Elasticities in AIDS Models: Comment

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*American Journal of Agricultural Economics* is currently published by American Agricultural Economics Association.

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Green and Alston have written two articles (1990, 1991) for the *Journal* on the correct method of calculating elasticities for the Linear Approximate Almost Ideal Demand System (LA/AIDS). Their articles are a departure from the typical approach to the LA/AIDS. The LA/AIDS is generally used to estimate the parameters of Deaton and Muellbauer’s Almost Ideal Demand System (AIDS). The elasticity formulae for the AIDS are known; Green and Alston’s formulae are relevant only if the LA/AIDS is treated as a system in its own right. Such a treatment has merit. However, after checking the implications of treating the LA/AIDS as a system in its own right, we discover that the LA/AIDS itself lacks merit.

The value of treating the LA/AIDS as a system in its own right comes from the econometric problems of relating AIDS and LA/AIDS parameters. As Green and Alston note, there are good reasons to suspect econometric problems in approximating the AIDS with the LA/AIDS, and the parameter estimates from the LA/AIDS need not conform closely to those from the AIDS. To put it more strongly, the LA/AIDS may be useless for estimating the AIDS parameters. However, a LA/AIDS specification is an appropriate method for estimating LA/AIDS models.

An ideal feature of the AIDS is that it can be estimated to be consistent with the “adding-up,” homogeneity, and symmetry restrictions of consumer demand theory. Those who estimate the AIDS generally want their demand functions to be consistent with theory. Approximate AIDS are at best only locally consistent with demand theory.

Green and Alston were skeptical of the LA/AIDS performance. In their first article, they noted that “it is not known whether the LA/AIDS has satisfactory theoretical properties.” Their statement could have been stronger and broader. The LA/AIDS and other approximations to the AIDS do not have satisfactory theoretical properties: they can violate the symmetry restrictions of consumer demand theory for most combinations of prices and expenditures.

### Structures of the AIDS and Its Approximations

Demand equations of the AIDS and its approximations generally are written in budget share form

\[
W_i = \frac{p_i q_i}{X} = A_i + \sum_{j=1}^{N} C_{ij} \ln(p_j) + B_i [\ln(X) - \ln(P)].
\]

In equation (1), \(p_i\), \(q_i\), and \(X\) are the price of good \(j\), the quantity consumed of good \(i\), and the total expenditures on the “\(N\)” goods in the system, while \(A_i\), \(C_{ij}\), and \(B_i\) are parameters. Term \(P\) is a price index. If all \(B\) coefficients are zero, the structure of the price index is irrelevant. However, zero \(B\)'s imply that the expenditure elasticities are all unity. One advantage of the AIDS is that it is a flexible functional form and can locally approximate any demand system. Restricting the \(B\)'s to zero clearly limits the flexibility of the AIDS and its approximations.

The following restrictions on the \(A\), \(B\), and \(C\) parameters insure that the AIDS meets the adding-up, homogeneity, and symmetry restrictions

\[
\sum_{j=1}^{N} A_j = 1
\]

\[
\sum_{j=1}^{N} B_j = 0
\]

\[
\sum_{j=1}^{N} C_{ij} = 0 \forall i
\]

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Review coordinated by Steven Buccola.

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The meaning and importance of these restrictions are outlined in Deaton and Muellbauer. These restrictions are generally imposed on the LA/AIDS and other approximate AIDS models and are purported to insure that the approximate models are also homogeneous and symmetric and meet adding-up conditions.

In the true AIDS, the price index is a function of the prices and the \( A_i \) and \( C_{ij} \) coefficients. Denote the true AIDS price index as \( P^a \). The AIDS price index is defined as

\[
(7) \quad \ln(P^a) = A_0 + \sum_{j=1}^{N} A_j \ln(p_j) + \frac{1}{2} \sum_{j=1}^{N} \sum_{k=1}^{n} C_{jk} \ln(p_j) \ln(p_k).
\]

This specification of the price index makes the AIDS a nonlinear econometric model, and this nonlinearity complicates AIDS estimation. Deaton and Muellbauer themselves appeared to have had problems getting their AIDS parameter estimates to converge.

