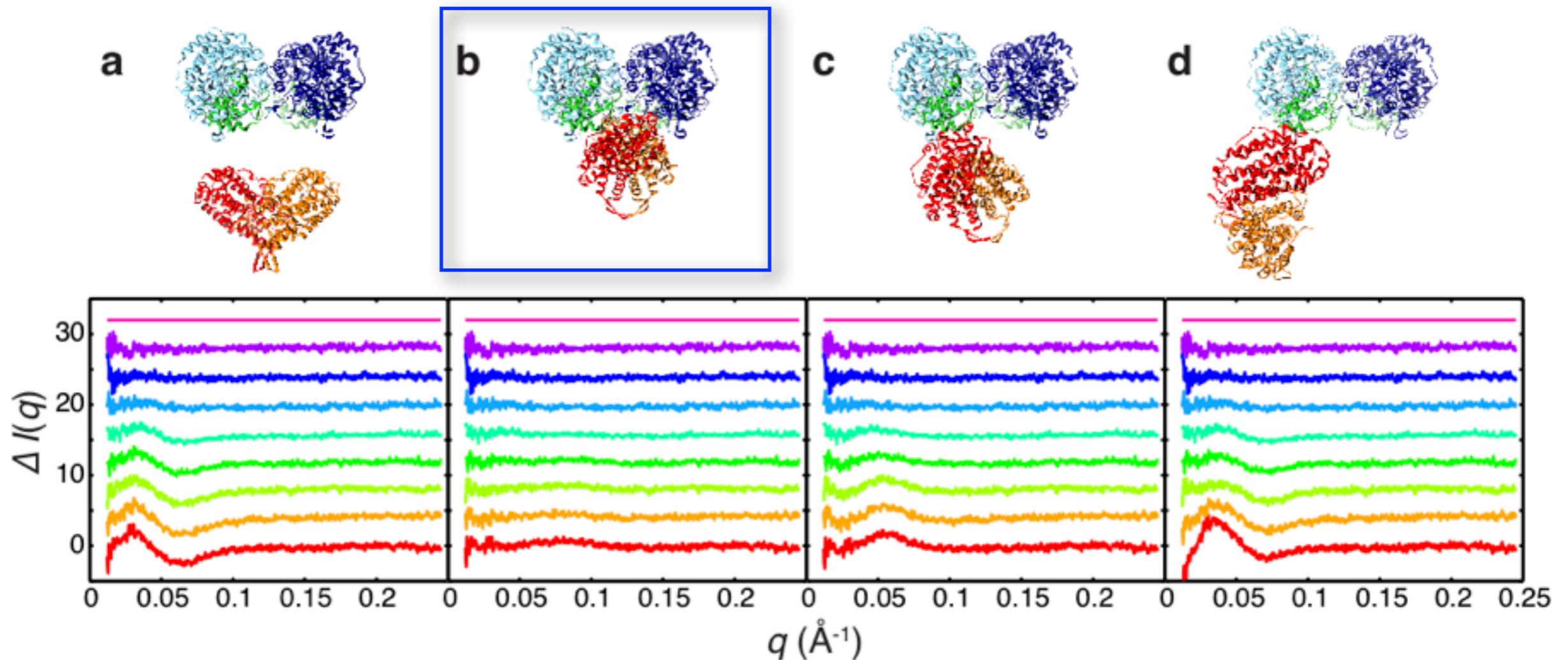
$$I(q) = f_1 I_1(q) + f_2 I_2(q)$$



# 11. Rigid body modeling of active complex

- Two-component fitting of dATP titration data with several candidate models for the active complex in equilibrium with inactive complex. Compare residuals.

