

Predicting Healthy Human and Amputee Walking Gait using Ideas from Underactuated Robot Control

Anne E. Martin¹ and James P. Schmedeler²

Abstract—The ability to predict human gait, particularly impaired human gait, has the potential to improve rehabilitation/training outcomes and to reduce prosthesis/orthosis development costs. This work presents a walking model of moderate complexity that accurately captures both sagittal plane joint kinematics and whole body energetics for healthy human walking. The six-link, left-right symmetric model with hips, knees, ankles, and rigid circular feet accurately predicts normal human walking over a wide range of speeds using a torque-squared objective function. For unilateral transtibial amputee gait, one ankle joint is eliminated, yielding a five-link, asymmetric model that is used to quantify the differences between amputee gaits optimized for symmetry and efficiency.

I. INTRODUCTION

For healthy humans, the optimal gait is both energetically efficient [1] and approximately left/right symmetric. Because most current commercially available prostheses are unpowered, though, it is unlikely that many unilateral amputees can walk in a manner that simultaneously maximizes both efficiency and symmetry [2]. Furthermore, the characteristics of each strategy and how the two differ are not well understood. Amputee gait research has typically focused on improving one of the metrics without consideration of the other. Quantifying the trade-offs between the two could lead to more informed patient care decisions from clinicians and more realistic modeling assumptions from biomechanists.

Most predictive models of human gait focus on healthy gait. Very basic models [3] typically approximate one aspect of gait using a simple mechanical system to clearly demonstrate the key underlying gait mechanics, and their straightforward mathematics tend to lead to easily verifiable predictions. At the other end of the spectrum are detailed, musculoskeletal models [4] that provide insights into muscle coordination. The complicated nature of these systems limits the number of cases examined because both the optimization and simulation are time consuming. In most cases, predictive simulations of normal human walking have also been limited to the self-selected walking speed (e.g. [5]). Because speed-related gait changes are well documented [6], however, the ability to predict gait at the self-selected speed does not necessarily translate into accurate predictions across a wide range of speeds. Such predictions are necessary for the design and selection of mechanical interventions, such as prostheses and orthoses, which motivates the predictive model development herein.

A model of moderate complexity can be detailed enough to capture multiple gait features simultaneously, yet simple enough to enable rapid motion optimization. Planar models controlled based on hybrid zero dynamics (HZD) [7] are one example. HZD-based control parameterizes the desired joint trajectories as functions of a phase variable that measures step progression, and not of time explicitly, which is consistent with human control [8]. Prior work has shown that an HZD-inspired model can accurately match human hip and knee kinematics at one walking speed [9] and can translate human gait to robotic gait [10]. To account for the human foot, a curved foot model is favored since the center of pressure traces a circular arc in a shank-fixed coordinate frame through most of stance [11]. In addition, ankle joints can capture the contributions of the ankle-foot complex in controlling whole body center-of-mass acceleration [12].

II. MODELS

The planar, left-right symmetric model [13] for healthy human gait consists of six links plus a point mass at the hip representing the mass of the head, arms and trunk (Fig. 1). Revolute joints at the hips, knees, and ankles connect two thighs, two shanks, and two feet. The thighs and shanks have both mass and inertia. The constant radius circular feet have mass but no inertia [11]. The five-link model for unilateral transtibial amputee gait is similar, but the ankle joint on the amputated leg is removed, so the model is not left-right symmetric. Representing the degrees of freedom in a model, $N = 6$ for the healthy model and 5 for the amputated model. Ideal torque generators control the leg joint angles ($q_2 - q_N$ in Fig. 1), with a single hip actuator controlling the angle between the thighs (q_2 in Fig. 1). The point of contact between the foot and ground is unactuated, and the unactuated motion is captured in the stance hip absolute angle (q_1 in Fig. 1).