Deaton and Muellbauer solved the convergence problem by replacing the price index in (7) with a predetermined price index. They chose Stone's price index, here denoted \( P^s \).

Green and Alston refer to the AIDS model with Stone's index instead of true price index \( P^a \) as the LA/AIDS. The LA/AIDS can be written as

\[
(8) \quad \ln(P^s) = \sum_{k=1}^{N} W_k \ln(p_k).
\]

All approximate AIDS models estimated subject to the restrictions in (2), (3), and (4) meet the adding-up conditions. These conditions require that the sum of the budget shares be unity. Summing (1) over all \( i \), then using (2), (3), and (4), gives

\[
(9) \quad W_i = A_i + \sum_{j=1}^{N} C_{ij} \ln(p_j) + B_i \left( \ln(X) - \sum_{j=1}^{N} W_j \ln(p_j) \right).
\]

Note that the budget shares appear in both the right- and left-hand side of (9). The LA/AIDS is technically a nonlinear, simultaneous equation system. This structure complicates the LA/AIDS elasticity formulae, as Green and Alston demonstrate. The structure also introduces the potential econometric problem of simultaneity bias. To avoid bias, the LA/AIDS should be estimated with simultaneous equation techniques. Another way around the simultaneity bias is to use a price index whose weights do not depend on current budget shares. Eales and Unnevehr approximated the AIDS using lagged shares to create the price index. Other formulations of the price index are also possible. Denote these other price indices as \( P^o \).

All approximate AIDS suffer from potential biases arising from the use of proxy variables. Also, coefficient \( A_0 \) is not identified in any of the approximate AIDS models. Because of these two problems, the use of approximate models to estimate AIDS coefficients has to be questioned.

Because approximate AIDS estimates may be poor estimates of AIDS parameters, Green and Alston proposed treating the approximate models as the actual demand system. However, in order for such approximate models to act as true demand systems, they must be able to meet the restrictions of consumer demand theory. In general, approximate systems do not meet all the restrictions. The problem with the approximate systems is that the price index does more than just deflate expenditures. The price index also affects system performance.

**Properties of Approximate Systems**

Demand theory implies three sets of restrictions on consumer demand: adding-up, homogeneity, and symmetry. The AIDS meets these three conditions for all sets of prices and expenditures as long as its coefficients meet the restrictions in (2)-(6).

All approximate AIDS models estimated subject to the restrictions in (2), (3), and (4) meet the adding-up conditions. These conditions require that the sum of the budget shares be unity. Summing (1) over all \( i \), then using (2), (3), and (4), gives

\[
(10) \quad \sum_{i=1}^{N} W_i = \sum_{i=1}^{N} A_i + \sum_{j=1}^{N} \ln(p_j) \left( \sum_{i=1}^{N} C_{ij} \right) + \left[ \ln(X) - \ln(P) \right] \sum_{i=1}^{N} B_i = 1.
\]
The adding up conditions are met by approximate AIDS models.

Homogeneity and symmetry conditions also restrict the derivatives of the demand function. There are numerous ways of expressing these restrictions. For our purposes, it is most convenient to work with the derivatives of the budget shares with respect to the logarithms of prices and expenditures.

Homogeneity restrictions imply that the budget shares will not change if all prices and expenditures are multiplied by the same positive constant. The homogeneity restrictions can be written as

\[ \sum_{i=1}^{N} \frac{\partial W_i}{\partial \ln(p_i)} + \frac{\partial W_i}{\partial \ln(X)} = 0, \forall i. \]  

The symmetry restrictions require that compensated demand effects be symmetric. Symmetry of the compensated demand effects has the following implications for the derivatives of the budget shares:

\[ \frac{\partial W_i}{\partial \ln(p_j)} + W_j \frac{\partial W_i}{\partial \ln(X)} = \frac{\partial W_j}{\partial \ln(p_i)} + W_i \frac{\partial W_j}{\partial \ln(X)}, \forall i, j. \]

In the appendix, (12) is derived from the compensated demand effects.