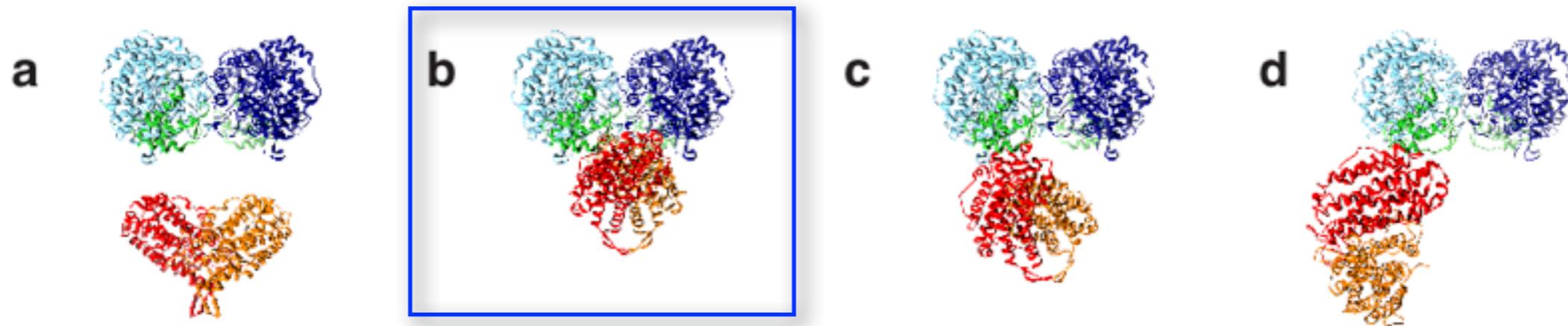
$$I(q) = f_1 I_1(q) + f_2 I_2(q)$$



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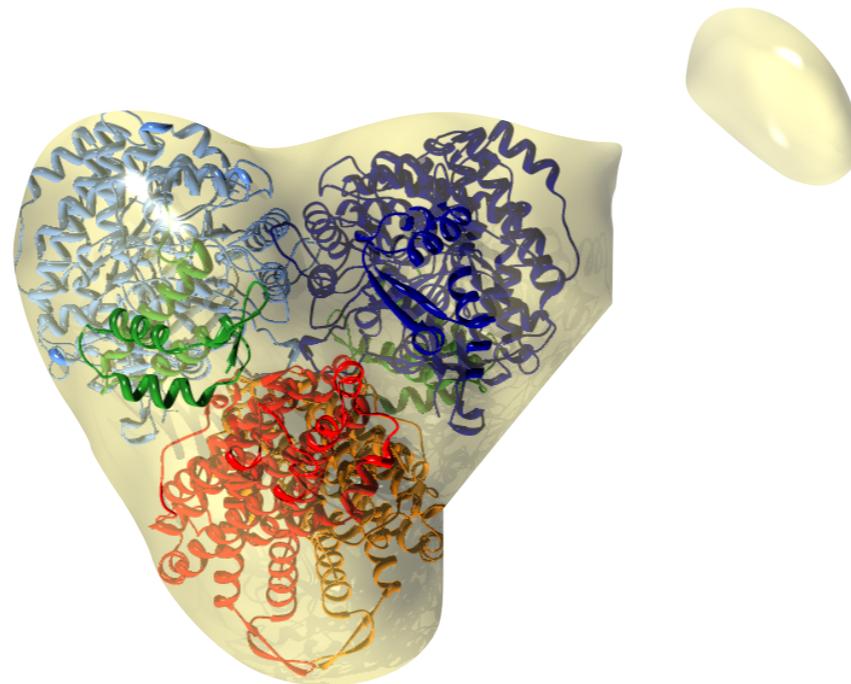


$\alpha_2\beta_2$  “docking” model  
[Uhlen and Eklund. Nature (1994)]

# 11. Rigid body modeling of active complex

- Two-component fitting of dATP titration data with several candidate models for the active complex in equilibrium with inactive complex. Compare residuals.

