THE ROTTERDAM DEMAND MODEL AND ITS APPLICATION IN MARKETING

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This paper shows how the system-wide approach to demand analysis can be utilized in marketing. In the context of the Rotterdam model, we describe how the approach can be applied to narrowly defined groups of goods (such as beer, wine and spirits) to estimate income and price elasticities of demand. The paper also provides extensions to deal with advertising and introduces a new way of identifying market structure.

(Consumer Demand; Advertising; Market Structure)

1. Introduction

The theory of the utility-maximizing consumer enjoys widespread acceptance and use in economics. The objective of this theory is to describe the way in which consumption of a good depends on the consumer's income and the prices he faces. In recent years much progress has been made in combining this theory with the empirical analysis of consumption data. The data are used to test the theory, and the theory is used to provide structure to the empirical analysis. This work is known as the system-wide approach in which demand equations for all n goods are estimated simultaneously. For surveys of these developments, see Barten (1977), Brown and Deaton (1972), Philips (1974), Powell (1974), Theil (1975/76, 1980a, b) and Theil and Clements (1987).

A leading example of a system of demand equations is the Rotterdam model, due to Barten (1964) and Theil (1965); the name “Rotterdam” comes from the location of Barten and Theil in the 1960s. This model has strong links with the economic theory of the consumer and its elegant simplicity has contributed to its popularity and influential role in the development of the system-wide approach.

The objective of this paper is to give a self-contained account of the Rotterdam model so that it is accessible to nonspecialists and to indicate how it may be of use in marketing research. The most frequent use of the model involves the demand for broad commodity groups such as food, clothing, housing and so on. For marketing purposes, however, these groups are much too broad. Fortunately, under certain conditions the model can be reformulated so that it can be applied to narrower groups of goods such as beer, wine and spirits or different brands of a certain product. But even with reformulation, to make the model of further use in marketing requires extensions to deal with...
advertising and other elements of the marketing mix. In this paper we show how all this can be done as well as how the model provides a new way of identifying market structure.

§§2 and 3 of the paper set out the economic theory of the consumer and the Rotterdam model. In §4 we use the convergence approach to aggregation to show how the microeconomic theory of the consumer can be applied with macro data. §§5 and 6 show how the model can be used to analyse the demand for narrowly defined goods, with the consumption of beer, wine and spirits used as an example. In §7 we introduce a new way of identifying market structure which is based on consumption theory. We then use the Rotterdam model to illustrate the application of this procedure. §§8 and 9 deal with advertising and alternatives to the Rotterdam model. Concluding comments are given in §10. Throughout the paper we draw on Theil (1975/76, 1980a) and Theil and Clements (1987).

2. Differential Demand Equations

We write \( p_i, q_i \) for the price and quantity of good \( i \) \((i = 1, \ldots, n)\). The consumer behaves as if he chooses \( q_1, \ldots, q_n \) to maximize his utility function, \( u(q_1, \ldots, q_n) \), subject to the budget constraint \( M = \sum_{i=1}^{n} p_i q_i \), \( M \) being income. The optimal quantities obviously depend on income and prices, so we write

\[
q_i = q_i(M, p_1, \ldots, p_n), \quad i = 1, \ldots, n. \tag{2.1}
\]

These are the demand functions.

The differential of (2.1) can be expressed as (Theil and Clements, 1987)

\[
w_i d(\log q_i) = \theta_i d(\log Q) + \sum_{j=1}^{n} \nu_{ij} d\left(\log \frac{p_j}{P^0}\right), \tag{2.2}
\]

where \( w_i = \frac{p_i q_i}{M} \) is the budget share of \( i \); \( \theta_i = \frac{\partial (p_i q_i)}{\partial M} \) is the \( i \)th marginal share; \( d(\log Q) = \sum_{i=1}^{n} w_i d(\log q_i) \) is the Divisia volume index of the change in the consumer’s real income; \( d(\log (p_j/P^0)) \) is the change in the deflated price of \( j \) with \( d(\log P^0) \) the Frisch price index; and

\[
\nu_{ij} = \frac{\lambda}{M} p_i u_{ij} p_j, \tag{2.3}
\]

is the \((i, j)\)th price coefficient, with \( \lambda \) the marginal utility of income and \( u_{ij} \) the \((i, j)\)th element of the inverse of the Hessian of the utility function, \( U = [\partial^2 u/\partial q_i \partial q_j] \). These \( \nu_{ij} \)'s satisfy

\[
\sum_{j=1}^{n} \nu_{ij} = \phi \theta_i, \quad i = 1, \ldots, n, \tag{2.4}
\]

where \( \phi = [\partial(\log \lambda)/\partial(\log M)]^{-1} \) is the income flexibility, the inverse of the income elasticity of the marginal utility of income.

Equation (2.2) is formulated in terms of relative (or deflated) prices. The corresponding absolute price version is

\[
w_i d(\log q_i) = \theta_i d(\log Q) + \sum_{j=1}^{n} \pi_{ij} d(\log p_j), \tag{2.5}
\]

where \( \pi_{ij} = \nu_{ij} - \phi \theta_i \theta_j \) is the \((i, j)\)th Slutsky coefficient. These coefficients satisfy demand homogeneity

\[
\sum_{j=1}^{n} \pi_{ij} = 0, \quad i = 1, \ldots, n; \tag{2.6}
\]
and Slutsky symmetry
\[ \pi_{ij} = \pi_{ji}, \quad i, j = 1, \ldots, n. \] (2.7)

The \( n \times n \) matrix \([\pi_{ij}]\) is negative semidefinite with rank \( (n - 1) \).

Demand homogeneity implies that an equiproportionate change in all prices has no effect on consumption, while Slutsky symmetry implies that the substitution effects of price changes are symmetric. Dividing both sides of (2.5) by \( w_i \), we obtain \( \theta_i/w_i \) as the income elasticity of demand for \( i \); and \( \pi_{ij}/w_i \) as the \((i, j)\)th price elasticity.

