ESTIMATING ELASTICITIES WITH THE LINEAR APPROXIMATE ALMOST IDEAL DEMAND SYSTEM: SOME MONTE CARLO RESULTS

Julian M. Alston, Kenneth A. Foster and Richard D. Green*

Abstract—The AIDS model has many desirable theoretical properties but usually it is estimated using a linear approximation. The quality of the approximation to the true AIDS depends on the parameters and the collinearity among the exogenous price variables. In the literature four alternative formulas have been used to compute elasticities of a demand system that is assumed to be of the AIDS form, using parameters estimated in the linear approximate AIDS. Monte Carlo experiments indicate that two of those four alternatives, as typically applied, are highly inaccurate but the other two are quite accurate.

The Almost Ideal Demand System (AIDS) of Deaton and Muellbauer is one of the most widely used flexible demand specifications. While the AIDS possesses many desirable properties, it may be difficult to estimate. To simplify the estimation problem, Deaton and Muellbauer (p. 316) suggested using a linear approximation. The linear approximate almost ideal demand system (LA/AIDS) has been employed in the vast majority of empirical applications of the AIDS model, with a variety of formulas to compute elasticities. This paper investigates how well the AIDS price elasticities are estimated using the LA/AIDS by conducting Monte Carlo experiments in which data are generated by the AIDS.

I. The AIDS and LA/AIDS Models

The AIDS budget share equations for estimation are given by (e.g., Deaton and Muellbauer)

\[ w_{it} = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_{jt} + \beta_i \ln(x_i/P_t) + u_{it}, \]  

where, in time t, \( w_{it} \) is the budget share of good \( i \), \( u_{it} \) is a random disturbance, \( p_{jt} \) is the price of commodity \( j \), \( x_i \) is total expenditure and \( P_t \) is a price index defined by

\[ \ln P_t = \alpha_0 + \sum_{k=1}^{n} \alpha_k \ln p_{kt} \]

+ \[ \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} \gamma_{jk} \ln p_{jt} \ln p_{kt}. \]

Adding up, homogeneity, and symmetry conditions are

(a) \( \sum_i \alpha_i = 1 \), \( \sum_j \gamma_{ij} = 0 \), \( \sum \beta_j = 0 \); and

(b) \( \gamma_{ij} = \gamma_{ji} \). Using the price index in (2) may make the estimation of the AIDS difficult. Thus, Stone's price index \((P^*)\) is often used instead of \( P \) where

\[ \ln P^*_t = \sum_{k=1}^{n} w_{kt} \ln p_{kt}. \]

The resulting linear approximate almost ideal demand system (LA/AIDS) is not an integrable demand system in general. Its widespread popularity appears to be based on the fact that it is comparatively easy to estimate, combined with a belief that it is a reasonably good approximation to the true AIDS. The purpose of this paper is to evaluate whether that belief is justified. This hinges in part on how well Stone's index approximates the AIDS price index. The relationship between the two price indexes may be represented as

\[ \ln P_t = \ln P^*_t + \xi_t, \]  

where \( \xi_t \) is a random variable with \( E(\xi_t) = \xi_0 \). Using \( P^*_t \) instead of the unobservable \( P_t \) causes an errors in variables problem and the estimates of the AIDS parameters (\( \gamma_{ij} \) and \( \beta_i \)) obtained by SUR or OLS will be inconsistent. This can be seen in the LA/AIDS estimating equation

\[ w_{it} = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_{jt} + \beta_i \ln(x_i/P^*_t) + u_{it}, \]  

with \( u_{it}^* = u_{it} - \beta_i(\xi_t - \xi_0) \) and \( \alpha_i^* = \alpha_i - \beta_i \xi_0 \) and \( \text{cov}(u_{it}^*, \ln P^*_t) \neq 0 \).

Equations (1) through (4) can be solved to express \( \xi_t \) as a function of parameters, prices, total expenditure, and the disturbances in the AIDS model:

\[ \xi_t = \alpha_0 \left(1 + \sum_{k=1}^{n} \beta_k \ln p_{kt}\right) \]

+ \[ \sum_{k=1}^{n} \beta_k \ln p_{kt} \left(\sum_{k=1}^{n} \alpha_k \ln p_{kt} - \ln x_i\right) \]

- \[ \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} \gamma_{jk} \ln p_{jt} \ln p_{kt} \]

\times \left[1 - \sum_{k=1}^{n} \beta_k \ln p_{kt}\right] - \sum_{k=1}^{n} u_{kt} \ln p_{kt}. \]  

There would be no problem if \( \xi_t \) were a constant, but from equation (6) it is clear that \( \xi_t \) can never be constant since the disturbances from all of the equations influence it; even the expected value of \( \xi_t \) need
not be constant, because it depends on the values taken by prices.