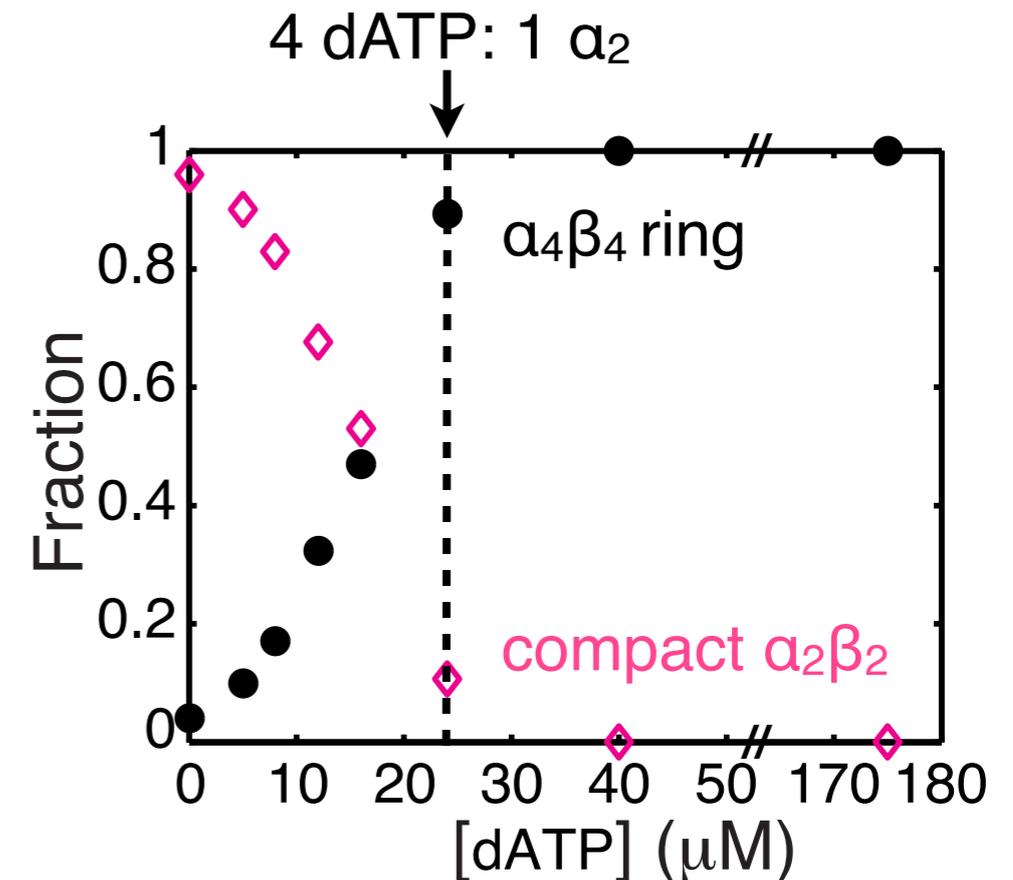
