Consumption, saving and habit formation

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Abstract

In this paper, we consider models of habit formation and derive closed-form solutions for consumption and saving under certainty equivalence and uncertainty. We find that consumption depends not only on permanent income and income risk, but also on past consumption. Similarly, saving depends not only on future income changes and income risk, but also on past saving. © 1997 Elsevier Science S.A.

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1. Introduction

One of the main advantages of working with the rational expectations-permanent income hypothesis (PIH) is that it is possible to derive a closed-form solution for both consumption and saving. However, many empirical studies have been carried out to test the predictions of the PIH, and rejections have been found both in macro and micro data.\textsuperscript{1} A promising extension to the model has been proposed by Caballero (1990), who considers preferences that allow for a precautionary saving motive. What distinguishes the work of Caballero from other papers on precautionary saving is that he is still able to derive a closed-form solution for saving and consumption.

Several other alternatives to the basic model have been considered in the literature. Habit formation has been proposed by a few authors as a way to reconcile the theory with the empirical findings.\textsuperscript{2} In this paper, we introduce habit formation in models with and without a precautionary saving motive, and we show that we can still derive a closed-form solution for consumption and saving.

There are many advantages to using closed-form solutions. First, we are able to work with levels

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\textsuperscript{1} See Deaton (1992) and the survey by Browning and Lusardi (1996).
\textsuperscript{2} See Carroll, Overland and Weil (Carroll et al., 1994), and the discussion in Deaton (Deaton, 1992, Chap. 1). Models with habit formation have been used more extensively in the asset-pricing literature to try to resolve the equity premium puzzle; see, for example, Abel (1990); Constantinides (1990); Campbell and Cochrane (1995).

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rather than first differences. This allows a better understanding of some of the issues concerning saving and consumption. Second, given the data sets at our disposal and the extent of measurement error in micro data, there are advantages in estimating the predictions of closed-form solutions rather than estimating Euler equations. Third, we are able to obtain a rich specification that extends some of the previous results in this literature.

The paper is organized as follows: In Section 2, we consider a model of habit formation under certainty equivalence. In Section 3, we extend our results to a model with precautionary saving. In Section 4, we provide some concluding remarks.

2. Habit formation under certainty equivalence

We consider a model in which the consumer maximizes the following utility function:

$$\max E_t \sum_{t=1}^{\infty} (1 + \rho)^{t-\tau} u_t(c_{\tau} - \gamma c_{\tau-1})$$

subject to an intertemporal budget constraint:

$$\sum_{t=1}^{\infty} (1 + r)^{t-\tau} c_{\tau} = (1 + r)A_{t-1} + \sum_{t=1}^{\infty} (1 + r)^{t-\tau} y_{\tau}$$

$$A_{t-1} \text{ given, } c_{t-1} \text{ given}$$

where $E_t$ is the expectations operator, $c_{\tau}$ indicates consumption in period $\tau$, $y_{\tau}$ is non-capital income, $A_{\tau}$ is non-human wealth, $r$ is real interest rate (which is assumed to be fixed) and $\rho$ indicates the rate of time preference. We assume a simple form of habit formation and model utility as depending not only on consumption at time $t$, but also on consumption at time $t-1$.

3. We show hereafter that, under some conditions, it is possible to derive a closed-form solution for consumption and saving for this habit-formation model.

Define $c^*_t = c_t - \gamma c_{t-1}$. We can solve the model by expressing everything in terms of $c^*_t$. We can re-write the intertemporal utility function as follows:

$$\max E_t \sum_{t=1}^{\infty} (1 - \rho)^{t-\tau} u_t(c^*_t)$$

Note that the utility function is additively separable in $c^*_t$. Due to the assumption of an infinite planning horizon, by substituting in $c^*_t$ for $c_{\tau}$ in Eq. (1b), the intertemporal budget constraint becomes:

$$\sum_{t=1}^{\infty} (1 + r)^{t-\tau} c^*_t = -\gamma c_{t-1} + \frac{(1 + r - \gamma)}{(1 + r)} \left( (1 + r)A_{t-1} + \sum_{t=1}^{\infty} (1 + r)^{t-\tau} y_{\tau} \right)$$

As in Campbell (1987), we also assume that the intra-temporal utility function is quadratic, and that