What are the determinants of the quality of the approximation? Suppose we ignore the disturbance component and conduct the argument "on average." Deaton and Muellbauer (p. 316) asserted that the approximation would be better if prices of the goods making up the group were highly collinear, but they provided no analytic results to justify the claim. Minimally one needs perfect collinearity of the form

$$\sum_k \beta_k \ln p_{kt} = 0$$

and

$$\sum_k \gamma_k j \ln p_{kt} = 0$$

to produce

$$E(\ln Qt) = \alpha_0.$$ 

Similarly, in the special case of homothetic separability ($\beta_i = \gamma_{ij} = 0 \forall i, j$), $\xi_i = \alpha_0 - \sum_k u_{kt} \ln p_{kt}$, and $E(\xi_i) = \alpha_0$. Extrapolating from these results, as either $\beta_k$ or $\gamma_{kj}$ tend to zero, or the correlation among the prices of goods increases, we would expect the approximation of $\ln P_t$ by $\ln P_t^*$ to get better and our estimates of parameters and elasticities should improve. The Monte Carlo experiments below are designed to explore this suggestion.

II. Price Elasticities

A general definition of the uncompensated price elasticities of demand from the AIDS and LA/AIDS ($\epsilon_{ij}$), suppressing $t$ subscripts for the time being, is:

$$\epsilon_{ij} = \frac{d \ln Q_i}{d \ln p_j} = -\delta_{ij} + \frac{d \ln w_j}{d \ln p_j},$$

where these elasticities refer to allocations within the group holding constant total group expenditures ($x$) and all other prices ($p_k, k \neq j$), $\delta_{ij}$ is the Kronecker delta ($\delta_{ij} = 1$ for $i = j$; $\delta_{ij} = 0$ for $i \neq j$) and for the LA/AIDS we use $\ln P^*$ from (3) instead of $\ln P$ from (2).

Four alternative formulas for price elasticities using LA/AIDS parameter estimates have appeared in the literature. The differences can be represented in terms of different expressions for the elasticity of the group price index with respect to the $j^{th}$ price (i.e., $d \ln P^*/d \ln p_j$ in the AIDS or $d \ln P^*/d \ln p_j$ in the LA/AIDS). In the AIDS model,

$$\frac{d \ln P}{d \ln p_j} = \alpha_j + \sum_{k=1}^n \gamma_{kj} \ln p_k.$$

Substituting (8) into (7) yields:

$$\epsilon_{ij}(AI) = -\delta_{ij} + \frac{\gamma_{ij}}{w_i} - \frac{\beta_i}{w_i} \left( \alpha_j + \sum_{k=1}^n \gamma_{kj} \ln p_k \right).$$

Several authors have used this equation to compute elasticities with LA/AIDS parameter estimates. However, Green and Alston (1990) have shown that substituting

$$\frac{d \ln P^*}{d \ln p_j} = w_j + \sum_{k=1}^n w_k \ln p_k \left( \frac{d \ln w_k}{d \ln p_j} \right)$$

$$= w_j + \sum_{k=1}^n w_k \ln p_k \left( \epsilon_{kj} + \delta_{kj} \right),$$

into (7) yields $n^2$ simultaneous equations for uncompensated LA/AIDS demand elasticities:

$$\epsilon_{ij}(LA) = -\delta_{ij} + \beta_i \left( \frac{\epsilon_{ij}}{w_i} - \frac{\beta_i}{w_i} \right) X \left( w_j + \sum_{k=1}^n w_k \ln p_k \left( \epsilon_{kj} + \delta_{kj} \right) \right).$$

Another common approach (e.g., Chalfant) is to use a special case of this formula, which obtains when $d \ln P^*/d \ln p_j = w_j$:

$$\epsilon_{ij}(LA') = -\delta_{ij} + \frac{\epsilon_{ij}}{w_i} - \frac{\beta_i}{w_i} w_j.$$