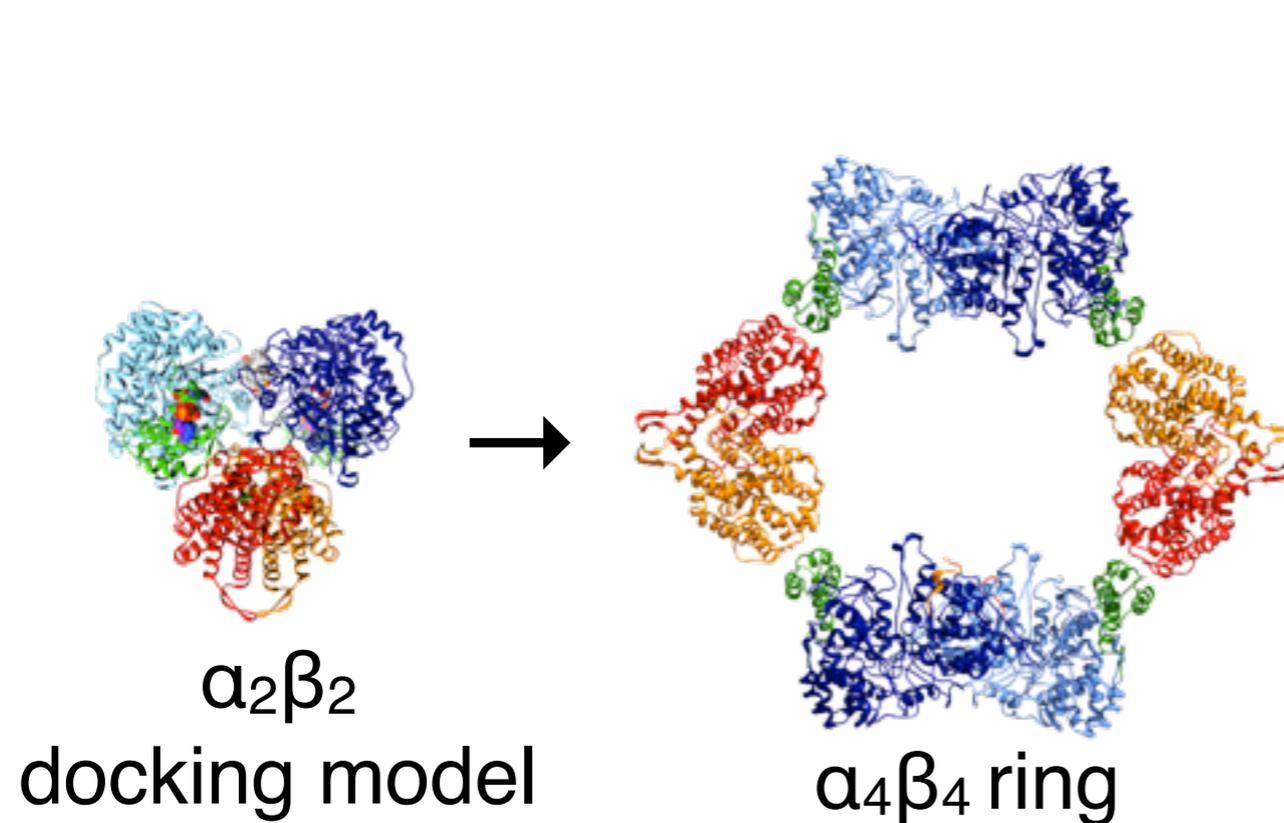
$$I(q) = f_1 I_1(q) + f_2 I_2(q)$$