3. The Rotterdam Model

The differential demand equations presented in the last section are in terms of infinitesimal changes. The Rotterdam model is simply a finite-change version of these demand equations. To obtain the relative price version of the Rotterdam model we make the following adjustments to (2.2). The budget share \( w_i \) is replaced with the arithmetic average of the budget share in \( t - 1 \) and \( t \), \( \bar{w}_it = \frac{1}{2}(w_{it} + w_{it-1}); d(\log q_i) \) is replaced with the change in the logarithm of \( q_i \) from period \( t - 1 \) to \( t \), \( Dq_{it} = \log q_{it} - \log q_{it-1}; d(\log Q) \) is replaced with \( DQ_t = \sum_{i=1}^{n} \bar{w}_itDq_{it}; d(\log p_j) \) is replaced with \( Dp_{jt} = \log p_{jt} - \log p_{jt-1} \); and \( d(\log P') \) is replaced with \( DP' = \sum_{i=1}^{n} \theta_iDP_{it} \).

The marginal share and price coefficients in (2.2) are not necessarily constant. In the Rotterdam model, as a simplification these are treated as constants. After making the above adjustments to (2.2) we obtain
\[ \bar{w}_itDq_{it} = \theta_iDQ_t + \sum_{j=1}^{n} \nu_{ij}(Dp_{jt} - DP'_t), \] (3.1)

which is the \textit{ith demand equation of the relative price version of the Rotterdam model.}

The complete model is made up of \( n \) equations of the form (3.1), one for each good. It follows from the definition of the price coefficient \( \nu_{ij} \) given in (2.3) that this coefficient gives information about the extent to which commodities \( i \) and \( j \) interact in the utility function. It will often be the case that when the number of goods \( (n) \) is large, it is reasonable to postulate that many of the \( \nu_{ij} \)'s are zero; more on this in §5. It also follows from the definition of \( \nu_{ij} \) that the \( n \times n \) matrix \([\nu_{ij}]\) is symmetric negative definite.

The \textit{ith demand equation of the absolute price version of the Rotterdam model} is the finite-change version of (2.5),
\[ \bar{w}_{it}Dq_{it} = \theta_iDQ_t + \sum_{j=1}^{n} \pi_{ij}Dp_{jt}. \] (3.2)

The Slutsky coefficients in (3.2) are subject to the homogeneity and symmetry constraints (2.6) and (2.7). As can be seen, (3.2) and the constraints are all linear in the parameters which makes estimation and hypothesis testing straightforward.

4. Aggregation over Individual Consumers

The demand equations derived above are based on the utility-maximizing behaviour of the individual consumer. As data in economics are usually available only in aggregate form, it is natural to raise the question, to what extent do the previous results carry over to the aggregate or market demand functions? In this section, which is mainly based on Barnett (1979), Selvanathan (1987) and Theil (1975/76), we use the convergence approach to aggregation to analyse this problem. This approach is a statistical tool which is used to analyse the market demand functions when the size of the consumer population becomes very large.
The Micro and Macro Demand Equations

Consider equation (2.5), the absolute price version of the differential demand equation for good \(i\). As that equation refers to an individual consumer we add a consumer subscript \(c\),

\[
w_{ic} d(\log q_{ic}) = \theta_{ic} d(\log Q_c) + \sum_{j=1}^{n} \pi_{ijc} d(\log p_{j}), \quad c = 1, \ldots, N, \tag{4.1}\]

where we have made the assumption that all consumers pay the same prices; and \(N\) is the number of consumers. The micro coefficients \(\theta_{ic}\) and \(\pi_{ijc}\) of the micro demand equation (4.1) are functions of income and prices and satisfy the homogeneity and symmetry restrictions (2.6) and (2.7).

We define the per capita variables which we shall refer to as the macro variables,

\[
\bar{q}_i = \frac{1}{N} \sum_{c=1}^{N} q_{ic}, \quad \bar{M} = \frac{1}{N} \sum_{c=1}^{N} M_{c} = \frac{1}{N} \sum_{c=1}^{N} p_{i} \bar{q}_i, \quad \bar{w}_i = \frac{p_{i} \bar{q}_i}{\bar{M}}.
\]

Our objective is to develop a macro demand equation in terms of macro variables and macro coefficients which is analogous to (4.1). To obtain this we first multiply both sides of that equation by \(M_c/\bar{M}\) and then take the summation over \(c = 1, \ldots, N,\)

\[
\sum_{c=1}^{N} \frac{M_c}{\bar{M}} w_{ic} d(\log q_{ic}) = \sum_{c=1}^{N} \frac{M_c}{\bar{M}} \theta_{ic} d(\log Q_c) + \sum_{c=1}^{N} \sum_{j=1}^{n} \frac{M_c}{\bar{M}} \pi_{ijc} d(\log p_{j}). \tag{4.2}
\]

The Left-Hand Variable of (4.2)

The left-hand term of (4.2) is

\[
\sum_{c=1}^{N} \frac{M_c}{\bar{M}} w_{ic} d(\log q_{ic}) = \sum_{c=1}^{N} \frac{M_c}{\bar{M}} p_{i} q_{ic} \frac{d q_{ic}}{M_c} = \frac{p_{i}}{\bar{M}} \sum_{c=1}^{N} d q_{ic} = p_{i} \frac{d \bar{q}_i}{\bar{M}} = \bar{w}_i d(\log \bar{q}_i). \tag{4.3}
\]

The last member of (4.3) is the macro analogue of the left-hand variable of equation (4.1).

Assumptions

We assume that \(M_1, \ldots, M_N\) are identically and independently distributed. We also assume that the macro coefficients, defined as

\[
\tilde{\theta}_i = \frac{E[M_c \theta_{ic}]}{E[M_c]}, \quad \tilde{\pi}_{ij} = \frac{E[M_c \pi_{ijc}]}{E[M_c]}, \quad i, j = 1, \ldots, n,
\]

exist and are finite. These macro coefficients are population versions of weighted-average micro coefficients with weights proportional to income \(M_c\). It can easily be shown that the definition of the macro Slutsky coefficients implies that they satisfy homogeneity and symmetry restrictions,

\[
\sum_{j=1}^{n} \tilde{\pi}_{ij} = 0, \quad i = 1, \ldots, n, \quad \tilde{\pi}_{ij} = \tilde{\pi}_{ji}, \quad i, j = 1, \ldots, n;
\]

that \(\sum_{i=1}^{n} \tilde{\theta}_i = 1;\) and that the matrix \([\tilde{\pi}_{ij}]\) is negative semidefinite with rank \((n - 1)\).