Eales and Unnevehr used a further special case of equation (11) that is correct only when preferences are homothetic ($\beta_i = 0 \forall i$):

$$\epsilon_{ij}(LA") = -\delta_{ij} + \frac{\epsilon_{ij}}{w_i}.$$

Equation (9) (hereafter AI) is correct when the AIDS price index is used. Equation (11) (hereafter LA) is correct when the LA/AIDS has been estimated, so long as consistent LA/AIDS parameter estimates have been obtained. Equations (12) (hereafter LA') and (13) (hereafter LA") are approximations to the LA formula. Corresponding formulas for compensated price elasticities ($\epsilon^c_{ij}$) are obtained by using the Slutsky equation ($\epsilon^c_{ij} = \epsilon_{ij} + \beta_i w_j \epsilon^e_{i,x}$) and formulas for the expenditure elasticities ($\epsilon^e_{i,x}$). For the $AI$, $LA'$ and $LA"$, $\epsilon_{i,x} = 1 + \beta_i / w_i$ is used. Green and Alston's (1991) result is

$$E = A - (BC \epsilon + I),$$

where the typical elements are

$$a_{ij} = -\delta_{ij} + (\gamma_{ij} - \beta_i w_j) / w_i \in A (an \ n \times n \ matrix); b_i = (\beta_i / w_i) \in \hat{B} (an \ n \times 1 \ vector); c_j = \ln p_j \in C (a \ 1 \times n \ vector); and \ \epsilon_{ij} \in E (an \ n \times n \ matrix).$$

Solving for the elasticities $\epsilon_{ij}$ yields, after some simplifications:

$$E = \left[ BC + I \right]^{-1} \left[ A + 1 \right] - I. \ Buse has derived an equivalent solution that is simpler.
NOTES

353

used for the LA formula. The formulas for compensated elasticities are:

\[
\epsilon^c_{ij}(AI) = -\delta_{ij} + \frac{\gamma_{ij}}{w_i} + w_j - \frac{\beta_{ij}}{w_i} \left( \sum_{k=1}^{n} w_k \ln p_k - w_j \right). \tag{9'}
\]

\[
\epsilon^c_{ij}(LA) = -\delta_{ij} + \frac{\gamma_{ij}}{w_i} + w_j - \frac{\beta_{ij}}{w_i} \left( \sum_{k=1}^{n} w_k \ln p_k \left( \epsilon^c_{kj} + \delta_{kj} - w_j \right) \right). \tag{11'}
\]

\[
\epsilon^c_{ij}(LA') = -\delta_{ij} + \frac{\gamma_{ij}}{w_i} + w_j. \tag{12'}
\]

\[
\epsilon^c_{ij}(LA'') = -\delta_{ij} + \frac{\gamma_{ij}}{w_i} + w_j + \frac{\beta_{ij}}{w_i} w_j. \tag{13'}
\]

III. Experimental Design

Data were generated for three commodities allowing three different settings each for the degrees of (i) multicollinearity \((\rho_{ij})\), (ii) homotheticity \((\beta_i)\), and (iii) substitutability \((\gamma_{ij})\). The sets of parameter values were chosen to satisfy theoretical restrictions and to result in a typical range of elasticities as found in food demand studies.\(^2\) At each of the 27 design points, 1,000 replications were performed, each with 35 (and also 70) observations.

Prices were drawn from a multivariate normal distribution with a covariance matrix:

\[
\Sigma = \begin{bmatrix}
\sigma_{12}^2 & \rho_{12}\sigma_1\sigma_2 & \rho_{13}\sigma_1\sigma_3 \\
\rho_{12}\sigma_1\sigma_2 & \sigma_2^2 & \rho_{23}\sigma_2\sigma_3 \\
\rho_{13}\sigma_1\sigma_3 & \rho_{23}\sigma_2\sigma_3 & \sigma_3^2
\end{bmatrix} \tag{14}
\]

where the \(\sigma_i\) are standard deviations and the \(\rho_{ij}\) represent the simple correlations between the prices. The “high,” “medium” and “low” values used for the \(\rho_{ij}\) are \((\rho_{ij} = 0.99), (\rho_{ij} = 0.75)\) and \((\rho_{ij} = 0.50)\). The final price series were generated by factoring \(\Sigma\) (the price covariance matrix) into a matrix \(\Delta\) such that \(\Sigma = \Delta \Delta'\) and computing

\[
P_i = \Delta Z_i + \mu \tag{15}
\]

where the vector \(P_i\) represents the \(i^{th}\) observation on the prices, \(Z_i\) is a vector of standard normal random variables and \(\mu\) is vector of mean prices.