$\alpha_2\beta_2$  “docking” model fits well in shape reconstruction at 0  $\mu\text{M}$  dATP

# 12. Fractions of species in transition

- Best-fit model for active complex is one in which the lobes of  $\beta_2$  are docked against the active sites of  $\alpha_2$  along symmetry axis.
- This so-called “docking model” was originally proposed by Uhlin and Eklund [Nature (1994)] based on shape complementarity of subunits.

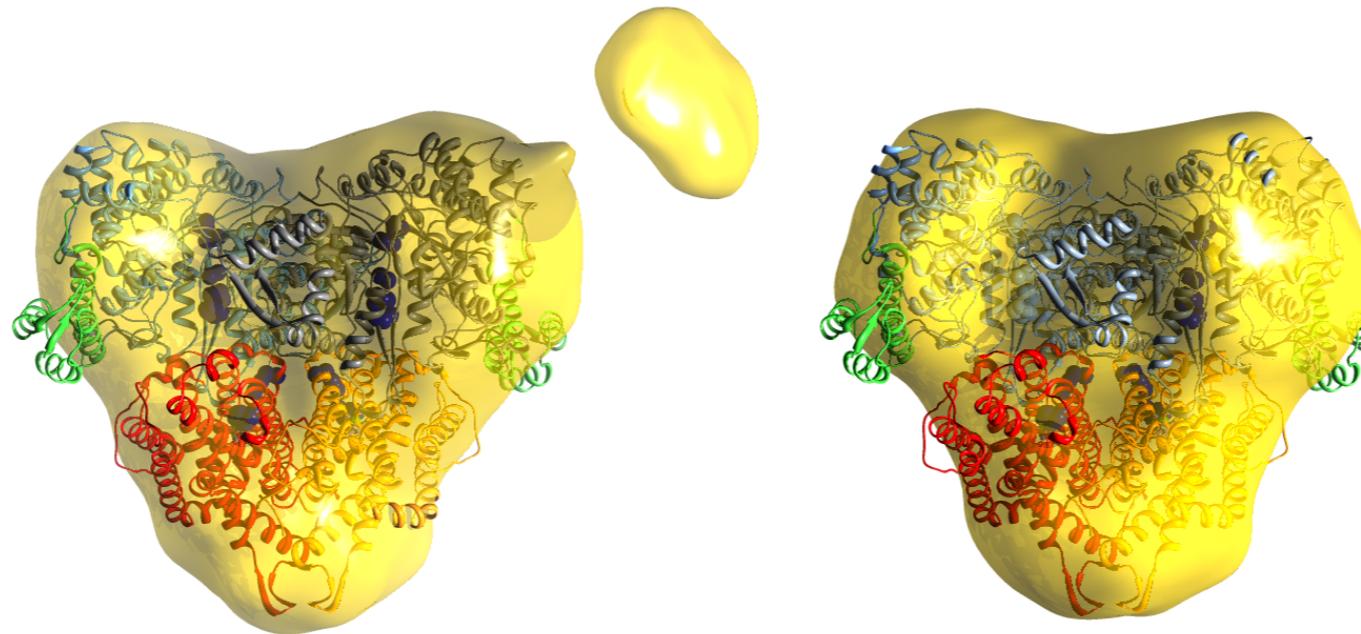


Even at 0  $\mu\text{M}$  dATP, there is 97%  $\alpha_2\beta_2$  and 3%  $\alpha_4\beta_4$ .

## 12. Fractions of species in transition

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Even at 0  $\mu\text{M}$  dATP, there is 97%  $\alpha_2\beta_2$  and 3%  $\alpha_4\beta_4$ .



Subtracting 3%  $\alpha_4\beta_4$  from 0  $\mu\text{M}$  dATP data gives shape reconstruction that is more representative of  $\alpha_2\beta_2$  complex.

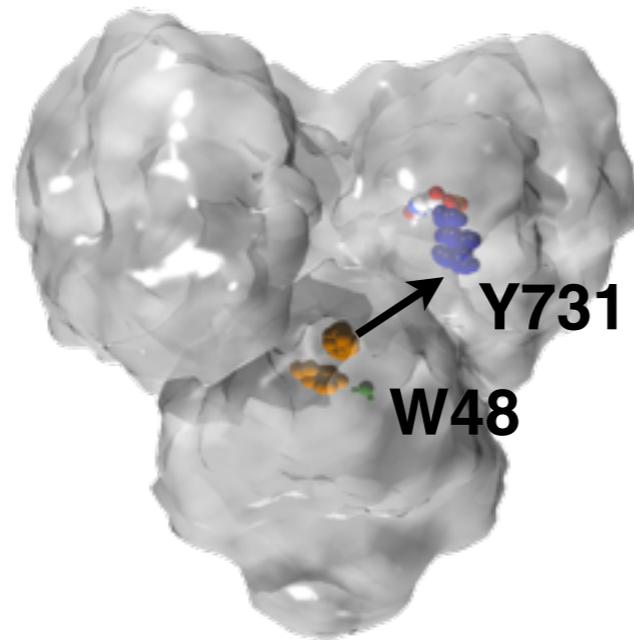
# 13. Conformation affects radical transfer path

