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HOUSEHOLD PRODUCTION AND CONSUMER DEMAND FUNCTIONS

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This paper considers the hypothesis that commodities purchased on the market by consumers are inputs into the production of goods within the household. Its implications for the family of consumer demand functions whose arguments are real income and relative prices are drawn and compared with those of the hypothesis of additive separability. The paper closes with some examples of differences in commodity demand elasticities which are qualitatively consistent with the household production hypothesis and some comments upon how the latter might be utilized in empirical work.

DESPITE THE attention which economists have so lavishly given it, the theory of consumer behavior has disappointingly few implications for empirical research. For this reason, the typical demand study proceeds largely unaided by theoretical considerations, and such studies rarely include more than one or two prices of apparently related commodities in addition to income and own price in the demand function to be estimated. Stimulated by the pioneering paper of Strotz [8], there has been a heartening tendency in recent years for some economists to consider restrictions in the form of various "separability" conditions on the consumer's utility function.[1] Separability means that marginal rates of substitution for certain pairs of commodities are functionally independent of the quantities of certain other commodities.[2] Restrictions such as these reduce the number of parameters that enter into the family of demand functions of a consumer and thus make estimation of all the demand function parameters more feasible.

In this paper I suggest the hypothesis that commodities purchased on the market by consumers are inputs into the production of goods within the household. Such production is characterized by conventional production functions. The goods produced, in turn, are arguments of a conventional utility function of the household. This common-sense view of the household is not particularly new, but its implications to my knowledge have never been worked out to any great extent. It was suggested to me in essentially two distinct ways. In studies of housing I have been making, it seemed both natural and convenient to view home owners as landlords who produce and sell housing services to themselves as tenants. Because of its usefulness in this particular area, I wondered whether other aspects of consumer demand might not be treated in the same way. In a totally unrelated project, I made some crude estimates of the income elasticity of expenditures for automobile operation and found values almost the same as the income elasticity of automobile demand found by Chow [3]. It occurred to me then that this might be the case if

[1] I was greatly stimulated in my own thinking by the papers of Strotz and Pearce [6].

[2] The notion of separability is discussed, clarified, and characterized by Goldman and Uzawa [4]. Their bibliography contains most of the pertinent citations.

automobiles and expenditures for automobile operation were inputs into the production of automobile services by the household.

The characterization of the household suggested here yields a utility function which is weakly separable when viewed as a function of commodities purchased on the market. However, viewing commodities purchased as inputs into household production functions suggests further sensible restrictions on the household's utility function. The most important is the tautological but highly fruitful condition that the production function for any good is homogeneous of degree one in all the relevant inputs, including labor used within the household. The latter assumption greatly reduces the number of consumer demand function parameters. It also permits separate study by the usual methods of demand analysis for goods such as food and housing, on the one hand, and the demand for commodities used to produce a particular good, such as food, without reference to income and to the prices of commodities used to produce other goods, on the other. While similar separations in demand analyses have been made many times and justified intuitively, the hypothesis suggested here implies that separation in the way described above involves no approximation error for infinitesimal changes when using conventionally weighted indexes of price and quantity relatives.

The implications of this analysis are similar to those of Strotz's **[8]** in that total money expenditure on a group of commodities depends only upon money income and indexes of money prices of commodities in each of the commodity groups. Strotz's conclusion follows from what he later characterized as the Charbydis of additivity of branch utility functions [9, p. 485], mine from the Scylla of their homogeneity.[3] In other respects, however, the implications of additivity and homogeneity are different. Provided only that the utility function is weakly separable and quasi-concave, the demand function for a commodity in any particular group may be written as a function of real income, the relative prices of all commodities in the particular group, and relative price indexes of all other commodity groups without approximation error for infinitesimal changes.[4] The appropriate price index weights involve the income elasticities of demand for all commodities as well as relative expenditures on them. In addition, the demand for a composite commodity which is an expenditure weighted average of quantity relatives of all commodities in a given group can be expressed in terms of real income and relative price indexes of all commodity groups only.

Additive separability implies that all the other commodity group relative price indexes can be combined into a single index, both for the composite group commodity and for any individual commodity in the group. However, the relative change

[3] Gorman [5] has demonstrated that, for separable utility functions, the result stated in the first sentence of this paragraph holds only under additivity, homogeneity, or a combination thereof.