The HZD-based control approach for curved foot bipeds [14] was applied to generate one- and two-step periodic gaits for the six- and five-link models, respectively. Each step consists of a finite-time single support phase and an instantaneous double support phase. While both healthy humans and amputees have a finite-time double support phase, condensing it to an instant simplifies the control problem significantly without compromising the model's ability to capture the dominant gait characteristics. Double support is modeled as an impulsive transition during which the pre-impact swing/stance leg contacts/lifts off the ground. This impact results in an instantaneous change in joint velocities

The authors are with the Aerospace and Mechanical Engineering Department of the University of Notre Dame

¹amarti29@nd.edu

²schmedeler.4@nd.edu

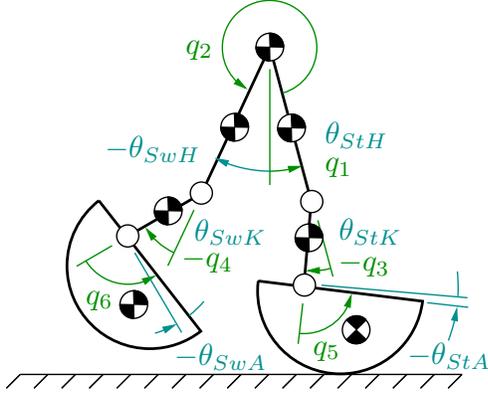


Fig. 1. Schematic of the healthy model. For the amputated model, the amputated foot is rigidly attached perpendicular to the amputated shank rather than jointed to it, but the contralateral leg is unaltered. The joint angles are indicated with the q_i 's. The unactuated angle is q_1 ; the actuated angles are q_2 through q_6 . The plots, however, transform the model angles into a more typical biomechanics convention as indicated with the θ_i 's.

and a swap of leg roles,

$$\mathbf{q}^+ = \mathbf{S}\mathbf{q}^- \quad (1)$$

$$\dot{\mathbf{q}}^+ = \mathbf{A}(\mathbf{q}^-)\dot{\mathbf{q}}^-, \quad (2)$$

where \mathbf{q} is the $N \times 1$ joint angle vector, \mathbf{S} is an $N \times N$ matrix that switches the angle definitions and \mathbf{A} is an $N \times N$ matrix relating the pre- and post-impact velocities. The superscripts '-' and '+' refer to the instants immediately before and after impact, respectively.

During single support, the dynamic equations can be integrated forward in time.

$$\mathbf{D}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \mathbf{u}, \quad (3)$$

where \mathbf{D} is the $N \times N$ inertia matrix, \mathbf{C} is the $N \times N$ matrix containing Coriolis and centripetal terms, \mathbf{G} is the $N \times 1$ vector containing gravity terms, and \mathbf{u} is the $N \times 1$ vector of joint torques. Using a nonlinear, gait-specific coordinate transformation and appropriate joint torques, Eq. 3 can be changed such that it is composed of a linear system involving the joint torques and a nonlinear, passive system [7]. The joint torques \mathbf{u} are chosen to achieve the desired motion \mathbf{y} of the actuated joints. It is critical that \mathbf{y} be a function of only the joint angles (not of time) and the unactuated angle. It is convenient to define \mathbf{y} using fifth order Bézier polynomials,

$$y_j = q_j - \sum_{k=0}^5 \alpha_{k,j} \frac{5!}{(5-k)!k!} s^k (1-s)^{5-k}, \quad (4)$$

where $j = 1, 2, \dots, N-1$, $\alpha_{k,j}$ denotes the control parameters to be chosen and $0 \leq s \leq 1$ is the phase variable, in this case a linearized approximation of the horizontal hip position, normalized so that $s = 0$ and $s = 1$ correspond to the start (impact) and end (lift-off) of the step, respectively. Since s is a function of the joint angles and the unactuated angle, the evolution of the desired joint angles is driven by hip progression, not by time explicitly. To ensure a periodic gait,

$\alpha_{j,0}$ and $\alpha_{j,1}$ are chosen as functions of $\alpha_{j,4}$ and $\alpha_{j,5}$ [7]. For the one-step periodic gaits of the healthy model, there is one set of control parameters, whereas the two-step periodic amputated model gaits have two sets of control parameters.

