Advances in Curved-Foot Dynamic Walking

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Motivation

Many researchers have successfully designed and built dynamic bipedal walkers with curved feet. The design process for these walkers, however, is typically based on intuition and/or poorly documented, suggesting the opportunity for improvements to be made through more systematic design. The most energy efficient bipeds have curved feet, as studies have shown that replacing point feet with curved feet can produce significant energy savings, both in simulation and in experiment [1, 2]. Additionally, during normal human walking, the center of pressure traces a nearly circular arc from heel contact until opposite heel contact when plotted in a shank-fixed frame [3]. It follows that understanding the design of curved feet has potential implications for lower limb protheses.

To take maximal advantage of curved feet, the shape of the curve, its placement relative to the leg, and associated walking controllers need to be selected in a systematic way. Thus, the objective is to develop a reduced order model that captures the key dynamics of walking with curved feet so as to inform the design and control of bipedal robots, aid understanding of human walking, and enhance the design of lower limb prostheses.

State of the Art

Dynamic bipeds having curved feet date back at least to McGeer's passive walkers [4, 5]. These designs have subsequently been enhanced to include passive 3D motion (side-to-side sway) [6] and minimal actuation to enable walking on flat surfaces [7, 8]. Near-fully actuated models [9] mimic the effects of a curved foot shape using ankle springs, which add degrees of freedom and increase design complexity. In all cases, mechanical design choices were made regarding foot shape and foot location relative to the shank. One might assume, unless explicitly stated otherwise, that foot shape is established by a strategy of "physical tinkering and little calculation [6]." Although no formal design tool has been reported, the foot's center of curvature is almost always forward of the shank, which tends to lock the knee for a larger percentage of the gait cycle. This minimizes energy usage [5] and also mimics human foot design [3]. It is desirable for these choices to be made in a systematic way that is more reliant on calculation than tinkering.

Even though curved feet designs have not been systematically determined, their benefits are readily apparent from an energy standpoint. Simple models clearly indicate the expected energy savings at impact for curved feet relative to point feet [1]. Additionally, the roll-over motion of curved feet can increase a biped's step length and modify the desired joint trajectories. These observations suggest that curved feet can provide energy savings not only at impact but during the gait as well. It is not yet well understood, however, how to characterize and exploit these potential additional energy savings.

This highlights the fact that in addition to mechanical design, control strategies that take full advantage of the benefits of curved feet are needed. Many existing curved foot bipeds are controlled with minimal actuation that is superimposed on simplified passive models. The Hybrid Zero Dynamics (HZD) approach provides a theoretical framework for using limit-cycle analysis tools on underactuated walkers. While these bipeds contain actuators to improve gait generation capabilities, they are unactuated at the ankle, allowing inverted pendulum analysis similar to passive-walkers. The HZD approach has been expanded to include series compliance [10, 11], theoretical results for 3D stabilization [12], and some consideration as a tool for modeling human motion [13]. Expansion to curved foot models, however, is still an open topic.

Like previous models [4, 14], HZD treats impacts as discontinuities in the state variables. In [4, 14], velocities were determined by conservation of angular momentum about the point of impact for the entire walker and about the hip for the trailing leg. Using the HZD approach, Lagrange multipliers are used to resolve the impact event, which also conserves angular momentum about the point of impact [15]; however, it makes no simplifying assumptions regarding the momentum of trailing leg. In all of these cases, the choice of impact map influences the simulated periodic gaits. Additionally in the case of HZD, it also changes the underlying reduced-order model used for control. A map that is too simple will not accurately capture the effects of impacts, while one that is too complex will make constructing a reduced-order model very difficult. The importance of the impact map is particularly evident when considering the impact event of curved feet. In [2], HZD-based control of a curved-foot biped using a point-foot impact model and curved foot dynamics led to success in experiment, but translation from simulation to experiment was not as reliable as when the biped had point feet [16]. The impact map presented in [17] attempts to address these issues, but has not been validated experimentally. Validation of a simple yet accurate impact map for curved feet remains a research need.

HZD-based control is limited in that it is driven by position sensing only, such that rejecting disturbances associated with relatively small deviations in terrain, velocity, or impact condition can be difficult. Knowledge of what measure of dynamic coupling exists between the actuated and unactuated dynamics could provide valuable insight for determining control action in such cases. A measure of this coupling has been developed for general mechanical systems [18]. Characterizing the ability to use this measure of coupling when constructing feedback control laws can lead to increased disturbance rejection capability.