The “high,” “medium,” and “low” values of \((\beta_1, \beta_2, \beta_3)\) were \((0.70, -0.40, -0.30), (0.40, -0.25, -0.15)\) and \((0.05, -0.03, -0.02)\), respectively. The parameter \(\alpha_0\) was set at unity. The \(\alpha_i\) were determined using the share equation and the median values for the shares (defined a priori as 0.4, 0.3, and 0.3 for goods 1, 2 and 3). Imposing symmetry and homogeneity restrictions leaves three free \((\gamma_{ij})\) parameters to be defined \((\gamma_{12}, \gamma_{13}, \gamma_{23})\). These were determined by defining the corresponding elasticities of substitution \((\sigma_{ij})\). The “high,” “medium,” and “low” values of \((\sigma_{12}, \sigma_{13}, \sigma_{23})\) were \((1.70, 1.70, -1.70), (1.00, 1.00, -1.00)\) and \((0.30, 0.30, -0.30)\), respectively. The logarithm of expenditures was generated from the AIDS expenditure function (e.g., see Deaton and Muellbauer, p. 313) with the utility index set at 0.5 and using the generated prices and established “true” parameters. One thousand sets of normal random disturbances were generated for each design point and added to the generated shares. The disturbance vectors were defined by: \(u_2 \sim N(0, .007), u_3 \sim N(0, .007), \) and \(u_1 = -u_2 - u_3\). The precision level of 0.007 was chosen to coincide with results obtained from previous estimations of the LA/AIDS (e.g., Blanciforti, Green, and King).\(^3\) The LA/AIDS was estimated by iterated Seemingly Unrelated Regression (ITSUR) and, considering (11) treats the shares as endogenous, by iterated Three Stage Least Squares (3SLS). For each set of estimates, demand elasticities were computed using the four formulas and we computed an overall “mean percent error” for each formula:

\[
\text{Mean Percent Error} = 100 \sum_{i=1}^{3} \sum_{j=1}^{3} \frac{1}{9} \left| \frac{\hat{\epsilon}_{ij} - \epsilon_{ij}}{\epsilon_{ij}} \right|. \tag{16}
\]

IV. Simulation Results

Table 1 shows the results for compensated elasticities at one of the 27 design points (HMM): “high” values for \(\beta_j\)'s and “medium” values for \(\gamma_{ij}\)'s and \(\rho\). This case was typical in that the LA and LA' were quite accurate, very similar to one another, and both more accurate than the LA'' and, especially, the AI. As shown in equation (5), the LA/AIDS estimates of the

\(^2\)Although u_2 and u_3 were drawn independently, the sample covariances were not exactly zero (i.e., \(\sigma_{23} = \text{cov}(u_2, u_3) \neq 0\)) and, by construction, \(\sigma_{12} = -\sigma_{23} - \sigma_{23} = -\sigma_{32} - \sigma_{33}\). Some limited experimentation with a bivariate normal distribution for \(u_2\) and \(u_3\) indicated that the size of the covariance between \(u_2\) and \(u_3\) did not affect results much, and introducing this as an additional design dimension appears to add unnecessary complications.
<table>
<thead>
<tr>
<th>Elasticity</th>
<th>True Value</th>
<th>Mean Calculated Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_{11}^*$</td>
<td>-0.58</td>
<td>-0.39</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.35)</td>
</tr>
<tr>
<td>$e_{12}^*$</td>
<td>0.24</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.28)</td>
</tr>
<tr>
<td>$e_{13}^*$</td>
<td>0.35</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.34)</td>
</tr>
<tr>
<td>$e_{21}^*$</td>
<td>0.38</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.45)</td>
</tr>
<tr>
<td>$e_{22}^*$</td>
<td>-0.02</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.98)</td>
</tr>
<tr>
<td>$e_{23}^*$</td>
<td>-0.36</td>
<td>-0.91</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.83)</td>
</tr>
<tr>
<td>$e_{31}^*$</td>
<td>0.39</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.38)</td>
</tr>
<tr>
<td>$e_{32}^*$</td>
<td>-0.26</td>
<td>-0.64</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.59)</td>
</tr>
<tr>
<td>$e_{33}^*$</td>
<td>-0.13</td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.72)</td>
</tr>
<tr>
<td>Mean Percent Error</td>
<td>434.74</td>
<td>412.96</td>
</tr>
</tbody>
</table>