III. PREDICTING HEALTHY HUMAN WALKING

Optimization is used to predict normal human walking at a specified speed without a priori knowledge of any other gait characteristics using a torque squared objective function [13]. To validate the results, the predicted gaits were compared to published experimental overground walking data from multiple sources over a wide range of speeds. Because of the range of subject sizes, all reported data were nondimensionalized using the factors proposed in [15]. For reference, the nondimensional self-selected and walk-to-run transition speeds are about 0.42 and 0.71, respectively.

To predict gaits, the control parameters $\alpha_{j,k}$ in Eq. 4 were determined via optimization with the objective function

$$g_{eff} = \sum_{k=1}^N c_k \cdot \tau_k^2 + g_p, \quad (5)$$

where

$$\tau_k^2 = \frac{\int_0^T u_k^2(t) dt}{T(Mg\ell_0)^2}, \quad (6)$$

T is the step duration, u is the joint moment, M is the subject mass, g is the gravitational acceleration, ℓ_0 is the subject leg length, c_k is a weighting factor and g_p is a penalty function used to avoid solutions requiring excessively long feet. It seems likely that torque squared is correlated to the square of muscle activations, which in turn, are generally believed to be correlated with metabolic cost [16]. Also, humans tend to minimize metabolic cost when walking [1]. Equal weighting did not accurately predict step length or knee kinematics, so the weights for the knee and ankle are 20% of that for the hip. Stance hip work is motion dominated, while stance knee and ankle work are torque dominated, so these weighting factors result in gaits that perform approximately equal magnitudes of work at each stance joint, consistent with normal human gait [17]. Speed is enforced as a constraint.

Step length increases linearly with speed at approximately the same rate for humans (0.58 ± 0.09 , $R^2 = 0.87$) and the model (0.59 ± 0.12 , $R^2 = 0.81$), indicating that the optimization successfully predicts the increase in step length with speed (Fig. 2). Similarly, the hip kinematics are well predicted, with the model capturing both the time-based trajectories and the peak magnitudes. The stance knee kinematics are acceptably captured, particularly at slower speeds. The main source of error is insufficient model knee flexion during the weight acceptance phase [18]. The peak stance knee flexion increases more rapidly with increasing speed in experiment than in simulation, leading to larger errors at faster speeds. The model slightly underestimates the energetic cost of walking as measured by the mean absolute power. Overall, the model can well approximate normal human walking as evidenced by the small errors in both the kinematics and the energetic requirements [13].

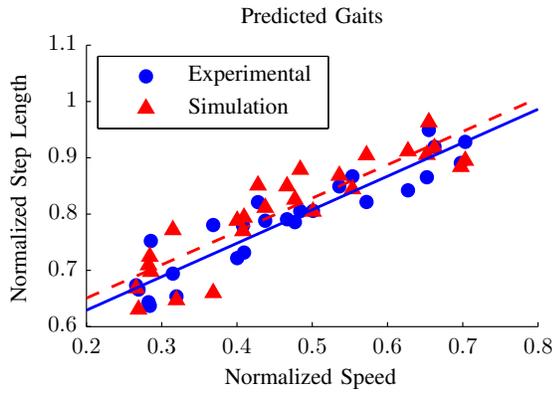


Fig. 2. Normalized step length vs. normalized average speed for the human experimental data and the predicted model gaits. The linear least-squares lines for the experimental (solid) and predicted (dashed) gaits are shown.