$\beta_2$

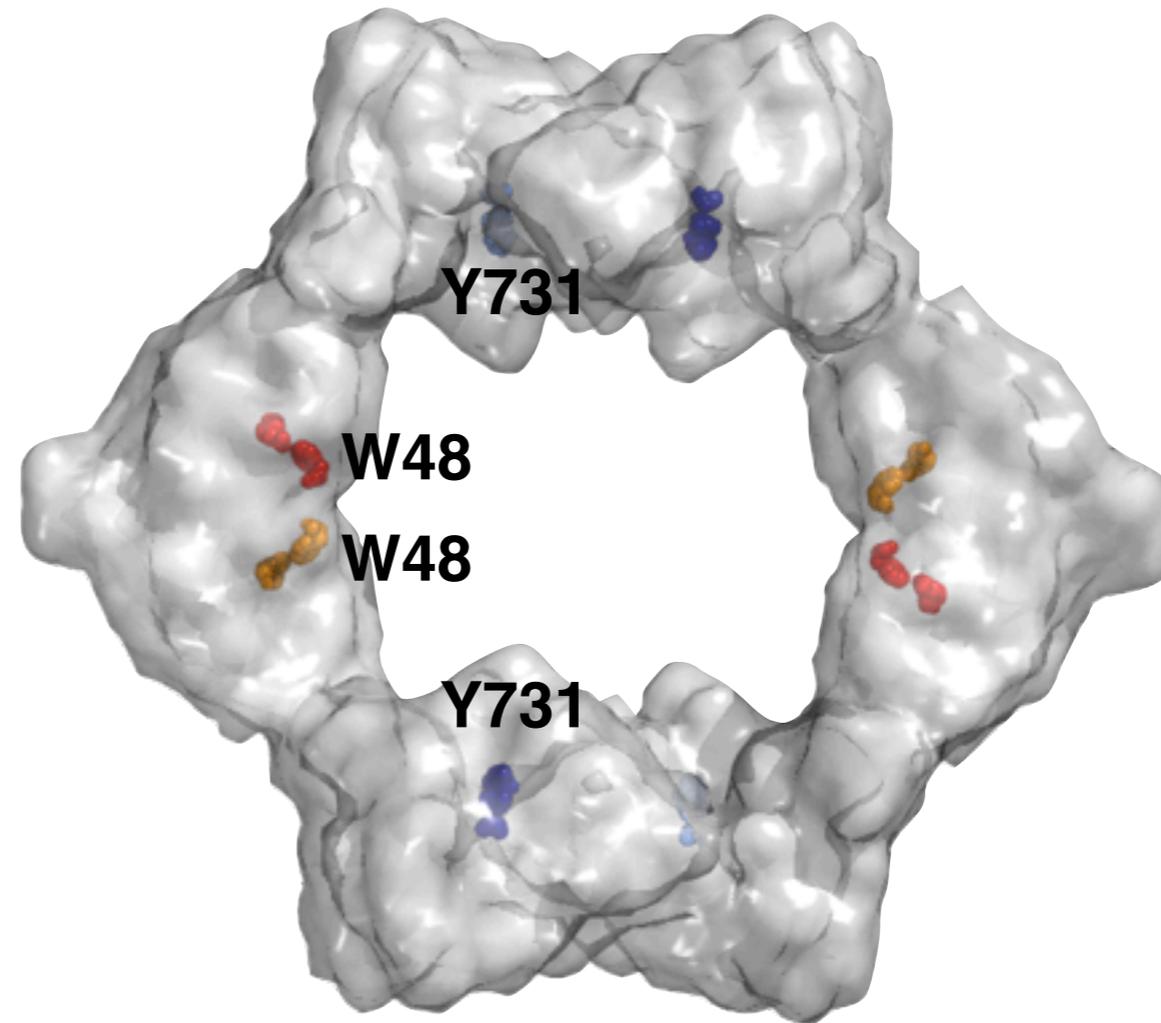
Y122 → W48 → Y356

Y731 → Y730 → C439

$\alpha_2$



$\alpha_2\beta_2$   
docking model