The Right-Hand Variables of (4.2)

Applying the convergence approach under the above assumptions, it can be shown that

\[
\sum_{c=1}^{N} \frac{M_c}{\bar{M}} \theta_{ic} d(\log Q_c) = \tilde{\theta}_i d(\log \bar{Q}) + \frac{\text{Cov}[K_{ic}, V_i]}{E[M_c]} + o_p(1), \tag{4.4}
\]

\[
\frac{E[M_c \theta_{ic}]}{E[M_c]}, \quad \frac{E[M_c \pi_{ijc}]}{E[M_c]}.
\]

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where \(d(\log \tilde{Q})\) is the change in per capita real income; \(K_{ic} = M_c(\theta_{ic} - \tilde{\theta}_i)\); \(V_c = d(\log Q_c)\); and \(o_p(1)\) denotes a random variable that converges in probability to zero as \(N\) (the number of consumers) tends to infinity. It can also be shown that

\[
\sum_{c=1}^{N} \sum_{j=1}^{n} \frac{M_c}{N} \pi_{jk} d(\log p_j) = \sum_{j=1}^{n} \tilde{\pi}_{ij} d(log p_j) + o_p(1). \tag{4.5}
\]

For derivations of these results, which are based on Khinchine’s theorem, see Barnett (1979) or Selvanathan (1987).

The Final Form of the Macro Demand Equations

Substituting (4.3), (4.4) and (4.5) in (4.2) we obtain

\[
\tilde{\theta}_i d(\log \tilde{Q}) = \tilde{\theta}_i d(\log \tilde{Q}) + \sum_{j=1}^{n} \tilde{\pi}_{ij} d(\log p_j) + \frac{\text{Cov} [K_{ic}, V_c]}{E[M_c]} + o_p(1).
\]

In Appendix A1 we argue that \(\text{Cov} [K_{ic}, V_c]/E[M_c]\) is likely to be negligibly small. Therefore, for sufficiently large \(N\), we can write the above equation as

\[
\tilde{\theta}_i d(\log \tilde{Q}) = \tilde{\theta}_i d(\log \tilde{Q}) + \sum_{j=1}^{n} \tilde{\pi}_{ij} d(\log p_j), \tag{4.6}
\]

where we have also used \(\text{plim}_{N \to \infty} o_p(1) = 0\).

Equation (4.6) is the macro analogue of the micro demand equation (4.1). The coefficients of (4.6) satisfy the homogeneity and symmetry restrictions (2.6) and (2.7). Consequently, the convergence approach reveals that all of the restrictions at the individual consumer level carry over to the aggregate equations. This is the justification for applying the Rotterdam model to aggregate data. In order to keep the notation as simple as possible, in what follows we shall not distinguish between micro coefficients and variables and their macro counterparts.

The above results are very appealing as they imply that the microeconomic theory of the consumer is directly applicable with aggregate data. The homogeneity and symmetry constraints play the role of reducing the number of free parameters to be estimated; and more often than not prices are highly collinear making it impossible to estimate parameters precisely without these constraints. It should nevertheless be acknowledged that the use of aggregate data can yield information about demand behaviour at the aggregate level only, not at the individual level. While some marketing applications no doubt require details of the heterogeneity at the individual level, for others aggregate level inferences are sufficient. An example of the latter is the case of alcoholic beverages in Australia where one of the major marketing decisions faced by brewers in recent years is whether or not beer and wine are closely competing for the drinker’s dollar. This example will be taken up below.

In concluding this section, we should note a caveat regarding the convergence approach. Equations (4.4) and (4.5) contain an \(o_p(1)\) term with \(\text{plim}_{N \to \infty} o_p(1) = 0\). Like all probability statements, this convergence result cannot of course guarantee that the actual value of this term is zero. It should also be pointed out that not all applications of the Rotterdam model have been with aggregate data. The use of micro data to estimate the model requires that prices vary across individuals, as is the case with panel data. A recent example of this approach is Kiefer (1984). These micro applications are still quite rare due to the lack of databases containing cross-sectional price variability.

5. The Demand for Groups of Goods and Conditional Demand Equations

The analysis thus far has dealt with the demand for all \(n\) goods. We now show how to analyse the demand for a subset or group of goods.
Block-Independent Preferences

Let the \( n \) goods now be divided into \( G \) groups, \( S_1, \ldots, S_G \), such that each good belongs to only one group. Further, let the consumer’s preferences be such that the utility function is the sum of \( G \) subutility functions, each involving the quantities of only one group,

\[
    u(q) = \sum_{g=1}^{G} u_g(q_g),
\]

where \( q = (q_1, \ldots, q_n)' \) and \( q_g \) is the vector of the \( q_i \)'s that fall under \( S_g \). Under (5.1), the marginal utility of a good depends only on the consumption of goods belonging to the same group. When the goods are numbered appropriately, the Hessian \( U \) and its inverse become block-diagonal. Accordingly, specification (5.1) is known as block-independent preferences.

Under block independence, \( v_{ij} = 0 \) for \( i \) and \( j \) in different groups. Thus (2.4) for \( i \in S_g \) becomes

\[
    v_{ii} = \phi_i, \quad g = 1, \ldots, G. \tag{5.2}
\]

The demand equation (3.1) for \( i \in S_g \) becomes

\[
    \tilde{w}_{it} Dq_{it} = \theta_i DQ_i + \sum_{j \in S_g} v_{ij} (Dp_{jt} - DP_i), \tag{5.3}
\]

so that the only deflated prices which appear are those of goods belonging to the same group as \( i \).

Composite Demand Equations

We write \( \tilde{W}_{gt} = \sum_{i \in S_g} \tilde{w}_{it}, \theta_g = \sum_{i \in S_g} \theta_i \) for the budget and marginal shares of group \( S_g \), and define the group volume and Frisch price indexes as

\[
    DQ_{gt} = \sum_{i \in S_g} \tilde{w}_{it} Dq_{it}, \quad DP_{gt} = \sum_{i \in S_g} \theta_i Dp_{it}.
\]

If we then add (5.3) over \( i \in S_g \) and use (5.2) as well as the symmetry of the matrix \([v_{ij}]\), we obtain the composite demand equation for \( S_g \) as a group (Theil and Clements 1987),

\[
    \tilde{W}_{gt} DQ_{gt} = \theta_g DQ_i + \phi_g (DP_{gt} - DP_i). \tag{5.4}
\]

Thus under block-independence, only the deflated price of the group, \( DP_{gt} - DP_i \), and income affect the demand for the group as a whole. The income and own-price elasticities for the group are \( \theta_g / \tilde{W}_{gt} \) and \( \phi_g / \tilde{W}_{gt} \), respectively.