IV. INVESTIGATING AMPUTEE GAIT

To quantify the trade-offs between symmetric and efficient unilateral transtibial amputee walking, gaits of the asymmetric amputee model were optimized using different objective functions. To quantify efficiency, the objective function for normal human gait was used (Eq. 5). To quantify symmetry, the objective function was

$$g_{sym} = 10^3(\Delta_v^2 + \Delta_S^2) + \sum_{k=1}^4 (1 - \rho(q_{k,amp}, q_{k,con})), \quad (7)$$

where Δ_v is the difference in average speed and Δ_S is the difference in step length between the amputated and contralateral sides. $\rho(q_{k,amp}, q_{k,con})$ is Pearson's correlation coefficient, used to evaluate similarity between the amputated and contralateral joint angle traces. Specifically, ρ measures how accurately a linear transformation would transform the amputated joint angle $q_{k,amp}$ into the contralateral joint angle $q_{k,con}$. A value of 1 indicates a perfect match, while 0 indicates no correlation. To find an array of gaits with varying weights on symmetry and efficiency, Eqs. 5 and 7 were also combined into a single objective function, and the weighting factors between the two were varied.

Indicative of successful optimization, maximally efficient gaits are more efficient than maximally symmetric gaits at a given speed, and maximally symmetric gaits are more symmetric than maximally efficient gaits at a given speed (Table I). For the maximally efficient gaits, the average step speed is slower when the amputated side is in stance, which is consistent with experimental data [19]. For maximally symmetric gaits, the average speed and step length are the same regardless of which leg is in stance. The energy requirements for maximally symmetric gaits are approximately 3-5 times higher than for maximally efficient gaits. None of the objective functions produced a gait that mimicked healthy human gait (Fig. 3). The most significant differences are in the swing knee flexion and the smoothness of the gait. All of the amputated gaits predicted much ($\approx 20^\circ$) smaller peak swing knee flexion than observed in healthy human

gait, which results in much less foot clearance for amputees. In addition, the joint angle trajectories for both strategies are not as smooth as for healthy humans, particularly for the more efficient gaits.

Overall, the gaits have significant differences in efficiency, amputated/contralateral symmetry, and joint motions, indicating the presence of trade-offs between maximizing efficiency and symmetry for unilateral transtibial amputees. These results inform current work to identify an objective function for accurate prediction of normal amputee walking.