Notes: The HMM design point consists of "high" values of $\beta_j$'s (0.7, -0.4, -0.3), a "medium" degree of multicollinearity (simple correlation coefficients between prices of $p_{ij} = 0.75$), and "medium" values of elasticities of substitution (1.0, 1.0, -1.0). The estimated correlation between $\ln P^*$ and $\ln P$ is $\rho_p = 0.960$. Entries are averages of 1,000 values of compensated price elasticities and numbers in parentheses are root mean square errors.

Intercepts are related to the intercepts in the true AIDS share equations according to $a_i^* = \alpha_i - \beta_i$. If we denote the elasticity using $a_i^*$ by $e_{ij}(AI^*)$ and the elasticity using $\alpha_i$ by $e_{ij}(AI)$, then we can write $\epsilon_{ij}(AI^*) = \epsilon_{ij}(AI) + \beta_i \beta_j \xi / \omega_j$. At unit prices $\xi = \alpha_0 > 0$ (the subsistence level), and $\epsilon_{ij}(AI^*)$ will systematically over-estimate the own-price elasticities whereas the direction of bias in the cross-price elasticities will be determined by the signs of $\beta_i$ and $\beta_j$. This effect can be seen in the results in table 1.

Table 2 summarizes the results across the 27 points in terms of Mean Percent Errors. Several patterns are apparent. First, the $LA$ and $LA'$ formulas are consistently more accurate than the $LA''$ and $AI$ formulas. Second, the $LA'$ (as used by Chalfant) provides a very good approximation to the $LA$ (advocated by Green and Alston, 1990) and, in fact, in twelve of the twenty-seven cases outperformed the $LA$. Third, as the degree of multicollinearity increases, accuracy decreases for all of the formulas, but especially for the $LA'$ and $AI$. While higher multicollinearity is expected to lead to an increase in the accuracy of the Stone's index as an approximation to the AIDS price index, it also makes estimation of parameters more difficult generally. Fourth, as preferences depart further from homotheticity (as the $\beta_i$'s increase) precision falls, and as substitutability among the goods rises (as the $\gamma_{ij}$'s increase) precision improves.

This last result corresponds to the idea that the precision of elasticity estimates would depend on the accuracy of the approximation using the Stone's index. At every level of multicollinearity, the errors were greatest when low $\gamma_{ij}$'s were combined with high $\beta_i$'s, the conditions under which precision of the Stone's index was lowest. The simple correlations ($\rho_{ij}$) between the AIDS price index ($\ln P$) and Stone's price index ($\ln P^*$) at each of the 27 design points are shown in table 2. Our analytic results suggested that low $\gamma_{ij}$'s, low $\beta_i$'s, or high multicollinearity among the goods' prices would improve the precision of the approximation. The results bear this out. The correlations were greater than 96% in two-thirds of the cases; greater than 99% in all of the cases when multicollinearity was "high" level and in all of the cases when $\beta_i$'s were "high." With "low" or "medium" multicollinearity, the correlations fell with increases in $\beta_i$'s and with decreases in $\gamma_{ij}$'s.

In summary, the $LA$ and $LA'$ formulas for compensated elasticities are always more accurate than the alternatives as they are commonly applied. The $LA'$ provides a very good approximation, and it might not be worth making the additional effort required to compute the $LA$ formulas. Increasing the sample size from 35 to 70 increased precision slightly but did not change the overall picture. Using 3SLS rather than SUR, in

---

\* This result, among others, is derived by Buse.
Table 2.—Mean Percent Errors in Compensated Elasticities, All Design Points
(N = 35, ITSUR estimates)