Conditional Demand Equations

Combining (5.3) and (5.4) to eliminate \( DQ_i \), we obtain (Theil and Clements 1987)

\[
    \tilde{w}_{it} Dq_{it} = \frac{\theta_i}{\theta_g} \tilde{W}_{gt} DQ_{gt} + \sum_{j \in S_g} v_{ij} (Dp_{jt} - DP_{gt}). \tag{5.5}
\]

This is the demand equation for \( i \in S_g \), given the demand for the group as a whole \( \tilde{W}_{gt} DQ_{gt} \). As the variables on the right of this equation are exclusively concerned with the group \( S_g \) to which the \( i \)th commodity belongs, it is known as a conditional demand equation. The term \( \theta_i / \theta_g \) is the conditional marginal share of \( i \) within the group \( S_g \), with \( \sum_{i \in S_g} \theta_i / \theta_g = 1 \). The conditional marginal share answers the question, if income increases by \$1, resulting in a certain additional amount spent on the group \( S_g \), what is the proportion of this additional amount that is allocated to commodity \( i \)?

Equation (5.5) can also be formulated in terms of absolute prices by using the definition of \( DP_{gt} \) to write the substitution term as
\[
\sum_{j \in S_g} v_{ij} \left[ Dp_{ji} - \sum_{k \in S_g} \theta_k \theta^k \right] = \sum_{j \in S_g} v_{ij} Dp_{ji} - \phi \theta_i \sum_{k \in S_g} \theta_k \theta^k \\
= \sum_{j \in S_g} \left( v_{ij} - \phi \theta_j \theta^j \right) Dp_{ji} = \sum_{j \in S_g} \pi^g_{ij} Dp_{ji},
\]
where the first step is based on (5.2); \( \theta^i = \theta_i / \theta^g \); and
\[
\pi^g_{ij} = v_{ij} - \phi \theta_j \theta^j, \quad i, j \in S_g,
\]
is the \((i, j)\)th conditional Slutsky coefficient. Substitution of the fourth member of (5.6) into (5.5) gives the absolute price version of the conditional demand equation for \( i \in S_g, \)
\[
\tilde{w}_{it} Dq_{it} = \theta^i \tilde{W}_g DQ_{gt} + \sum_{j \in S_g} \pi^g_{ij} Dp_{jt}.
\]
Our previous assumptions imply that \( \theta^i \) and \( \pi^g_{ij} \) in this equation are constant coefficients.

By dividing both sides of (5.8) by \( \tilde{w}_{it} \), we find that \( \theta^i \tilde{W}_g / \tilde{w}_{it} = (\theta_j / \tilde{w}_{it}) / (\theta_j / \tilde{W}_g) \) is the ratio of the income elasticity of \( i \) to that of the group to which the good belongs. We shall refer to this ratio as the conditional income elasticity of \( i \). We also find that \( \pi^g_{ij} / \tilde{w}_{it} \) is the conditional price elasticity, i.e., the elasticity of \( q_i \) with respect to the absolute price \( p_j \) \((i, j \in S_g)\). These conditional elasticities hold constant total consumption of the group, as measured by the Divisia volume index \( DQ_{gt} \).

As \( \sum_{j \in S_g} \theta^j = 1 \), it follows from (5.2) and (5.7) that
\[
\sum_{j \in S_g} \pi^g_{ij} = 0, \quad i \in S_g.
\]
This reflects the homogeneity proposition that a proportionate change in all prices in the group does not affect the demand for any good in the group, total consumption of the group remaining unchanged. If \( S_g \) consists of \( n_g \) commodities, the \( n_g \times n_g \) matrix of price coefficients referring to \( S_g \) is a principal submatrix of the \( n \times n \) price coefficient matrix \([v_{ij}]\). As the latter matrix is symmetric, so is the former. It then follows from (5.7) that the conditional Slutsky matrix \([\pi^g_{ij}]\) is also symmetric,
\[
\pi^g_{ij} = \pi^g_{ji}, \quad i, j = 1, \ldots, n_g.
\]
The negative definiteness of \([v_{ij}]\) together with (5.7) and (5.9) imply that \([\pi^g_{ij}]\) is negative semidefinite with rank \((n_g - 1)\).

A Two-Level Decision Hierarchy

It is to be noted that the variables of (5.8) deal only with the group to which the good belongs. Consequently, conditional demand functions allow us to ignore the prices of the goods outside the group in question. As will be seen in the next section, this allows us to analyse the demand for quite narrowly defined goods.

Block-independent preferences imply that the consumer’s problem can be solved in two steps. The first decision involves the allocation of income to the \( G \) groups, as described by the \( G \) group demand equations (5.4). Each of these demand equations contains real income and the relative price of the group in question, but not the prices of the individual goods. Then in the second decision, for each of the groups, expenditure is allocated to the goods within the group. The conditional demand equations (5.8) describe this allocation and they contain total consumption of the group, as determined by the previous decision, and relative prices within the group. Accordingly, there is a two-level decision hierarchy under block independence. In many applications such a separation into levels of the consumer’s decision problem makes good intuitive sense. This is illustrated below with the demand for alcoholic beverages. Under block inde-
pendence, the consumer determines expenditure on total alcohol in the first step; and then in the second step that expenditure is allocated to beer, wine and spirits as a function of the relative prices of the beverages.

6. An Example: Beer, Wine and Spirits

Clements and Johnson (1983) used annual per capita Australian data, 1956–1977, to estimate the conditional Rotterdam model (5.8) for beer, wine and spirits. At means, beer accounts for 72 percent of total alcohol expenditure, wine 13 percent and spirits 15 percent. The maximum likelihood estimates are presented in Table 1 (constant terms are included to take account of trend-like changes in tastes, etc., but these are not reported in the table). These estimates are constrained by the homogeneity and symmetry restrictions (5.9) and (5.10). For the details of the estimation procedure see Appendix A3, available on request.

The conditional marginal shares given in Table 1 indicate that when alcohol expenditure increases by $1, expenditure on beer rises by $0.59, wine by $0.10, with the remaining $0.31 being spent on spirits. We give only the upper triangle of the matrix of conditional Slutsky coefficients as it is symmetric. All the diagonal elements of this matrix are negative, as they should be, and the off-diagonal elements are all positive. The characteristic roots of the conditional Slutsky matrix are all nonpositive, which verifies that this matrix is negative semidefinite. In Appendix A2 we show that the data are not inconsistent with the assumption that the alcoholic beverages are block independent.

