Non-Equilibrium Physics with Quantum Gases

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Intro: cold atoms as isolated quantum systems
Mean field experiments at various $T$
Gases with correlations
1D gases with $\delta$-interactions: thermalization

Outline

Time scales, Integrability, Correlations, Thermalization
Cold gas experiments

Cold gases are metastable

trapped gas

⇒
inelastic 3-body collisions

⇒ (typically) metal coating the vacuum chamber

This is usually unimportant

Any study of dynamics has to be concerned with timescales

Cold gases are very well isolated quantum systems

light trapping

\[ p \propto E, \quad U = -p \cdot E \]

\[ U_{AC} \propto \text{Intensity} \]

magnetic trapping

\[ U = -\mu \cdot B \]

small complications: intensity, current, position fluctuations; background gas collisions; spontaneous emission
The Mean Field

S-wave interactions can be accounted for with the Huang pseudo-potential

\[ V(r) = \frac{4\pi\hbar^2}{m} a \delta^3(\vec{r}) \]

- Long range behavior correct \( R \propto 1 - \frac{a}{r} \)
- Enforces boundary condition \( \Psi(r = a) = 0 \)

This leads to the Gross-Pitaevskii equation (non-linear S.E.)

\[
\left[ -\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r}) + \frac{4\pi\hbar^2}{m} a|\Psi|^2 \right] \Psi = E\Psi = \mu\Psi \quad \psi = \frac{1}{\sqrt{N}} \phi_0
\]

The effects of collisions are in the mean field term.
There is nothing irreversible about it!

The evolution is integrable, with excitations of \( \Psi \) the only degrees of freedom
Prepare atoms in a superposition of number states at each lattice site

\[ |\alpha(t)\rangle = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} e^{-i2Un(n-1)t/\hbar} |n\rangle \]

These collisions are coherent
3D BEC Integrable Evolution

**Dalibard**

breathing mode

\[
\frac{A(t)}{A} \approx Q^{-2000}
\]

Time (ms)

**Ketterle**

dipole mode

\[
\text{mean-field frequency scale} = \frac{4\pi\hbar a}{m} n_{\text{BEC}}
\]

\[
\text{thermal collision rate} = n_{\text{th}} \pi a^2 v
\]

It's GP-simple when

\[
\lambda_{dB} \gg a \frac{n_{\text{th}}}{n_{\text{BEC}}}
\]

for short enough time scales

10 milliseconds per frame

200 μm
The coherence grows faster than the $N_{\text{BEC}}$
Some Mean Field Non-Eq. Expts.

Solitons

Tkachenko oscillations

Josephson Oscillations

Hulet

Magnetic dipole + mean field

Cornell

Oberthaler

Also:
Cold neutral plasmas
Rydberg blockaded gases
Cold molecules...
**Coupling strength**

In a Bose gas, the ratio of two energies, $\gamma$, governs the extent of correlations in a quantum gas:

$$E_{int} = \frac{4\pi \hbar^2 a}{m} n_{3D}$$

Mean field energy

$$E_K = \frac{\hbar^2 k_F^2}{2m}$$

Kinetic energy

$$\gamma = \frac{E_{int}}{E_K}$$

For low $\gamma$, it is less energetically costly for single particle wavefunctions to overlap than be separated $\Rightarrow$ Mean field theory & the G-P equation apply: **weak coupling**

For high $\gamma$, it is less energetically costly for wavefunctions to avoid each other $\Rightarrow$ **strong coupling**

As $\gamma$ increases, dynamics becomes a quantum many-body problem
Significance of correlations

Weak correlations allow long range phase coherence, and macroscopic wavefunction phenomena \( \Rightarrow \) Eg., interference, superfluidity, vortices

Strongly correlated systems are much harder to calculate, especially out of equilibrium

\( \gamma \uparrow \) for bosons at high density in 3D, in optical lattices, or in reduced dimensions
Single atom dynamics

Mobile spin impurities

MI → SF

coherence grows fast

Greiner

Bloch
1D Bose gases with variable point-like interactions

Elliot Lieb and Werner Liniger, 1963: Exact solutions for 1D Bose gases with arbitrary $\delta(z)$ interactions

A Bethe ansatz approach yields solutions parameterized by

$$\gamma = \frac{m}{\hbar^2} \frac{g_{1D}}{n_{1D}}$$

$$H_{1D} = \sum_{j=1}^{N} -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z_j^2} + \sum_{i<j} g_{1D} \delta(z_i - z_j)$$

Lieb & Liniger, Phys Rev 130 1605 (1963)

Wavefunctions and all other (local and non-local) properties are exactly calculable.

Maxim Olshanii, 1998: Adaptation to real atoms

$$\gamma = \frac{4a_{3D}}{a_{\perp}^2 n_{1D}} \left(1 - \frac{Ca_{3D}}{a_{\perp}}\right)^{-1}$$

$a_{3D} = 3D$ scattering length
$a_{\perp} = \text{transverse oscillator length}$
$C \approx 1.46$
The Lieb-Liniger limits

\( \gamma \gg 1 \)
Tonks-Girardeau gas

\( \gamma \ll 1 \)
mean field theory (Thomas-Fermi gas)

\[ \gamma \text{eff} \]

Integrable systems have \( N \) constants of motion
\[ \Rightarrow \text{they cannot thermalize} \]

Weiss

(no "diffractive" collisions)
Experimental 1D gases

For 1D: all energies $< \hbar \omega_{\perp}$; negligible tunneling
1D Evolution in a Harmonic Trap

Position (μm)

ms

0

5

10

-500 0 500

40 μm

1st cycle average

15τ  195 ms

30τ  390 ms

Quantum Newton’s Cradles
Steady-state Momentum Distributions

After dephasing (prethermalization), the 1D gases reach a steady state that is not thermal equilibrium.

Generalized Gibbs Ensemble- Rigol/Olshanii

Each atom continues to oscillate with its original amplitude

Lower limit: thousands of 2-body collisions without thermalization
What happens in 3D?

Thermalization is known to occur in ~3 collisions.

How does thermalization begin in a slightly non-integrable systems? Will it always eventually thermalize?
A lot of non-equilibrium physics can be studied with cold atoms.

When integrability is built into the interaction Hamiltonian, the system is robust against thermalization. But how robust?

Mean field integrability is fragile. When integrability is built into the interaction Hamiltonian, the system is robust against thermalization. But how robust?

From many diverse phenomena, perhaps universal behavior can be identified.