Household Production Theory, Quality, and the "Hedonic Technique"

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A feature of many of the empirical papers which use the "hedonic technique" of correcting prices for quality change is their reference to work on household production by Kelvin Lancaster and Richard Muth (and sometimes Gary Becker) as providing a theoretical foundation. However, the exact nature of this theoretical support has never been adequately spelled out.

Roughly speaking, exponents of the hedonic technique (see Zvi Griliches (1971) for a very useful survey and also Jack Triplett and Robert Gordon) take the view that the quality of a good is related to measurable specification variables or characteristics such as size, performance, etc. All the empirical applications regress prices or the logs of prices of the different varieties or models of a type of good on these characteristics. However, there are two main variants of the empirical forms which have been used: single year cross-section regressions, and pooled (over at least two years) time-series/cross-section regressions.

The first variant claims to estimate the "shadow prices" of the "characteristics" of the goods for a given year. These shadow prices are then used to price some average bundle of characteristics in the two years which are being compared. If, say, the base year's level of characteristics is used, an index which looks similar to a Laspeyres price index can be constructed. In the second method, the time dummy variables which link the cross-sections are supposed to pick up differences in general price levels at different times. The inclusion of the specification variables then controls for the effects of changes in quality, i.e., constrains quality to be a constant. In other words, the variation in the dependent variable (price or log of price) is broken up into two parts. One reflects changes in the average price level over time, the other reflects differences in characteristics.

The main purpose of this paper is to relate the hedonic technique to a theoretical constant utility price index when quality changes are taken into account. It shows that the conditions under which precise statements can be made about the relationship of the usual empirical approximations to such a theoretical price index are rather restrictive. These conditions impose interesting and quite strong restriction on the classes of empirical techniques of quality correction which can thus be rigorously justified.

The approach is unashamedly one-sided; only the demand side is treated. The orientation of the paper is towards the analysis of the welfare of individual consumers and less towards the explanation of market phenomena. Its subject matter is therefore rather different from that of the recent paper by Sherwin Rosen. The supply side and the simultaneity problems which may arise are ignored. This is done partly for reasons of space given that the analysis of demand is prior to market analysis. How-

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ever, sometimes it may also be realistic, as with good second-hand markets, to assume that consumers are price takers and adjust fully.

The structure of the paper is as follows. After the summary and conclusions, which are hopefully self-contained, Section II spells out the basic utility maximizing model within the household production framework. A true (constant utility) price index is defined. Section III introduces quality change and discusses the difficulty of taking account of this in the true price index. Section IV discusses conditions under which these difficulties can be overcome. Section V discusses some special household production models, in particular that of Lancaster. Section VI introduces an alternative approach outside the household production framework. This is based on a generalization of the one-good model proposed by Fisher-Shell. Section VII deals with a quite different strand in the literature originating in Hendrik Houthakker's 1952 paper. Here shadow prices are assumed to exist as part of the supply conditions which are external to the household. One of the major conclusions, therefore, of the present paper is that there are three separate classes of theories that can be said to underly empirical work on quality change.

I. Summary and Conclusions

The development of the theory and the conclusions of the paper as regards the first of these approaches may be summarized as follows.

A true price index is defined as the relative expenditure under two price regimes required to reach a given level of utility. If tastes are constant and there is no quality change, the form of the functional relationship between expenditure on the one hand and prices and utility on the other is constant. In this case the traditional Laspeyres and Paasche approximations to a true index are valid and well known. However, if quality changes, the form of this functional relationship changes. It then follows that the traditional approximations are in general no longer valid. The solution that the household production approach then proposes is to hypothesize that a household derives utility from some basic goods (Z) whose qualities do not change and that these are produced from market inputs (x) whose qualities can change. If one can at least identify the shadow costs (\( \pi \)) of Z, one should be able to approximate the true price index without necessarily knowing the full household technology and tastes.