The upper part of Table 2 gives the conditional demand elasticities implied by the Australian estimates contained in Table 1. These are conditional in that total expenditure on alcohol is held constant, so that they deal with inter-beverage competition. The conditional income elasticities are 0.8, 0.8 and 2.0 for beer, wine and spirits, respectively. These indicate that, within alcohol, beer and wine are necessities (having elasticities less than one), while spirits is a strong luxury. The high elasticity for spirits agrees well with the notion that in Australia the more affluent tend to be spirits drinkers. All the conditional price elasticities are less than one in absolute value, the own-price elasticities being –0.1, –0.4 and –0.4 for beer, wine and spirits. The conditional cross-price elasticities are all positive, indicating that the three beverages are pairwise substitutes. As expected, there is only a moderate amount of substitutability between the three beverages. The corresponding unconditional elasticities are

| TABLE 1 |
| Conditional Demand Equations for Alcoholic Beverages: Australia, 1956–1977 |

\[
\tilde{w}_{Dq_i} = \theta_1 \tilde{W}_{Dq} + \sum_{j=1}^{3} \pi_{ij} Dp_{jq} 
\]

(Asymptotic standard errors are in parentheses)

<table>
<thead>
<tr>
<th>Beverage</th>
<th>Conditional Marginal Share $\theta_i$</th>
<th>$\pi_{ij} \times 100$</th>
<th>$\pi_{iq} \times 100$</th>
<th>$\pi_{pq} \times 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beer</td>
<td>0.590</td>
<td>–0.390</td>
<td>0.150</td>
<td>0.240</td>
</tr>
<tr>
<td></td>
<td>(0.045)</td>
<td>(0.145)</td>
<td>(0.114)</td>
<td>(0.097)</td>
</tr>
<tr>
<td>Wine</td>
<td>0.099</td>
<td>–0.293</td>
<td>–0.293</td>
<td>0.144</td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
<td>(0.124)</td>
<td>(0.072)</td>
<td></td>
</tr>
<tr>
<td>Spirits</td>
<td>0.311</td>
<td>–0.384</td>
<td>–0.384</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.042)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### TABLE 2

**Conditional Demand Elasticities for Alcoholic Beverages in Three Countries**

(Asymptotic standard errors are in parentheses)

<table>
<thead>
<tr>
<th>Beverage</th>
<th>Conditional Income Elasticity</th>
<th>Conditional Price Elasticities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beer</td>
<td>0.83 (0.06)</td>
<td>0.41 (0.08)</td>
</tr>
<tr>
<td>Wine</td>
<td>0.78 (0.28)</td>
<td>1.93 (0.19)</td>
</tr>
<tr>
<td>Spirits</td>
<td>1.97 (0.27)</td>
<td>1.80 (0.14)</td>
</tr>
</tbody>
</table>

These elasticities hold real income constant and allow total alcohol expenditure to vary.

The middle and lower parts of Table 2 present the same elasticities for the U.K. and the U.S. These are from Clements and Selvanathan (1987) who estimated alcohol demand equations similar to (5.8) with British and American data. A comparison of the income elasticities across countries reveals that they are fairly similar. In particular, beer is always a necessity and spirits a luxury. Although the price elasticities for the U.S. are insignificant, there is still a broadly similar pattern of substitutability in the three countries. Clements and Selvanathan (1987) introduce ways of improving the precision of the price elasticities estimates.

### 7. A Marketing Application: Identifying Market Structure

The analysis of market structure in marketing is concerned with identifying closely-competing brands of the same product. This analysis refers to those factors determining the probability of the consumer choosing one brand of a certain product, rather than another. These factors include the degree of consumer loyalty, which determines the pattern of repeat-buying; the nature of competition from other brands, which in part relates to the probability of switching brands; the evolution of consumer perception of...
the various brands; and so on. For examples of this literature, see Grover and Dillon (1985), Kalwani and Morrion (1977), Rao and Sabavala (1981), Rubinson et al. (1980), Srivastava et al. (1981), Srivastava et al. (1984) and Urban et al. (1984).

Consumption theory provides an alternative way of identifying market structure. The approach is to define brands \(i\) and \(j\) as being closely related if additional purchases of \(j\) cause the marginal utility of \(i\) to change significantly. If the marginal utility of \(i\) falls with increased consumption of \(j\), then it could be said that these two brands both satisfy a common want of the consumer; \(i\) and \(j\) are then competitive or substitutes for each other. This was basically the procedure used in §6 where we partitioned the \(n\) goods into \(G = 2\) groups, (i) alcoholic beverages and (ii) all other goods. The marginal utility of each of the alcoholic beverages, \(\partial u / \partial q_i, i \in S_g\), depends only on the consumption of those beverages and not on that of goods outside the group. This means that

\[
\frac{\partial^2 u}{\partial q_i \partial q_j} = \begin{cases} 
0, & i, j \in S_g, \\
-0, & i \in S_g, \quad j \not\in S_g,
\end{cases}
\]

which implies that the price coefficient \(v_{ij} = 0\) for \(i \in S_g, j \not\in S_g\). This blocking of goods amounts to postulating that the alcoholic beverages, on the one hand, and all other goods, on the other, are not closely competitive.

The same approach can also be used to test whether goods are closely related. We can analyse the existence of partitions within the alcoholic beverages group by asking, how does the marginal utility of one beverage change with increased consumption of the others? The most convenient way to do this is via the \(v_{ij}\)'s \((i, j \in S_g)\) whose definition we repeat here:

\[
v_{ij} = \frac{\lambda_i p_j}{M} u_i^j,
\]

where \(u_i^j\) is the \((i, j)\)th element of the inverse of the Hessian matrix of the utility function. If, for example, the marginal utility of wine is independent of the consumption of beer and spirits, \(\partial^2 u / \partial q_i \partial q_j = 0\) for \(i = \text{wine}\) and \(j = \text{beer and spirits}\); and it follows from the above definition that the corresponding \(v_{ij}\)'s are also zero. We would then say that wine is not closely competitive with the other two beverages, so that the market structure so identified partitions alcoholic beverages into two groups: (i) wine and (ii) beer and spirits.