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3. See also Deaton (Deaton, 1992, p. 30). We consider a simple specification of habit formation, but our results can be generalized to the case of an intra-temporal utility with $n$ lagged consumption terms. Proofs for this general case are available from the authors upon request.
the interest rate is equal to the rate of time preference. Given these assumptions and given the additive nature of the optimization problem, it is easy to show that the closed-form solution for consumption is:

\[ c_t = \frac{\gamma}{(1 + r)} c_{t-1} + \left(1 - \frac{\gamma}{(1 + r)}\right) Y_{pt} \]  

(3)

where

\[ Y_{pt} = \frac{r}{(1 + r)} \left((1 + r)A_{t-1} + \sum_{\tau=t}^{\infty} (1 + r)^{\tau-t} E_t y_\tau\right) \]  

(3a)

Eq. (3) shows that in the case of no habits (\(\gamma = 0\)), consumption is equal to permanent income (\(Y_{pt}\)), which is simply the annuity value of lifetime resources. When there is habit formation (\(\gamma > 0\)), the closed-form solution for consumption is basically a weighted average of past consumption and permanent income. The stronger the habit, the more weight will be put on past consumption.

We can rewrite the model in terms of saving rather than consumption. Campbell (1987), for example, rewrites the predictions of the model in terms of saving and he shows that, under the PIH, saving is equal to the present discounted value of future income changes. With habit formation, the saving equation has the following form:

\[ s_t = \gamma s_{t-1} + \frac{\gamma}{(1 + r)} \Delta y_t - \left(1 - \frac{\gamma}{(1 + r)}\right) \sum_{\tau=t}^{\infty} (1 + r)^{\tau-t} E_t \Delta y_\tau \]  

(4)

Saving depends on past saving and on a convex combination of current income changes and the discounted value of future income changes. Again the weights depend on the importance of habits. The stronger the habit, the lower the importance of future income changes (and the higher the importance of past saving). Note that with no habits (\(\gamma = 0\)), we get back the simple ‘saving for a rainy day’ equation, where saving depends only on expected future income changes.

3. Habit formation and precautionary saving

So far, we have assumed that preferences are quadratic, and, therefore, uncertainty has no effect. We now extend the model and consider the case where there is a precautionary saving motive. We follow Caballero (1990) who uses a negative exponential utility function. He considers the case where non-capital income is the only source of uncertainty (the return on assets is certain and is set equal to the rate of time preference) and follows any ARMA process. In other words, the process of non-capital income can be represented by a moving-average process with \(\psi_i\) representing the \(i^{th}\) MA coefficient. Hence

\[ E_t y_\tau - E_{t-1} y_\tau = \psi_{t-1} w_\tau \]  

(5)

\(^4^\) We derive the first-order condition for consumption (\(c^*_t\)) and use it in the intertemporal budget constraint.

\(^5^\) There are several advantages to working with saving rather than consumption. For a discussion and application using aggregate or micro data, see Campbell (1987); Alessie and Lusardi (1995).
\[ \psi_0 = 1, \ \sum_{t=1}^{\infty} (1+r)^{i-t} \psi_t c_t < \infty, \] and \( w_t \) an i.i.d. innovation disturbance. Given these assumptions, Caballero proves that:

1. the stochastic process of consumption is a martingale with drift;
2. the disturbance of the stochastic process of consumption is equal to the annuity value of the contemporaneous innovation in income;
3. the consumption function can be decomposed, additively, in two terms: a first term analogous to that of the certainty equivalence case, and a second term which measures precautionary saving and which depends on the properties of the income process.

We extend Caballero’s model by allowing for habit formation. We write the utility function as follows:

\[
\max E_t \sum_{\tau=t}^{\infty} (1+r)^{i-\tau} \left( -\frac{1}{\theta} e^{-\theta(c_t - \gamma c_{t-1})} \right)
\] (6)

The maximization is subject to the same constraints seen previously, i.e. the constraint in Eq. (1b). We apply the same transformation as before and redefine the problem in terms of \( c_t^g = c_t - \gamma c_{t-1} \).