[4] As noted by Pearce [6, p. 512], index number problems are simplified when the demand function is written in terms of real income and relative prices.

in quantity demanded in response to the change in the index of all other prices will be different for the different commodities in the group. For this reason, knowledge of the change in quantity of, say, food demanded, says little about the change in the demand for hamburger. If weak separability plus homogeneity is assumed, on the other hand, separate price indexes are required for each of the commodity groups in both individual and composite commodity demand functions. In contrast to additivity, the relative change in quantity demanded which results from a change in any other group price index is the same for all commodities in the given group, and hence the same for the composite commodity as for any individual one.

I

Let the household possess a utility function $U = U(x_1, \ldots, x_G)$ with the usual properties of differentiability and curvature, where x_g is the gth good, such as food, nourishment, or protein. Also let the household possess G production functions $x_g = x_g(y_{g1}, \ldots, y_{gn_g})$, where the y_{gi} are commodities, such as semi-processed food, raw meat or raw hamburger, generally purchased on the market by the household and used by it in the production of goods. (In what follows I shall generally designate commodities with a single subscript and use the phrase "j in g" to mean the commodity y_j is an input into the production of good x_g.) The functions x_g are assumed to be homogeneous of degree one and also to possess the usual differentiability and curvature properties of production functions.

Since $U_i = U_g x_{g,i}$ for i in g, where $U_i = \partial U/\partial y_i$ and $x_{g,i} = \partial x_g/\partial y_i$,

$$(U_i/U_j) = (x_{g,i}/x_{g,j}), \quad i, j \text{ in } g$$

and

$$(U_i/U_j) = (U_g/U_h)\,(x_{g,i}/x_{h,j}), \quad i \text{ in } g,\ j \text{ in } h \neq g.$$

Thus, for any pair of commodities used to produce the same good, the marginal rate of substitution in consumption as usually defined is merely the marginal rate of substitution in household production of the good. As such, it is functionally independent of all commodities not used in the production of good g by the household. The conventionally written utility function $U = U(y_1, \ldots, y_j)$ is thus weakly separable in the sense of Goldman and Uzawa [4].[5]

The elasticities of the consumer demand functions can be readily derived. Since the functions x_g are homogeneous of degree one, equiproportional increases in all j in g lead to the same percentage increase in the output of good g and leave all

[5] I note that, by viewing groups of commodities as inputs into household production, one would not expect the utility function to be separable in Pearce's sense [6], namely that the marginal rate of substitution between a pair of inputs used to produce g is also functionally independent of the rate of inputs of all other commodities used to produce g. The substitutability of better stoves for household labor in producing food, for example, is likely to depend upon the relative quantities of different raw or semi-processed food items purchased at the grocery.

marginal rates of substitution in production unchanged. Hence, where E_i stands for the income elasticity of demand for the commodity y_i,

$$(1) \qquad E_i = E_j = E_g\,, \quad \text{all } i,\, j \text{ in } g.$$

It is primarily this implication that permits the simplifications in the consumer demand function achieved below.

The Slutsky compensated or Friedman real-income-constant price elasticities, H_{ij} denoting the elasticity of demand for commodity i with respect to the price of commodity j, where i, j in g, follow immediately from the theory of demand for a factor of production. In the form presented by Allen [**1**, pp. 502–509],

$$(2) \qquad H_{ij} = s_{g,j}(S_{g,ij} + H_{gg})$$

where

$$(3) \qquad s_{g,j} = p_j y_j / \sum_{k \text{ in } g} p_k y_k = s_j / s_g$$

is the share of commodity j in producing good g, $S_{g,ij}$ is the partial elasticity of substitution of commodity i for commodity j in the production of good g, and H_{gg} is the own price elasticity of demand for good g. The first term of (2) results from the fact that a rise in the price of commodity j leads to a change in the least costly way to produce a given output of good g. The second arises because if p_j rises, good g becomes more expensive relative to other goods and, hence, relatively less of good g is produced and consumed by the household.

Where i in g, j in $h \neq g$, the compensated cross-partial demand elasticities follow immediately from Theorem 5 of Goldman and Uzawa [**4**]. They prove that for a weakly separable, quasi-concave utility function, the compensated cross-partial derivative

$$\frac{\partial y_{i,c}}{\partial p_j} = K^{gh} \frac{\partial y_i}{\partial I} \frac{\partial y_j}{\partial I}$$

where I denotes income and K^{gh} is defined for all $g \neq h$ and is a function of the quantities of the various commodities constituting the initial position. Of course, $K^{gh} = K^{hg}$.