It turns out that the Australian data are in approximate agreement with the idea that wine is in a class by itself. If we use in (5.7) the estimates of the \(\pi_{ij}\)'s and the \(\theta_{ij}\)'s contained in Table 1, together with the estimates \(\phi = -0.595\) and \(\theta_g = 0.0576\) (see Appendix A3, available on request), we obtain the following estimates \((\times 100)\) of the \(v_{ij}\)'s for alcohol:

\[
\begin{bmatrix}
\text{Beer} & \text{Wine} & \text{Spirits} \\
\text{Beer} & -1.583 (0.315) & -0.050 (0.127) & -0.389 (0.151) \\
\text{Wine} & -0.327 (0.133) & 0.038 (0.082) & \\
\text{Spirits} & & -0.716 (0.134) \\
\end{bmatrix}
\]

Here we have only given the upper triangle of the \([v_{ij}]\) matrix as it is symmetric. The numbers in parentheses are the asymptotic standard errors. As can be seen, the two off-diagonal elements for wine, \(v_{12}\) and \(v_{23}\), are both very small in comparison with the other elements. Moreover, these two elements are not significantly different from zero so that we cannot reject the hypothesis that wine is independent of the other two beverages. In other words, the marginal utility of wine is independent of beer and spirits consumption.
A similar analysis of the British data reveals the following structure of the $[v_{ij}]$ matrix:

\[
\begin{array}{ccc}
\text{Beer} & \text{Wine} & \text{Spirits} \\
\text{Beer} & \text{Significant} & \text{Insignificant} & \text{Insignificant} \\
\text{Wine} & \text{Significant} & \text{Significant} & \\
\text{Spirits} & \text{Significant} & & \\
\end{array}
\]

This indicates that beer is in a class by itself, so that the British drinker survives by beer alone. For the U.S., this matrix takes the form

\[
\begin{array}{ccc}
\text{Beer} & \text{Wine} & \text{Spirits} \\
\text{Beer} & \text{Significant} & \text{Insignificant} & \text{Significant} \\
\text{Wine} & \text{Significant} & \text{Insignificant} & \\
\text{Spirits} & \text{Significant} & & \\
\end{array}
\]

which has the same structure as the Australian matrix, i.e. wine is in a class by itself. These results for the U.K. and the U.S. are from Clements and Selvanathan (1987).

As can be seen, using the degree of utility interaction of the beverages (as measured by the $v_{ij}$'s) is a very appealing way to analyse the degree of competition between beverages $i$ and $j$. In contrast to the approaches in the marketing literature, it is a theory-based procedure for identifying market structure.

8. Extending the Rotterdam Model to Deal with Advertising

In the previous sections we used the Rotterdam model to analyse the effect of income and prices on the demand for the alcoholic beverages. The model can be extended to incorporate the role of other elements of the marketing mix (e.g. advertising, product quality, distribution) in determining consumption. In this section we restrict ourselves to advertising. As our objective is to illustrate how the Rotterdam model can be extended to deal with advertising, we mainly consider only a very simple case and just mention more general formulations.

Advertising and Utility

We postulate that a one percent increase in advertising of good $i$ changes the logarithm of the marginal utility of that good, $\log (a_i/q_i)$, by a positive constant $\mu$,

\[
\frac{\partial \left( \log \frac{\partial u}{\partial q_i} \right)}{\partial \left( \log a_i \right)} = \mu,
\]

where $a_i$ is a measure of the volume of advertising for good $i$. The coefficient $\mu$ is interpreted as the elasticity of the marginal utility of good $i$ with respect to its advertising.

Conditional Demand Equations with Advertising

The conditional demand equation for $i \in S_g$ under block independence is given by (5.5). When the model is extended to deal with advertising by using (8.1), it can be shown (Theil 1980b, pp. 42–44, 126–129) that this demand equation becomes

\[
\tilde{w}_{gi} Dq_{it} = \theta_i \tilde{W}_{gi} DQ_{gi} + \sum_{j \in S_g} v_{ij} [(Dp_{ij} - DP_{ig}) - \mu(Da_{ij} - DA'_{ig})],
\]

where $Dp_{ij}$ is the log-change in $p_{ij}$ and $DA'_{ig} = \sum_{k \in S_g} (\theta_k/\theta_g) Da_{ik}$ is the Frisch index of advertising for the group $S_g$. This index is a weighted average of the change in advertising of each member of the group, the weights being the conditional marginal shares.
Equation (8.2) is a simple, one-parameter extension of the Rotterdam model. The deflation of each $D_{ajt}$ by $D_{Agt}$ means that an equiproportional increase in advertising of each member of the group has no effect on consumption; it is only relative advertising that matters. Consider the situation when all prices change proportionately, so that $D_{pjt} - D_{Pgt} = 0$ in (8.2). As long as $D_{ajt} - D_{Agt} \neq 0$, it still may be possible to estimate the $v_{ij}$s of (8.2). This is likely to be important in marketing research where the prices of different brands tend to move in tandem.

Using (5.7) in (8.2) we can write the absolute price version of the conditional demand equation for $i \in S_g$ as

$$\tilde{w}_itD_{qit} = \theta'_i\tilde{W}_{gt}D_{Qgt} + \sum_{j \in S_g} \pi^x_{ij}(D_{pjt} - \mu D_{ajt}),$$

(8.3)

where $\theta'_i = \theta_i/\theta_g$ is the conditional marginal share of $i$. Equation (8.3) is the extended version of (5.8). By dividing both sides of (8.3) by $\tilde{w}_it$, we find that $-\mu\pi^x_{ij}/\tilde{w}_it$ is the conditional advertising elasticity of $i$, i.e. $\delta(\log q_i)/\delta(\log a_j)$ for $i, j \in S_g$. This elasticity is proportional to the corresponding conditional price elasticity with $-\mu$ the factor of proportionality.

Application to U.K. Alcohol Consumption

We use annual per capita data for the U.K., 1955–1975, to estimate (8.3) for beer, wine and spirits. These data are from McGuinness (1979, App. 2). The maximum likelihood estimates are presented in Table 3. As before, these estimates are constrained by the homogeneity and symmetry restrictions.