<table>
<thead>
<tr>
<th>Design Point</th>
<th>LA</th>
<th>LA'</th>
<th>LA''</th>
<th>AI</th>
<th>( \rho_P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>LLL</td>
<td>6.16</td>
<td>3.94</td>
<td>77.26</td>
<td>325.55</td>
<td>0.998</td>
</tr>
<tr>
<td>LLM</td>
<td>2.46</td>
<td>3.97</td>
<td>8.01</td>
<td>29.90</td>
<td>0.996</td>
</tr>
<tr>
<td>LLH</td>
<td>0.57</td>
<td>0.70</td>
<td>16.78</td>
<td>18.39</td>
<td>0.905</td>
</tr>
<tr>
<td>MLL</td>
<td>3.64</td>
<td>14.09</td>
<td>34.44</td>
<td>51.07</td>
<td>0.935</td>
</tr>
<tr>
<td>MLM</td>
<td>6.15</td>
<td>5.30</td>
<td>37.81</td>
<td>75.76</td>
<td>0.933</td>
</tr>
<tr>
<td>MLH</td>
<td>1.02</td>
<td>1.44</td>
<td>53.67</td>
<td>82.49</td>
<td>0.830</td>
</tr>
<tr>
<td>HLL</td>
<td>9.75</td>
<td>5.03</td>
<td>275.72</td>
<td>595.57</td>
<td>0.999</td>
</tr>
<tr>
<td>LMM</td>
<td>2.25</td>
<td>10.29</td>
<td>102.33</td>
<td>300.48</td>
<td>0.999</td>
</tr>
<tr>
<td>LML</td>
<td>0.85</td>
<td>0.48</td>
<td>22.59</td>
<td>93.18</td>
<td>0.999</td>
</tr>
<tr>
<td>MML</td>
<td>1.33</td>
<td>0.95</td>
<td>42.29</td>
<td>177.29</td>
<td>0.967</td>
</tr>
<tr>
<td>MMM</td>
<td>1.75</td>
<td>2.27</td>
<td>14.48</td>
<td>27.55</td>
<td>0.955</td>
</tr>
<tr>
<td>MMH</td>
<td>2.66</td>
<td>3.42</td>
<td>26.44</td>
<td>49.19</td>
<td>0.983</td>
</tr>
<tr>
<td>HML</td>
<td>43.68</td>
<td>68.22</td>
<td>820.69</td>
<td>2322.74</td>
<td>0.954</td>
</tr>
<tr>
<td>HMM</td>
<td>21.48</td>
<td>47.63</td>
<td>142.96</td>
<td>434.74</td>
<td>0.960</td>
</tr>
<tr>
<td>HMH</td>
<td>2.97</td>
<td>3.33</td>
<td>37.91</td>
<td>145.73</td>
<td>0.920</td>
</tr>
<tr>
<td>LHL</td>
<td>32.06</td>
<td>2.64</td>
<td>3208.50</td>
<td>67928.71</td>
<td>0.999</td>
</tr>
<tr>
<td>LHM</td>
<td>5.75</td>
<td>0.75</td>
<td>551.09</td>
<td>10257.75</td>
<td>0.999</td>
</tr>
<tr>
<td>LHl</td>
<td>19.10</td>
<td>22.13</td>
<td>681.58</td>
<td>13147.08</td>
<td>0.999</td>
</tr>
<tr>
<td>MHL</td>
<td>27.46</td>
<td>21.54</td>
<td>1328.24</td>
<td>15867.05</td>
<td>0.999</td>
</tr>
<tr>
<td>MHH</td>
<td>5.62</td>
<td>4.32</td>
<td>246.41</td>
<td>7996.77</td>
<td>0.999</td>
</tr>
<tr>
<td>MHH</td>
<td>1.65</td>
<td>1.24</td>
<td>365.42</td>
<td>6766.09</td>
<td>0.999</td>
</tr>
<tr>
<td>HHL</td>
<td>103.51</td>
<td>169.59</td>
<td>4119.20</td>
<td>88055.89</td>
<td>0.994</td>
</tr>
<tr>
<td>HHH</td>
<td>28.40</td>
<td>7.46</td>
<td>1644.05</td>
<td>42653.96</td>
<td>0.997</td>
</tr>
<tr>
<td>HHH</td>
<td>7.58</td>
<td>7.82</td>
<td>254.90</td>
<td>3180.52</td>
<td>0.997</td>
</tr>
</tbody>
</table>

Notes: See notes to table 1. L, M, and H denote “low,” “medium,” and “high” settings of parameters. For example, LMH denotes “low” values for \( \beta_i \)’s, “medium” values for \( \rho_i \)’s, and “high” values for \( \gamma_i \)’s. \( \rho_P \) is the estimated correlation between In \( P^* \) and In \( P \).