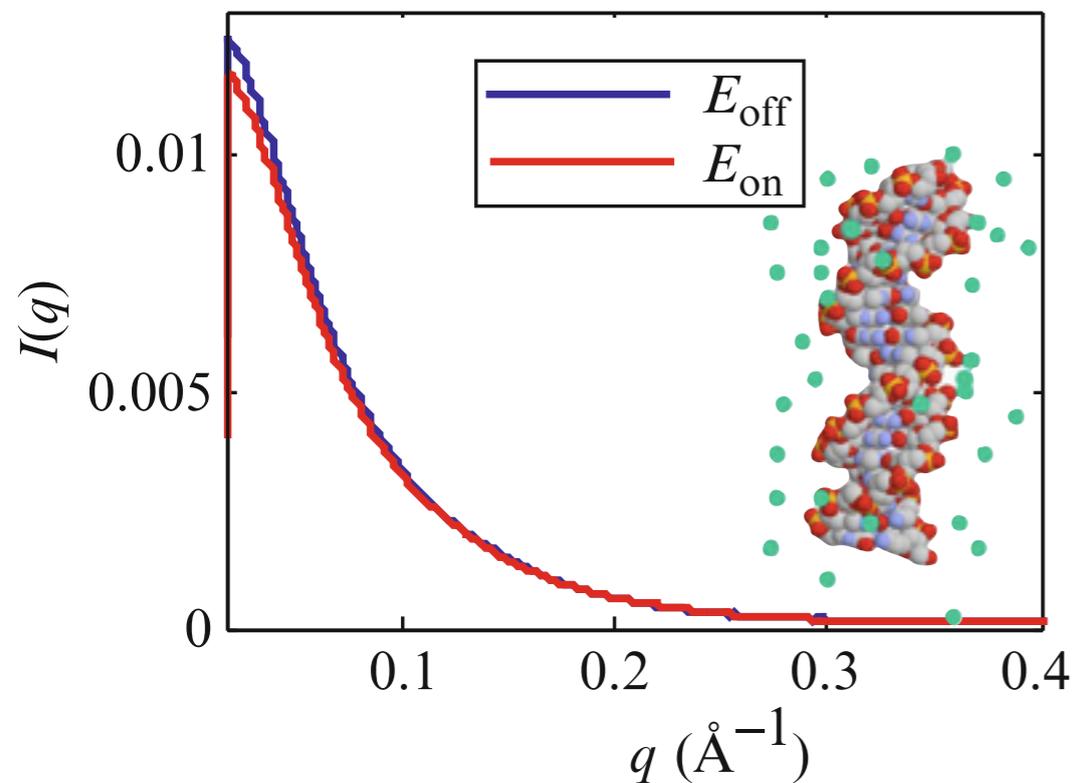


$\alpha_4\beta_4$  ring

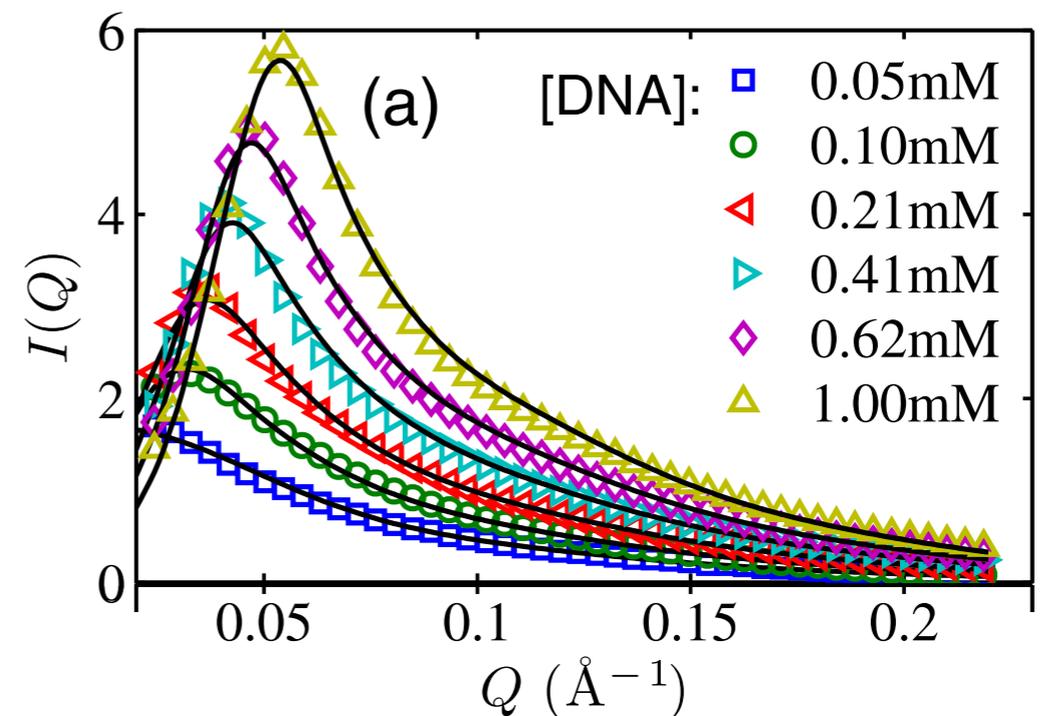
Ando, et al. (2011). PNAS, 108(52), 21046–21051.