In elasticity form,

$$(4) \qquad H_{ij} = s_j E_i E_j \left(\frac{K^{gh}}{I}\right) = s_j E_g E_h \left(\frac{K^{gh}}{I}\right),$$

the latter following from (1). Thus, the compensated cross-partial elasticities for i in g, j, k in $h \neq g$ differ only in the relative importance of commodities j and k in producing good h. That this should be the case can best be understood by noting that, where an asterisk means the natural log of the variable so designated,

$$(5) \qquad \sum_{j \text{ in } h} H_{ij} dp_j^* = \sum_{j \text{ in } h} s_j E_g E_h \left(\frac{K^{gh}}{I}\right) dp_j^*$$

$$= s_h E_g E_h \left(\frac{K^{gh}}{I}\right) \sum_{j \text{ in } h} s_{h,j} dp_j^* = H_{gh} dp_h^* .$$

In (5), the cross-partial elasticity of demand of good g with respect to the price of good h is of exactly the same form with respect to goods as H_{ij} is for commodities. The price of good h is defined naturally as a conventional index of price relatives using expenditure weights. As so defined, H_{gh} is independent of the composition of relative price changes of commodities used to produce good h and depends only upon their weighted total.

The demand function for commodity i in g can be written

$$dy_i^* = E_g dI_r^* + \sum_{j=1}^{J} H_{ij} dp_j^*$$

$$= E_g dI_r^* + \sum_{j \text{ in } g} H_{ij} dp_j^* + \sum_{j \text{ not in } g} H_{ij} dp_j^* ,$$

where $dI_r^* = dI^* - \Sigma_{j=1}^{J} s_j dp_j^*$ measures changes in real income as conventionally defined.

Considering only the contribution of prices of commodities used in producing good g,

$$(6) \qquad \sum_{j \text{ in } g} H_{ij} dp_j^* = \sum_{j \text{ in } g} s_{g,j} (S_{g,ij} + H_{gg}) dp_j^*$$

$$= H_{gg} dp_g^* + \sum_{j \text{ in } g} s_{g,j} S_{g,ij} dp_j^* .$$

Letting $dp_{g,j}^* = dp_j^* - dp_g^*$, the second term in (6) can be rewritten

as $\Sigma_{j \text{ in } g} s_{g,j} S_{g,ij} dp_{g,j}^*$ since $\Sigma_{j \text{ in } g} s_{g,j} S_{g,ij} = 0$.

Using (5) and (6) the demand function for commodity i becomes

$$(7) \qquad dy_i^* = E_g dI_r^* + \sum_{h=1}^{G} H_{gh} dp_h^* + \sum_{j \text{ in } g} s_{g,j} S_{g,ij} dp_{g,j}^* .$$

The demand function for commodity i therefore depends only upon real income, the relative prices of the G goods, and the ratios of relative prices of all commodities used to produce good g to the price of good g. The number of parameters of the family of demand functions such as (7) is thus drastically reduced from the total of all parameters of the family of demand functions for all commodities when written in its most general form. Moreover, the following section suggests a comparatively simple two-stage procedure for estimating this reduced number of parameters.

II

Since it is assumed that the function x_g is homogeneous of degree one, it is quite natural to define the quantity of the gth good consumed as[6]

$$x_g = \sum_{j \text{ in } g} x_{g,j} y_j = \frac{1}{p_g} \sum_{j \text{ in } g} p_j y_j$$

or,

$$dx_g^* = d(p_g x_g)^* - dp_g^* = \sum_{j \text{ in } g} s_{g,j} dy_j^* . \tag{8}$$

The relative change in the gth good is thus the same weighted average of relative commodity changes as used for computing the price index for the gth good, and so is readily obtainable empirically. In performing the summation of (7) which is indicated by (8), the first two terms reproduce exactly since they are the same for all j in g and $\Sigma_{j \text{ in } g} s_{g,j} = 1$. The weighted summation of the third term in (7) is

$$\sum_{j \text{ in } g} s_{g,j} \sum_{k \text{ in } g} s_{g,k} S_{g,jk} dp_{g,k}^* = \sum_{k \text{ in } g} s_{g,k} \Big(\sum_{j \text{ in } g} s_{g,j} S_{g,kj} \Big) dp_{g,k}^* = 0$$

because of the symmetry of the partial elasticity of substitution and because the weighted sum in parentheses is equal to zero. The demand function for the good g is therefore

$$dx_g^* = E_g dI_r^* + \sum_h H_{gh} dp_h^* . \tag{9}$$

On the hypothesis I am suggesting, then, the common-sense and commonly used procedure of grouping and applying the usual demand formula to analyzing group demand involves no error of approximation for infinitesimal changes when using conventional price and quantity index numbers. (A specification error may exist, of course, if the group is incorrectly specified.) An approximation error, or index number problem, still exists for finite changes. If one knows the parameters of (9) and is willing to assume that tastes for goods and production possibilities for combining commodities into good are unchanged, one can correctly revise the general price index number weights since,

$$ds_g^* = dx_g^* + dp_g^* - dI_r^* = (E_g - 1) dI_r^* + (1 + H_{gg}) dp_g^* + \sum_{h \neq g} H_{gh} dp_h^* . \tag{10}$$