First examine the homogeneity and symmetry conditions of the approximate models using a price index, \( P^0 \), that is independent of the current budget shares. Substituting the derivatives of (1) into (11) and using \( P^0 \) for \( P \) gives

\[ C_{ij} - B_j \frac{\partial \ln(P^0)}{\partial \ln(p_j)} + B_i \left\{ A_j + \sum_{k=1}^{N} C_{jk} \ln(p_k) + B_j [\ln(X) - \ln(P^0)] \right\} = 0, \forall i. \]

Using (6) to cancel the \( C_{ij} \) and \( C_{ji} \), and further simplifying (15), gives

\[ -B_j \left\{ \frac{\partial \ln(P^0)}{\partial \ln(p_j)} - \left[ A_j + \sum_{k=1}^{N} C_{jk} \ln(p_k) \right] \right\} = 0, \forall i, j. \]

The only way that (16) can be true for all possible combinations of \( A \), \( B \), and \( C \) coefficients is for the following equation to hold:

\[ \sum_{i=1}^{N} \frac{\partial \ln(P^0)}{\partial \ln(p_i)} = A_j + \sum_{i=1}^{N} C_{ij} \ln(p_i), \forall j. \]

Equation (17) holds for the true AIDS index, \( P^0 \). Constructing an appropriate \( P^0 \) requires prior estimates of the \( A \) and \( C_{ij} \) coefficients. At best, a price index with arbitrary weights would meet condition (17) at some points, but not at others.

Finding the derivatives of the LA/AIDS is more complex. Green and Alston solved for these derivatives of the LA/AIDS in their derivations of its elasticity formulae. The total differential of the LA/AIDS can be written
Equation (21) holds because of (5) and the adding-up condition that forces the budget shares to sum to one. Since the sum of the elements of each row of matrix \((C - BW')B\) is zero, the LA/AIDS meets the homogeneity conditions.

Demonstrating the violation of the symmetry conditions is more difficult due to the complicated formula in (19). It happens that the LA/AIDS is symmetric if all prices are identical, but that changing even one price can make all symmetry conditions invalid.

First, consider the total differential of the LA/AIDS when all prices are identical and equal to \(p_0\). In this case the total differential, equation (18), can be written as

\[
dW_i = \sum_{j=1}^{N} C_{ij} dLn(p_j) + B_i \left[ dLn(X) - \sum_{j=1}^{N} W_j dLn(p_j) - \sum_{j=1}^{N} dW_j dLn(p_j) \right].
\]

Following Green and Alston, the solution to (18) is

\[
dW = (I + BLnP')^{-1}[(C - BW')B]dY
\]

where

\[
B = \begin{bmatrix}
B_1 \\
B_2 \\
\vdots \\
B_N
\end{bmatrix},
\]

\[
LnP = \begin{bmatrix}
Ln(p_1) \\
Ln(p_2) \\
\vdots \\
Ln(p_N)
\end{bmatrix},
\]

\[
C = \begin{bmatrix}
C_{11}, C_{12}, \ldots, C_{1N} \\
C_{21}, C_{22}, \ldots, C_{2N} \\
\vdots \\
C_{N1}, C_{N2}, \ldots, C_{NN}
\end{bmatrix},
\]

\[
W = \begin{bmatrix}
W_1 \\
W_2 \\
\vdots \\
W_N
\end{bmatrix}
\]

and

\[
dY = \begin{bmatrix}
dLn(p_1) \\
dLn(p_2) \\
\vdots \\
dLn(p_N) \\
dLn(X)
\end{bmatrix}
\]

The homogeneity condition can be written in matrix form as

\[
\begin{bmatrix}
0 \\
0 \\
\vdots \\
0
\end{bmatrix} = (I + BLnP')^{-1}[(C - BW')B] \begin{bmatrix}
1 \\
1 \\
\vdots \\
1
\end{bmatrix}
\]

Equation (24) is identical to (6), so the typical AIDS restrictions insure that the LA/AIDS is symmetric when all prices are equal.