# Part 5: Variations

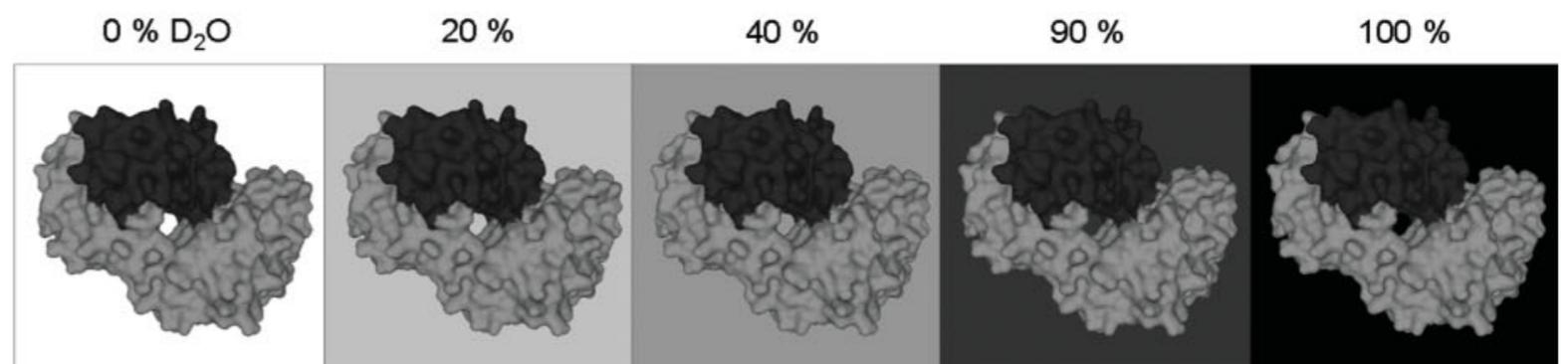
**Anomalous SAXS:** Tune to absorption edge of atom ( $Z > 24$ )



**Structure factor:** Study strongly non-ideal solutions to probe inter-particle potentials



**Contrast Variation:** Tune relative scattering strength (neutron scattering)



Pabit, et al. (2009). Meth. Enz. 469, 391–410.

Qiu, et al. (2006). PRL, 96(13), 138101.

Jacques & Trewhella (2010). Protein Science, 19(4), 642–657.

# Literature

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**A great introduction to SAXS with practical advice:**

Jacques & Trewhella (2010). *Protein Science*, 19(4), 642–657.

**An extensive review with comparison to crystallography:**

Putnam, Hammel, Hura, & Tainer (2007). *Q Rev Biophys*, 40(3), 191–285.

**Review with discussion of scattering of basic geometric shapes:**

Svergun & Koch (2003). *Reports on Progress in Physics*, 66, 1735–1782.

**Data collection protocols and trouble-shooting guide:**

Skou, Gillilan, & Ando (2014) *Nature Protocols*, *in press*.

**Discussion of SAXS publication guidelines:**

Jacques, Guss, Svergun, & Trewhella (2012). *Acta Cryst D*, 68(Pt 6), 620–626.

**Essential reading on *ab initio* shape reconstructions:**

Volkov & Svergun (2003). *Journal of Applied Crystallography*, 36(3), 860–864.

**On anomalous SAXS of metal ion distributions:**

Pabit, Finkelstein, & Pollack (2009). *Methods in Enzymology*, 469, 391–410.

**Classic text on SAXS for the serious SAXS user:**

Glatter & Kratky, editors. *Small Angle X-ray Scattering*. Academic Press, New York, 1982.