Since (9) is likely to contain relatively few parameters of substantial practical importance, relatively frequent revision of a general price index would be feasible

[6] It might be objected here that, since the household is at once a monopolistic buyer from and seller to itself, a bilateral monopoly problem is involved in price determination and hence the measurement of quantity. But since there are no barriers to vertical integration, any difficulties which result from my schizophrenic assumption relate only to division of the gains and not the proper price to charge. The former presents no problems for this analysis since it is not concerned with who shall wash the dishes but only with how many dishes to use and how to wash them.

without recourse to expensive consumer expenditure surveys. As equation (12) below demonstrates, this is also true for the group price indexes.

The first two terms of the individual commodity demand function, (7), are seen from (9) to be the quantity of good g demanded, so substitution of (9) into (7) yields

$$dy_i^* = dx_g^* + \sum_{j \text{ in } g} s_{g,j} S_{g,ij} dp_{g,j}^*$$

or

$$(11) \qquad d(y_i/x_g)^* = \sum_{j \text{ in } g} s_{g,j} S_{g,ij} dp_{g,j}^* \, .$$

The second component of the demand for any individual commodity, the third term of (7), is the differential change in demand for commodity i relative to good g which results from a change in the least costly way to produce good g. The latter, of course, depends only upon the relative prices of commodities used in the production of good g. Thus, having estimated the appropriate group demand function (9), one can then complete the analysis of the demand for any particular commodity without reference to income or to the price of any commodity not used in the production of good g. Again, this can be done with no approximation error using conventional index numbers. To study the demand for meat, for example, as a first stage, one would study the demand for food or protein. Using the first stage estimates, the second stage would be to study the demand for meat relative to, say, protein using only price indexes of meat, fish, fowl, and dairy products as explanatory variables. The group index weights, like the general price index weights, can be readily revised knowing the relevant parameters since

$$(12) \qquad \begin{aligned} ds_{g,i}^* &= (dp_i^* - dp_g) + (dy_i^* - dx_g^*) \\ &= s_{g,i}(1 + S_{g,ii}) dp_{g,i}^* + \sum_{\substack{j \text{ in } g \\ j \neq i}} s_{g,j} S_{g,ij} dp_{g,j}^* \, . \end{aligned}$$

III

The degree of simplification of demand analysis achieved above depends heavily, though not entirely, on the implication of linear homogeneity that income elasticities of demand are the same for all commodities j used to produce good g. Indeed, while casual observation and common sense might be helpful in grouping commodities properly, such grouping might be most easily done objectively on the basis of similarity among estimated income elasticities. At first glance, however, it would seem that the implication of equality of income elasticities within commodity group is readily contradicted by available evidence.[7] Reid argues in her study of housing [7] that the income elasticity of demand for houses exceeds that for house-

[7] Strotz [9] and Gorman [5] are very suspicious of homogeneity, presumably for this reason.

hold utilities, and one might argue that both are used to produce housing. Similarly, Burstein [2] has estimated the income elasticity of demand for household refrigeration at between +1 and +2, which is certainly greater than that for food purchased for use in the home, yet both can be seemingly interpreted as commodities used in the production of food served in the home.

Many such seeming inconsistencies can readily be disposed of by noting that the demand for some commodities purchased by the household is really a composite demand. The household's demand for fuel, say, is a composite of the derived demand for fuel for cooking and fuel for heating, or for fuel used to produce food and fuel used to produce housing. In cases such as these, the commodity inputs y_j used in the preceding equation should be interpreted as commodities used for a particular purpose. Letting

$$y_i' = \sum_{g=1}^{H} y_{gi}, \qquad H \leqslant G,$$

where y_i' is total fuel used by the household, say, y_{g1} is fuel used for cooking, and y_{g2} is fuel for heating,

$$dy_i'^* = \sum_{g=1}^{H} s_{i,g} dy_{gi}^*,$$

where

$$s_{i,g} = s_{gi} / \sum_{h=1}^{H} s_{hi} .$$

Hence,

$$\frac{\partial y_i'^*}{\partial I_r^*} = \sum_{g=1}^{H} s_{i,g} E_g . \tag{13}$$

It follows from (13) that if fuel is used to produce both food and housing and the income elasticity of demand for food served in the home is less than that for housing, the income elasticity of demand for housing will exceed that for all fuel.