Contributions

This research aims to develop an experimentally validated impact map for planar bipeds with curved feet and use that map to help quantify the dynamic coupling present throughout walking gaits. Development of the impact map follows the procedure outlined in [15]. An n-degree-of-freedom biped is described by an extended set of n+2 coordinates: n joint angles and the x, y position of a point on the biped. The robot is then modeled using Lagrange's constrained equations of motion,

$$\mathbf{D_e}(\mathbf{q_e})\ddot{\mathbf{q}}_\mathbf{e} + \mathbf{C_e}(\dot{\mathbf{q}}_\mathbf{e}, \mathbf{q_e})\dot{\mathbf{q}}_e + \mathbf{G_e}(\mathbf{q_e}) = \tau + \mathbf{E}(\mathbf{q_e})^T\mathbf{F}, \quad (1)$$

where $\mathbf{D_e}$, $\mathbf{C_e}$, and $\mathbf{G_e}$ are the inertia, Coriolis, and gravity terms, τ is the vector of joint torques, $\mathbf{F}$ is the vector of Lagrange multipliers, and $\mathbf{E}$ is a matrix of terms from the constraint equations. The impact is modeled by integrating the dynamics (1) over the infinitesimal duration of impact. Under the assumptions in [15], there is no change in coordinates $\mathbf{q_e}$ at impact, and the impulsive forces at impact are much larger that the motor and gravity torques. These assumptions result in

$$\mathbf{D_e}(\mathbf{q_e})(\dot{\mathbf{q}}_\mathbf{e}{}^+ - \dot{\mathbf{q}}_\mathbf{e}{}^-) = \mathbf{E}(\mathbf{q_e})^\mathbf{T}\mathbf{f}, \quad (2)$$

where $\mathbf{f}$ is the integrated impulsive force, and the superscripts '-' and '+' refer to the state of the robot just before and just after impact, respectively. Equation (2) represents n+2 equations in n+4 unknowns. The final two equations are obtained from the no-slip condition for the impacting foot,

$$\mathbf{E}\dot{\mathbf{q}}_\mathbf{e}{}^+ = \mathbf{0}. \quad (3)$$

This impact map was first verified by comparing simulation results and experimental results of McGeer's passive walker [4]. The experimental step duration ranged from 0.56 to 0.67 seconds and the leg splay angle at impact from 0.54 to 0.66 radians (30° to 38°). Using the above impact map, the simulated gait has a step duration of 0.67 seconds and a leg splay angle of 0.58 radians (34°). The agreement with experiment suggests that the map accurately captures the dominant dynamics of curved foot walking. For comparison, McGeer's original model predicted a leg splay angle of 0.82 radians (47°) and a step duration of 0.55 seconds, with better agreement, 0.66 radians (38°) and 0.61 seconds, achieved after inclusion of a rolling friction term in the model.

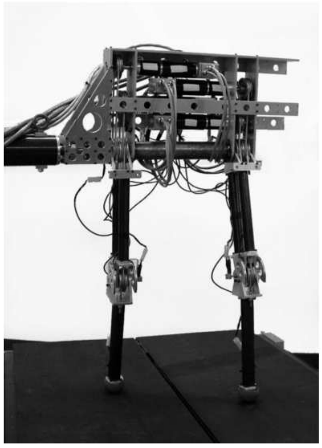

Figure 1: The planar biped ERNIE prior to retrofitting with curved feet

The impact map is currently being further verified on the hardware platform ERNIE [16] shown in Fig. 1. The point feet have been replaced by curved feet designed to allow adjustment of the fore-aft position of the foot center of curvature. Initial experiments indicate good correlation between simulation and experiment. In 12 of 14 gaits with distinct average speeds of 0.40 - 0.60 m/s, the simulated step duration was within 1 standard deviation of the experimentally measured values. This correlation was obtained with no system identification and inertias only roughly calculated from CAD models. Since the impact map readily permits variations in the fore-aft offset of the foot center of curvature, further study of center of curvature location and curve radius are currently being conducted in simulation and expected in experiment with ERNIE in the near future.

These recent advances in impact map modeling allow quantification of the dynamic coupling present throughout the experimental gaits. This leads to investigation first in simulation and subsequently in experiment of the relationship between the coupling and robustness to velocity disturbances with the goal of developing an enhanced HZD-based controller for disturbance rejection.

Open Questions

1. In addition to decreased energy loss at impact, what characteristics of gaits associated with curved feet introduce energy savings?
2. How does a designer systematically select an optimal foot shape and foot location relative to the shank for walking? How do optimal foot shape and location vary with speed?
3. How do curved feet affect the disturbance rejection capabilities of a biped walking under HZD-based control?

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