REFERENCES

- [1] H. J. Ralston, "Energy-speed relation and optimal speed during level walking," *European Journal of Applied Physiology and Occupational Physiology*, vol. 17, no. 4, pp. 277–83, 1958.
- [2] R. W. Selles, J. B. Bussmann, L. Klip, B. Speet, A. Vansoest, and H. J. Stam, "Adaptations to mass perturbations in transtibial amputees: Kinetic or kinematic invariance?" *Archives of Physical Medicine and Rehabilitation*, vol. 85, no. 12, pp. 2046–2052, 2004.
- [3] A. D. Kuo, "The six determinants of gait and the inverted pendulum analogy: A dynamic walking perspective," *Human Movement Science*, vol. 26, no. 4, pp. 617–56, 2007.
- [4] M. Q. Liu, F. C. Anderson, M. H. Schwartz, and S. L. Delp, "Muscle contributions to support and progression over a range of walking speeds," *Journal of Biomechanics*, vol. 41, no. 15, pp. 3243–52, 2008.
- [5] L. Ren, R. K. Jones, and D. Howard, "Predictive modelling of human walking over a complete gait cycle," *Journal of Biomechanics*, vol. 40, no. 7, pp. 1567–74, 2007.
- [6] N. Lythgo, C. Wilson, and M. Galea, "Basic gait and symmetry measures for primary school-aged children and young adults. ii: walking at slow, free and fast speed," *Gait & Posture*, vol. 33, no. 1, pp. 29–35, 2011.
- [7] E. R. Westervelt, J. W. Grizzle, C. Chevallereau, J. H. Choi, and B. Morris, *Feedback Control of Dynamic Bipedal Robot Locomotion*. CRC Press, 2007.
- [8] R. D. Gregg, E. J. Rouse, L. J. Hargrove, and J. W. Sensinger, "Evidence for a time-invariant phase variable in human ankle control," *Public Library of Science ONE*, vol. In press, 2014.
- [9] S. Srinivasan, I. A. Raptis, and E. R. Westervelt, "Low-dimensional sagittal plane model of normal human walking," *Journal of Biomechanical Engineering*, vol. 130, no. 5, p. 051017, 2008.
- [10] R. W. Sinnet and A. D. Ames, "Bio-inspired feedback control of three-dimensional humanlike bipedal robots," *Journal of Robotics and Mechatronics*, vol. 24, no. 04, pp. 595–601, 2012.
- [11] A. H. Hansen, D. S. Childress, and E. H. Knox, "Roll-over shapes of human locomotor systems: Effects of walking speed," *Clinical Biomechanics*, vol. 19, no. 4, pp. 407–14, 2004.
- [12] F. C. Anderson and M. G. Pandy, "Individual muscle contributions to support in normal walking," *Gait & Posture*, vol. 17, no. 2, pp. 159–69, 2003.
- [13] A. E. Martin and J. P. Schmiedeler, "Predicting human walking gaits with a simple planar model," *Journal of Biomechanics*, 2014. [Online]. Available: <http://dx.doi.org/10.1016/j.jbiomech.2014.01.035>
- [14] A. Martin, D. Post, and J. Schmiedeler, "Design and experimental implementation of a hybrid zero dynamics-based controller for planar bipeds with curved feet," *International Journal of Robotics Research*, to appear. [Online]. Available: http://ame-robotics.nd.edu/LAB/Publications/IJRR_2013.Preprint.pdf
- [15] A. L. Hof, "Scaling gait data to body size," *Gait & Posture*, vol. 4, no. 3, pp. 222–3, 1996.
- [16] A. Silder, T. Besier, and S. L. Delp, "Predicting the metabolic cost of incline walking from muscle activity and walking mechanics," *Journal of Biomechanics*, vol. 45, no. 10, pp. 1842–9, 2012.
- [17] P. DeVita, J. Helseth, and T. Hortobagyi, "Muscles do more positive than negative work in human locomotion," *The Journal of Experimental Biology*, vol. 210, no. Pt 19, pp. 3361–73, 2007.
- [18] L. Worthen-Chaudhari, J. Bing, J. Schmiedeler, and D. Basso, "A new look at an old problem: Defining weight acceptance in human walking," *Gait & Posture*, vol. 39, no. 1, pp. 588–592, 2014.
- [19] E. Isakov, O. Keren, and N. Benjuya, "Trans-tibial amputee gait: Time-distance parameters and emg activity," *Prosthetics and Orthotics International*, vol. 24, no. 3, pp. 216–20, 2000.

TABLE I

AMPUTEE MODEL GAIT CHARACTERISTICS FOR GAITS OPTIMIZED WITH DIFFERENT OBJECTIVE FUNCTIONS AT TWO NORMALIZED SPEEDS.

Gait Speed	Objective Function	Step Length		Speed		$\sum_1^4(1 - \rho(\cdot))$	Eq. 5
		Amputated	Contralateral	Amputated	Contralateral		
0.33	Max Efficiency	0.49	0.82	0.26	0.40	0.74	0.004
	Equal Weighting	0.56	0.57	0.33	0.33	0.93	0.005
	Max Symmetry	0.53	0.53	0.33	0.33	0.04	0.011
0.27	Max Efficiency	0.54	0.74	0.23	0.30	0.55	0.001
	Max Symmetry	0.57	0.57	0.27	0.27	0.02	0.006