Other seemingly inconsistent income elasticities may be explained by noting that household labor is an important input into the household production process. Food served in the home is an obvious example. As typically measured, however, income elasticities include the confounded effects of an increase in opportunities open to the household and an increase in the price of household labor. If the price of household labor is not held constant, then from (11)

$$\frac{\partial (y_i/x_g)^*}{\partial I_r^*} = s_{g,L} S_{g,iL} \frac{\partial p_L^*}{\partial I_r^*}, \tag{14}$$

where $\partial p_L^* / \partial I_r^* > 0$, represents the differential effect on the income elasticity of the ith commodity as usually measured. Now, one would certainly expect refrigerators as well as stoves and certain other appliances to be substitutes for household labor in the production of food served in the home or that $S_{g,iL} > 0$ for refrigerators. On the other hand, it seems quite possible that $S_{g,iL} < 0$ for food purchased for preparation in the home, since it is reasonable to anticipate that an increase in the amount of purchased food would increase the marginal physical productivity of household labor used in food preparation. Under these conditions, the income elasticity of refrigerator demand would be greater than, and that for purchased food less than, the income elasticity of demand for food served at home as these commodity elasticities are usually measured.[8]

Thus, by recognizing the composite nature of demand for many commodities and the importance of household labor in production in the home, many seemingly inconsistent demand elasticities might be explained. Indeed, one of the principal uses of the way of looking at consumer demand I am suggesting would be to explain differences in measured demand elasticities of different commodities. However, it is quite possible that, even after taking composite demand and household labor into account, the income elasticity of demand for evening clothes might still exceed that for blue jeans, to use Gorman's [5, p. 475] example. To explain differences such as these using the model I am suggesting, one might introduce an additional good "ostentation" or "luxury," which, presumably, would be a part of the composite demand for evening clothes but not for blue jeans. In like manner, to explain differences in income elasticities of demand for, say, hamburger and beefsteak, one could suppose that the demand for beefsteak is a composite of the derived demands for nourishment and palatability. Presumably, the finer the breakdown of commodities the more such composite demands would have to be introduced, since for all semi-processed food purchased at retail the distinction between nourishment and palatability tends to cancel. And, of course, the finer the breakdown of commodities the greater the needed number of goods in general. When considering all semi-processed food purchased at retail, a single good nourishment might suffice; with a finer breakdown one might need several categories of nourishment such as protein, carbohydrate, etc.

Now it will probably be objected at this point that, by defining a sufficient number of goods and interpreting the proviso that the function x_g is homogeneous of degree one in *all* relevant inputs sufficiently broadly, the analysis becomes purely tautological. I readily grant the truth of the statement but not the force of the objection. Tautologies are exceedingly useful if they lead to a simplification of theory in application. The relevant consideration is whether using a sufficient number of tautological definitions of goods and inputs enables one empirically to

[8] The fact that the income elasticity of demand for food in restaurants appears to exceed that for food served at home may well result both from composite demand and a differential effect operating through the price of household labor. Food eaten away from home provides both nourishment and entertainment, the latter probably having a higher income elasticity, and food eaten away from home is less household labor intensive than food eaten at home.

estimate families of consumer demand functions more accurately and easily than by using other models and their attendant tautologies.

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REFERENCES

[1] ALLEN, R. G. D.: *Mathematical Analysis for Economists*, London: MacMillan and Co., Ltd., 1950.

[2] BURSTEIN, MEYER L.: "The Demand for Household Refrigeration in the United States," Arnold C. Harberger (ed.), *The Demand for Durable Goods*, Chicago: University of Chicago Press, 1960, pp. 99–145.

[3] CHOW, GREGORY C.: *The Demand for Automobiles in the United States: A Study in Consumer Durables*, Amsterdam: North-Holland Publishing Co., 1957.

[4] GOLDMAN, S. M., AND H. UZAWA: "A Note on Separability in Demand Analysis," *Econometrica*, Vol. 32 (1964), pp. 387–98.

[5] GORMAN, W. M.: "Separability and Aggregation," *Econometrica*, Vol. 27 (1959), pp. 469–81.

[6] PEARCE, I. F.: "An Exact Method of Consumer Demand Analysis," *Econometrica*, Vol. 29 (1961), pp. 499–516.

[7] REID, MARGARET G.: *Housing and Income*, Chicago: University of Chicago Press, 1962.

[8] STROTZ, R. H.: "The Empirical Implications of a Utility Tree," *Econometrica*, Vol. 25 (1957), pp. 269–80.

[9] ———: "The Utility Tree—A Correction and Further Appraisal," *Econometrica*, Vol. 27 (1959), pp. 482–88.