Using the same method as Caballero (1990), it can be shown that in a precautionary saving model with habit formation the following results obtain:\(^6\)

(1a) The stochastic process of \( c_t^g \) is a martingale with drift:

\[
c_t^g = c_t^g + \Gamma_{t-1} + \nu_t
\] (7)

where \( \nu_t \) denotes the innovation in consumption \( c_t^g \) and \( \Gamma_{t-1} \), measures the effect of precautionary saving. Note that the latter term depends on the properties of income shocks. For example, \( \Gamma_{t-1} \), is increasing with the riskiness and persistence of income shocks. In the simple case where shocks are normally distributed, \( \Gamma_{t-1} \), becomes a function of the variance of income.

(2a) The relation between the consumption innovation \( \nu_t \) and the income innovation \( w_t \) can be written as follows:

\[
\nu_t = \psi^* w_t
\] (8)

where

\[
\psi^* = \left( 1 - \frac{\gamma}{1+r} \right) \frac{r}{(1+r)} \sum_{i=0}^{\infty} \psi_i (1+r)^{-i}
\] (8a)

Eqs. (8) and (8a) say that, if there is no habit formation (\( \gamma = 0 \)), then the consumption innovation is equal to the annuity value of the contemporaneous innovation in income. In the presence of habit

\(^6\) The solution procedure goes as follows: we make a guess on the form of the stochastic process of consumption \( c_t^g \) and use the Euler equation to pin down the functional form of that process. We have to make some assumptions about the stochastic process of income, and, as mentioned above, we consider a moving average representation of income and model shocks as i.i.d. innovation disturbances. We then use the intertemporal budget constraint and solve for consumption.
formation, the consumption innovation is equal to \[1-(\gamma/(1+r))\] times the revision in permanent income, i.e. consumption is less sensitive to income shocks than in the case of no habits.

(3a) The consumption function corresponding to this optimization problem has the following form:

\[c_t = \frac{\gamma}{(1+r)} c_{t-1} + \left(1 - \frac{\gamma}{(1+r)}\right) Y_{pt} - \frac{r}{(1+r)} \sum_{\tau=t+1}^{\infty} (1+r)^{t-\tau} \sum_{j=t+1}^{\tau} I_{j-1}\]  

(9)

and

\[I_{j-1} = \frac{1}{\theta} \ln E_{j-1} \exp(-\theta\psi^* w_j)\]  

(9a)

\(\psi^*\) is defined in Eq. (8a). Eq. (9) is similar to the previous Eq. (3). Consumption depends on permanent income and on past consumption, but there is an additional third term that measures the precautionary saving motive. As in Caballero (1990), the closed-form solution is additive and precautionary saving depends on the properties of income risk. Note, however, that the parameter \(\gamma\) affects the relative importance of all three terms. In particular, habit formation affects the size of the precautionary saving term. As shown in Eq. (9a) the stronger the habit, the lower the effect of income uncertainty on consumption.

As before, we can rewrite the model in terms of saving rather than consumption. Saving has the following form:

\[s_t = \gamma s_{t-1} + \frac{\gamma}{(1+r)} \Delta y_t - \frac{r}{(1+r)} \sum_{\tau=t+1}^{\infty} (1+r)^{t-\tau} E_{t} \Delta y_t + \frac{r}{1+r} \sum_{\tau=t+1}^{\infty} (1+r)^{t-\tau} \sum_{j=t+1}^{\tau} I_{j-1}\]  

(10)

In the precautionary saving model with habit formation, saving depends on past saving, on current and future income changes, and on the properties of the income process. The size of each coefficient depends critically on the strength of habit, i.e. on the size of \(\gamma\).

4. Concluding remarks

In this paper we have shown that, using a simple model of habit formation, we are able to derive a closed-form solution for consumption and saving in the case of certainty equivalence and uncertainty. While we have to use some restrictive assumptions in our derivation, the advantages of using closed-form solutions are many. For example, we are able to assess how habit formation affects the level of consumption and saving. Furthermore, we obtain a richer specification, which extends some of the previous results. Our solutions indicate that consumption depends not only on permanent income and income risk, but also on past consumption. Similarly, saving depends not only on current and future income changes and income risk, but also on past saving. These models are a promising extension and may be able to reconcile some of the ‘puzzles’ of the empirical evidence on consumption and saving.
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References