V. Conclusion

The primary conclusion of this paper is that vastly different values can be obtained for AIDS elasticities when the LA/AIDS parameter estimates are substituted in various elasticity expressions. Two of the formulas that have been used in previous studies (one of which is the elasticity formula for the true AIDS model) result in very poor estimates, especially when multicollinearity among prices is high. The best results are obtained from either the elasticity expression that assumes that budget shares are endogenous on the right hand side of the demand equations or the one that assumes they are constant. Using these formulas for elasticities, the LA/AIDS provides quite accurate estimates of elasticities when the true data generating process is AIDS. Demand analysts can consequently have a certain degree of confidence when estimating the LA/AIDS and using either of these formulas for obtaining estimates of the “true” AIDS elasticities. Alternatively, as shown by Buse, good estimates can be obtained by correcting the LA/AIDS estimates of the intercepts in the AIDS model and using the \( AI \) formula.

REFERENCES


5 Results for these other cases are available from the authors.
ESTIMATING A SYSTEM OF RECREATION DEMAND FUNCTIONS USING A SEEMINGLY UNRELATED POISSON REGRESSION APPROACH

Teofilo Ozuna, Jr. and Irma Adriana Gomez*

Abstract—In this article, a seemingly unrelated Poisson regression model (SUPREME) is presented as an alternative to using Zellner’s seemingly unrelated regression model for estimating a system of recreation demand functions. SUPREME provides estimates that are asymptotically more efficient than equation-by-equation Poisson estimates and circumvents the bias and inconsistency problems that result when using Zellner’s seemingly unrelated regression model. Additionally, SUPREME is applied to an empirical problem dealing with the value of recreational boating and the findings indicate that the SUR consumer surplus estimates are substantially different from those of SUPREME.

I. Introduction

Prior recreation demand research acknowledges the importance of substitute sites for the estimation of recreation demand functions. Research in this area shows that omitted variable bias can occur if substitute sites are not incorporated in the estimation process. Hence, researchers either include substitute site variables in the recreation demand function or jointly estimate a system of recreation demand functions which include substitute sites (Burt and Brewer (1971); Sellar, Stoll, and Chavas (1985)).

Current research also recognizes the importance of adequately modeling the count nature of trip demand. With respect to this issue, Creel and Loomis (1990), Grogger and Carson (1991), and Hellerstein (1991) suggest the use of count distributions for the estimation of recreation demand functions. Failure to account for this distributional issue will result in inconsistent parameter estimates (Mullahy (1986)) and, consequently, invalid consumer surplus estimates.

This article adds to the aforementioned literature in that it proposes a seemingly unrelated Poisson regression model (SUPREME) for the estimation of a system of recreation demand functions. The model provides parameter estimates that are asymptotically more efficient than equation-by-equation Poisson estimates and circumvents the bias and inconsistency problems that result when using Zellner’s (1962) seemingly unrelated regression model (SUR) with endogenous count variables (King (1989)). Finally, the model permits the testing of cross equation economic hypothesis.

The rest of the article is organized as follows. The next section briefly reviews SUPREME. The third section applies the model to a recreational boating study. The final section provides some concluding comments and possible extensions of SUPREME.

II. The Seemingly Unrelated Poisson Regression Model

Consider the following recreation demand functions:

\[ Q_{1i} = \beta_0 + \beta_1 P_{1i} + \beta_2 P_{2i} + \epsilon_{1i}, \]  
\[ Q_{2i} = \alpha_0 + \alpha_1 P_{1i} + \alpha_2 P_{2i} + \epsilon_{2i}, \]

where \( Q_{1i} \) and \( Q_{2i} \), \( P_{1i} \) and \( P_{2i} \), and \( \epsilon_{1i} \) and \( \epsilon_{2i} \), are, respectively, the number of trips, surrogate prices, and disturbance terms for recreation sites 1 and 2. If the disturbances between the functions are uncorrelated, there is no relationship between the functions, and consequently, invalid consumer surplus estimates.

Received for publication October 15, 1991. Revision accepted for publication December 16, 1992.

* Texas A & M University.

The authors are grateful to Lonnie L. Jones and Badi Baltagi for their helpful comments and to John R. Stoll for providing the data for the empirical illustration.