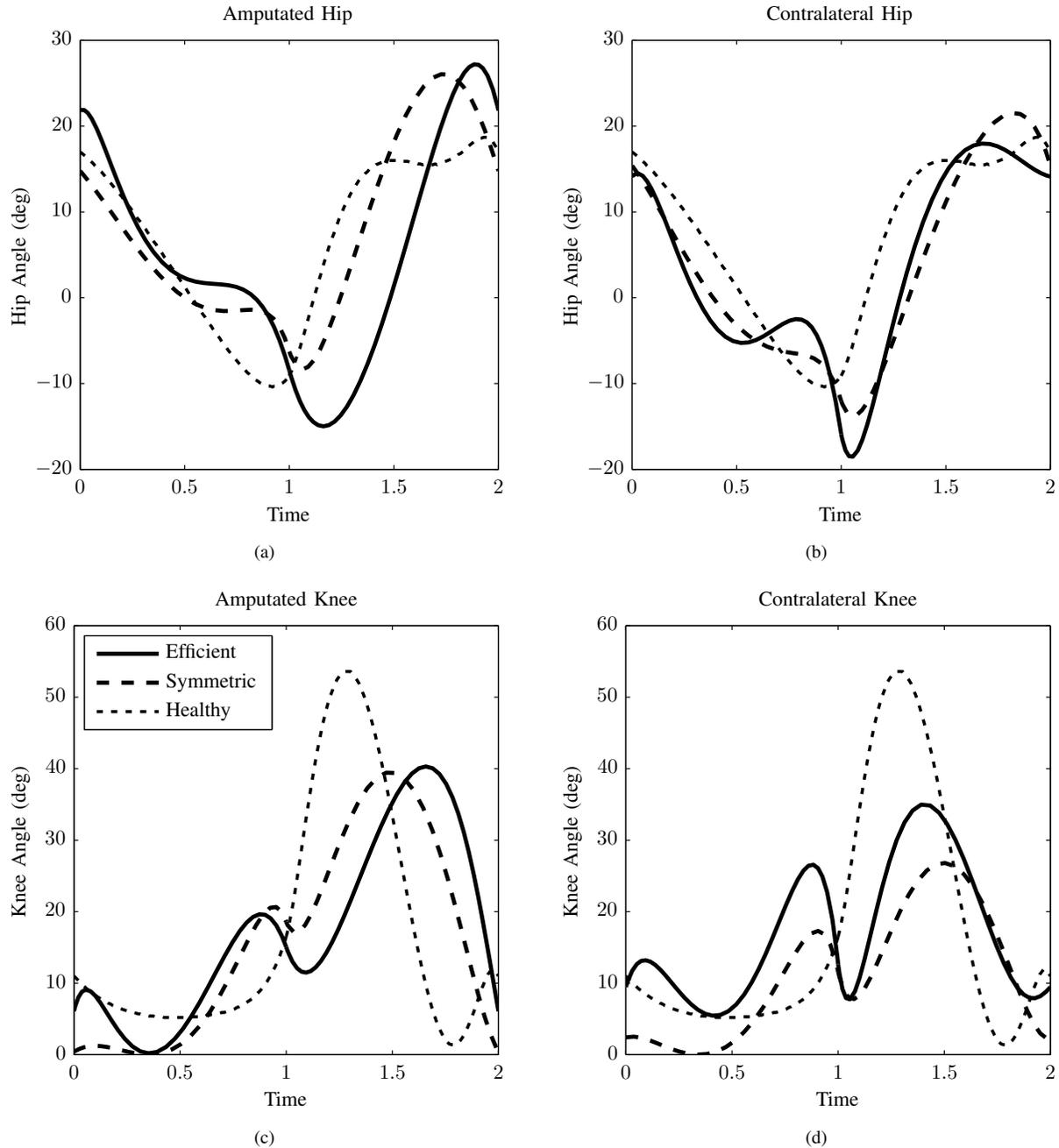


Fig. 3. Comparison of healthy human and unilateral transtibial amputee model joint angle trajectories for the (a) amputated hip, (b) contralateral hip (c) amputated knee and (d) contralateral knee for walking gaits at a nondimensional speed of 0.33. Model gaits were optimized for either maximal efficiency or maximal symmetry. The stance phase occurs from 0 to 1, and the swing phase occurs from 1 to 2.