The estimate of $\mu$ given in Table 3 is significantly positive. Its value indicates that when advertising on beverage $i$ is increased by ten percent then the marginal utility of that beverage increases by 5.6 percent, all other variables remaining unchanged. Table 4 explores the implications of this estimate by presenting the conditional advertising elasticities. Consider the first row of this table, that for beer. The first entry is 0.10; this indicates that a ten percent increase in the advertising of beer raises beer sales by one percent (all other variables held constant). The second entry of $-0.06$ implies that a ten percent rise in wine advertising depresses beer sales by 0.6 percent, so that beer and wine are competitive, as expected. The last entry of $-0.04$ indicates that beer sales fall by 0.4 percent as a result of a 10 percent increase in spirits advertising. Note that the

| TABLE 3
| Conditional Demand Equations for Alcoholic Beverages: United Kingdom, 1955–1975 |
|----------------|----------------|----------------|
|                  | Conditional Marginal Share $\theta'_i$ | Conditional Slutsky Coefficients $\pi^x_{ij} \times 100$ | $\pi^z_{ij} \times 100$ |
|----------------|----------------|----------------|
| **Beverage**    |                  |                  |                  |
| Beer            | 0.231            | -0.695          | 0.398            |
| (0.041)         | (0.292)          | (0.160)         | (0.175)          |
| Wine            | 0.271            | -0.248          | 0.149            |
| (0.020)         | (0.116)          | (0.106)         |                  |
| Spirits         | 0.498            | -0.148          |                  |
| (0.042)         | (0.151)          |                  |                  |
| **Advertising elasticity of the marginal utility of consumption** | $\mu = 0.556$ |                  |
|                  |                  | (0.242)         |                  |
sum of these effects is zero \((1 - 0.6 - 0.4 = 0)\), reflecting the fact that an equiproportional increase in advertising of all beverages has a cancelling effect. In this application it turns out that the corresponding unconditional elasticities are approximately equal to their conditional counterparts.

**Further Extensions**

There are two generalizations of the above extension of the Rotterdam model. First, equation (8.1) specifies that a one percent increase in advertising of good \(i\) changes the logarithm of that good’s marginal utility by \(\mu\), which is the same for all goods. As some goods are more sensitive to advertising than others, this specification can be generalized by replacing \(\mu\) with one parameter for each good in the group, \(\mu_1, \ldots, \mu_N\). This can be extended still further by allowing advertising of good \(j\) to directly affect the marginal utility of \(i\), so that (8.1) becomes

\[
\frac{\partial \left( \log \frac{\partial u}{\partial a_i} \right)}{\partial (\log a_j)} = \mu_{ij}, \quad i, j = 1, \ldots, N. 
\]

For the corresponding demand model, see Selvanathan (1987).

Second, it is reasonable to postulate that in addition to current advertising, past advertising also affects current sales. This can be handled by interpreting the variable \(a_i\) as the stock of accumulated advertising, rather than the current flow of advertising expenditure. This approach is used by Selvanathan (1987). Using the U.K. data, he is, however, unable to reject the hypothesis that the effects last for one year only.

**9. Alternatives to the Rotterdam Model**

In addition to the Rotterdam model there are a number of other system-wide demand models, the most popular being the linear expenditure system (LES), the translog model and the almost ideal demand system (AIDS); see Theil and Clements (1987) for a survey. While these share some features with the Rotterdam model, there are a number of differences.

The utility function underlying the LES is the sum of \(n\) subutility functions, one for each good. This form of the utility function implies that each marginal utility is independent of the consumption of the other goods, a characteristic known as preference independence. As most narrowly defined goods could not be expected to exhibit this property, the LES is probably too rigid for marketing applications.

The translog model and AIDS are more general than the LES as they are capable of dealing with the interaction of goods in the utility function. Like the Rotterdam model,
these models can be used to test the hypotheses of homogeneity and symmetry. The translog and AIDS are both based on the algebraic specification of the form of the utility function (strictly speaking, the indirect utility function and the cost function), a property shared by the LES. By contrast, the Rotterdam model is a first-order approximation to completely general demand equations and is thus approximately consistent with a wide variety of forms of the underlying utility function. In this sense, the model is more attractive than its competitors. However, until more comparative studies are available, one cannot be dogmatic and state definitely that one model will always dominate the others.

The idea of the Rotterdam model as an approximation to the true demand equations leads to a wider class of model, known as differential demand equations (Theil 1980a). Within this class the “coefficients” are not necessarily constant and can vary with income and prices. For example, the marginal share \( \theta_i \) in the Rotterdam model is specified as a constant; as an alternative, we could specify that this share equals a constant plus the corresponding budget share to give another member of the class, known as Working’s (1943) model which has been recently used by Theil et al. (1981). It is possible to devise numerous other members of the class which may be useful in marketing research. For further details, see Theil (1979).

10. Concluding Comments

In this paper we have presented a self-contained exposition of the Rotterdam model which is a leading example of a system of consumer demand equations. The model has strong links with the economic theory of the consumer and is attractive for its elegant simplicity. We showed that although the model is based on the theory of the individual consumer, it can nonetheless be applied to aggregate data.

To illustrate how the model could be employed in marketing research, we used the consumption of beer, wine and spirits. When the alcoholic beverages are block independent in the consumer’s utility function, the analyst can confine his attention to variables pertaining to those goods only. One of the features of the Rotterdam model is that the hypothesis of block independence can be tested in a straightforward way. The alcohol application included the marketing topics of identifying market structure and advertising. Our hope is that this paper will inspire marketing researchers to use the Rotterdam model in their work. (For other recent examples of the use of the Rotterdam model in marketing, see Lee 1987, and Vilcassim 1987.)

This paper was received March 1985 and has been with the authors 14 months for 3 revisions.

Appendix A1. The Covariance Term in the Macro Demand Equations

In §4 we showed that the macro demand equations contained the term \( \text{Cov} [K_{ic}, V_c] / E[M_c] \), where \( K_{ic} = M_i(\theta_i - \bar{\theta}_i); \bar{\theta}_i = E[M_i\theta_i] / E[M_i]; \) and \( V_c = d(\log Q_c) \). In this appendix we use three different approaches to show that this term is likely to be negligibly small.

The First Approach

As \( E[K_{ic}] = 0 \), we have \( \text{Cov} [K_{ic}, V_c] = E[K_{ic}V_c] \). If we assume \( M_i, (\theta_i - \bar{\theta}_i) \) and \( V_c \) are uncorrelated, we obtain

\[
E[K_{ic}V_c] = E[M_iE[\theta_i - \bar{\theta}_i]E[V_c]] \quad \text{and} \quad E[\theta_i] = \bar{\theta}_i.
\]

Therefore \( E[\theta_i - \bar{\theta}_i] = 0 \), which implies that \( E[K_{ic}V_c] = 0 \). Hence \( \text{Cov} [K_{ic}, V_c] = 0 \). It is to be noted that the assumptions required for this approach are quite strong. In the next two sections we relax these assumptions.