Suppose one price is not equal to \(p_0\). Let the price of good "I" be such that the log of its price is "u" greater than the log of \(p_0\). In this case the total differential is

\[
dW_i = \sum_{j=1}^{N} (C_{ij} - B_i W_j) dLn(p_j)
\]

Note that the sum of the elements in a row of matrix \((C - BW')B\) can be written

\[
dW_i = \sum_{j=1}^{N} (C_{ij} - B_i W_j) dLn(p_j)
\]
Equation (25) implies the following solutions for the price and expenditure derivatives:

\[
\frac{\partial W_i}{\partial \ln(p_j)} = C_{ij} - B_i W_j - uB_i \left[ \frac{C_{ij} - W_i B_j}{1 + uB_i} \right]
\]

and

\[
\frac{\partial W_i}{\partial \ln(X)} = \frac{B_i}{1 + uB_i}.
\]

Substituting (26) and (27) into the symmetry restriction (12) gives

\[
C_i - W_i B_i = uB_i \left[ \frac{C_{ij} - W_i B_j}{1 + uB_i} \right] + \frac{B_i W_i}{1 + uB_i}, \quad \forall i, j.
\]

Using (6) to cancel \(C_{ij}\) and \(C_{ji}\) out of (28), and further simplifying, gives

\[
B_i C_{ij} = B_j C_{ji}, \quad \forall i, j.
\]

Equation (29) will hold for all \(i, j\) and \(l\) if the following restrictions are imposed on the C coefficients

\[
C_{ij} = KB_i B_j, \quad \forall i, j.
\]

where “\(K\)” is some real number. Equation (30) is sufficient to insure symmetry when only one price is different than the others. More restrictions may or may not be required to make the LA/AIDS symmetric if more than one price differs from the others. In any case, (30) is a nonlinear restriction, and imposing it destroys the linearity of the LA/AIDS, complicating its estimation. Further, these additional restrictions limit the flexibility of LA/AIDS.

Conclusions

Because of deficiencies of approximations to the AIDS model, I have two pieces of advice for applied demand analysts. First, if you wish to estimate the AIDS, estimate it, not one of its linear approximations. Second, if you want a model linear in parameters and consistent with utility maximization, a number of models that are specified in terms of first differences are available. Keller and van Driel discuss three such demand systems: the Rotterdam system, a differential version of the AIDS, and a hybrid system they developed which they called the CBS system. Barten and Bettendorf discuss the inverse forms of these three demand systems.

[Received April 1993; final revision received February 1994.]

References


Appendix

Deriving the Demand Restrictions in Terms of Budget Shares

The symmetry restrictions for demand functions are

\[
\frac{\partial q_i}{\partial p_j} + q_j \frac{\partial q_i}{\partial X} = \frac{\partial q_i}{\partial p_i} + q_i \frac{\partial q_i}{\partial X}.
\]

Multiplying both sides of equation (A1) by \(P, P/X\) and using identities

\[
\frac{\partial q_i}{\partial \ln(p_j)} = p_i, \quad \frac{\partial W_i}{\partial p_j} = \frac{\partial q_i}{p_i}, \quad \frac{\partial q_i}{\partial p_j X}.
\]

References


\[
\frac{\partial X}{\partial \ln(X)} = X, \frac{\partial W_i}{\partial X} = \frac{\partial q_i}{\partial X} p_i - \frac{W_i}{X}
\]
gives

\[
(A3) \quad \frac{\partial W_i}{\partial \ln(p_i)} + W_iW_j + W_i \frac{\partial W_i}{\partial \ln(X)} = \frac{\partial W_j}{\partial \ln(p_i)} + W_iW_j + W_i \frac{\partial W_j}{\partial \ln(X)}.
\]

Terms \(W_iW_j\) can be cancelled from both sides of (A3) to give (12).