The Second Approach

To derive equation (4.4) of the text, we have used Khinchine’s theorem in the form

\[
\frac{1}{N} \sum_{i=1}^{N} K_{ic}V_c = \text{Cov} [K_{ic}, V_c] + o_p(1),
\]
where we have also used \( E[K_c] = 0 \). We write the left-hand side of the above equation as

\[
\frac{1}{N} \sum_{c=1}^{N} M_c(\theta_{ic} - \bar{\theta}_i)d(\log Q_c),
\]

which we denote by \( A \). If we assume \( M_c(\theta_{ic} - \bar{\theta}_i) \) and \( d(\log Q_c) \) are uncorrelated, then

\[
E[A] = \frac{1}{N} \sum_{c=1}^{N} E[M_c(\theta_{ic} - \bar{\theta}_i)]E[d(\log Q_c)] = 0,
\]

where we have used \( E[M_c(\theta_{ic} - \bar{\theta}_i)] = 0 \).

Consider the conditional variance of \( A \), conditioned by \( d(\log Q_c) \),

\[
\text{var} \left[ A | d(\log Q_c), c = 1, \ldots, N \right] = \frac{1}{N} \sum_{c=1}^{N} \text{var} [M_c(\theta_{ic} - \bar{\theta}_i)] \frac{1}{N} \sum_{c=1}^{N} [d(\log Q_c)]^2.
\]

It is reasonable to assume that \( (1/N) \sum_{c=1}^{N} [d(\log Q_c)]^2 \) does not increase systematically when \( N \) increases. Therefore \( \text{var} \left[ A | d(\log Q_c), c = 1, \ldots, N \right] \to 0 \) as \( N \to \infty \). This implies that \( A \to 0 \) as \( N \to \infty \). For more details of this approach, see Theil (1971).

The Third Approach

By writing \( d(\log Q_c) = d(\log M_c) - \sum_{j=1}^{n} w_{jc} d(\log p_j) \), we obtain

\[
\frac{\text{Cov} \left[ K_{ic}, DQ_i \right]}{E[M_c]} = \frac{\text{Cov} \left[ K_{ic}, d(\log M_c) \right]}{E[M_c]} - \frac{\text{Cov} \left[ K_{ic}, \sum_{j=1}^{n} w_{jc} d(\log p_j) \right]}{E[M_c]}
\]

\[
= \rho_{ic} \left[ \text{var} \left[ K_{ic} \right] \frac{1}{E[M_c]} \right]^{1/2} \sum_{j=1}^{n} \rho_{ij} \left[ \text{var} \left[ K_{jc} \right] \text{var} \left[ w_{jc} \right] \right]^{1/2} \frac{1}{E[M_c]}
\]

where \( \rho_{ic} = \text{correlation} \left[ K_{ic}, d(\log M_c) \right] \); and \( \rho_{ij} = \text{correlation} \left[ K_{ic}, w_{jc} \right] \).

For a developed economy it is reasonable to assume that (a) \( \theta_{ic} \approx \bar{\theta}_i, c = 1, \ldots, N \), so that \( \text{var} \left[ K_{ic} \right] \approx 0 \); (b) the variation in the change in income across consumers is small in comparison with their average income, so that \( \text{var} \left[ d(\log M_c) \right] / E[M_c] \approx 0 \); and (c) the variation in budget shares across consumers is small in comparison with their average income, so that \( \text{var} \left[ w_{jc} \right] / E[M_c] \approx 0 \). It is therefore likely that \( \text{Cov} \left[ K_{ic}, DQ_i \right] / E[M_c] \) is negligibly small. For more details of this approach, see Barnett (1979).

Appendix A2. Testing Block Independence

In the text the alcoholic beverages group was taken to be block independent of all other goods. We now test this assumption with the Australian data. The \( i \text{th} \) (unconditional) demand equation of the relative price version of the Rotterdam model is given by (3.1) which we repeat here,

\[
\tilde{w}_i Dq_{it} = \theta_i DQ_t + \sum_{j=1}^{n} v_{ij}(DP_j - DP_i).
\]

The price coefficients \( v_{ij} \) are symmetric and satisfy (2.4). Let there be \( n = 4 \) goods, beer (\( i = 1 \)), wine (\( i = 2 \)), spirits (\( i = 3 \)) and all other goods (\( i = 4 \)). When alcohol is block independent the off-diagonal \( v_{ij} \)'s involving all other goods are zero,

\[
v_{14} = v_{24} = v_{34} = 0.
\]

Our objective is to estimate (A2.1) to test restriction (A2.2).

To estimate (A2.1) we add constant terms. To identify the \( v_{ij} \)'s we set the income flexibility \( \phi \) equal to \(-0.6 \) (see Appendix A3, available on request). This \( \phi \) appears in constraint (2.4). The ML estimates \((\times 100)\) of the \( v_{ij} \)'s are as follows (asymptotic standard errors are in parentheses):

<table>
<thead>
<tr>
<th></th>
<th>Beer</th>
<th>Wine</th>
<th>Spirits</th>
<th>All other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beer</td>
<td>-1.444 (0.365)</td>
<td>0.047 (0.140)</td>
<td>-0.483 (0.154)</td>
<td>-0.246 (0.585)</td>
</tr>
<tr>
<td>Wine</td>
<td>-0.307 (0.131)</td>
<td>0.054 (0.080)</td>
<td>-0.232 (0.216)</td>
<td></td>
</tr>
<tr>
<td>Spirits</td>
<td>-0.805 (0.127)</td>
<td>0.235 (0.296)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>All other</td>
<td></td>
<td>-56.193 (1.308)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As can be seen, the off-diagonal \( v_{ij} \)'s involving all other goods are insignificant. A likelihood ratio test of (A2.2) yields a \( \chi^2 \) value of 1.46 which is not significant. Accordingly, we are unable to reject block independence.
Acknowledgements. Part of this research was carried out while the first author was McKethan-Matherly Research Fellow at the University of Florida, Gainesville, and he is grateful for the partial financial support of the McKethan-Matherly Eminent Scholar Chair. We would like to acknowledge the assistance of Peter Goldschmidt; and the helpful comments of Subrata Sen, the Area Editor and three referees of this journal.

References