However, if the index is to make much sense, the shadow value of Z must have a stable relationship with the market value of x at a given technology. This "consistency condition" is satisfied if the technology is homogeneous of any positive degree in inputs and outputs. A Laspeyres index is then defined on \( \pi \) (and the prices of other market goods, if any).

In conventional theory without quality change, a Laspeyres index is an upper bound on the "true" index using base utility as reference. The reason why this may not be true in the presence of quality change is that the relative \( \pi \) which correspond to the actual current period purchases may be different from the relative \( \pi \) which are not directly observable and which correspond to current market prices and base utility. There are two alternative conditions which will guarantee that this problem does not arise. The first is a restriction on the technology (specified as a form of separability of the cost function for Z) which implies a linear transformation locus in Z and in addition, constant returns to scale in the production function relating Z to x. This restriction is equivalent to nonjointness. Nonjointness implies that the level of each input \( x_i \) can be split into separate parts, each of which is used...
in only one branch of production, i.e., for only one $Z_j$. This special case is an example of Paul Samuelson's well-known "non-substitution theorem."

Nonjointness, however, is unrealistic for many examples of household production and poses serious problems in practical identification of $Z$. For example, the use of an automobile produces travel services and psychic rewards jointly. The theoretical advantage that consumers with different tastes and incomes will then face the same shadow costs (given the same technology for each consumer) is therefore bought at a high price. But this is the only case when the "aggregation problem" cannot arise. The theorems which I shall prove to establish this proposition imply a criticism of some of the work of the household production theorists. Much of the alleged usefulness of their approach stems from the ability to argue about likely consequences on the $Z$ of changes in $r$. However, if different consumers face a different $r$, little information in the form of empirical restrictions on behavior is gained by making the usual a priori assumptions of the household production theorists.

The second alternative is to assume that the utility function is homothetic in $Z$. It then follows that, at given prices and under constant returns, the shadow costs will be the same regardless of the consumer's utility level. However, homotheticity which implies unitary income elasticities is not supported by empirical evidence.

These general problems can be illustrated by a discussion of some special cases. A good example is the Lancaster model, earlier proposed by W. M. Gorman, with, in addition, a variation such that $Z$ is defined as the sum of characteristics with the logs of $x$ instead of $x$ as weights. For goods purchased by the consumer, the Lancaster model implies a linear relationship between the market prices and the characteristics with $\pi$ as parameters. This can be interpreted as a justification of the first version of the empirical hedonic technique mentioned in the introduction. Gorman's variation implies that $x$ enters the price-characteristics relationship. I prove a simple theorem which shows that a semilog price-characteristic relationship cannot be directly interpreted in the household production framework. However, a partial semilog relationship between the price and any single characteristic is permitted. This is an important empirical restriction, since the semilog form has been widely used in applications.

Moreover, the Lancaster model is not altogether plausible, particularly with regard to durable goods which are the most frequent objects of study in the empirical hedonic literature. For, although the divisibility assumption is just as implausible in other branches of the consumer literature, in Lancaster's model it does play a central role. Further, it is often true that when multiple varieties are available, the consumer buys amounts of just one at a time. Although this is not excluded by the Lancaster model (this being the case of an optimum at a vertex), the model predicts that this is not a frequent occurrence unless the indifference curves are linear.

Section VI introduces an alternative model of consumer behavior which overcomes this criticism and provides an alternative framework for the analysis of quality change. Basically, this involves the imposition of strong separability-type restrictions on the utility function. These are derived from generalization to a group of goods of the Fisher-Shell "simple repackaging hypothesis." A homothetic category utility function is assumed to exist for a group of goods, defined as the weighted sum of the $x$, where the weights are interpreted as quality indices. This implies straight line indifference curves between the $x$. The quality indices are defined to be an a priori unrestricted function of the
characteristics. If a section of the market can be identified where consumers have similar tastes, then the relative values of the quality indices for the purchased goods must be approximately equal to their relative prices. Market competition enforces this. Thus, there is a multiplicative relationship of the form $p_n = p_t \cdot g(\text{characteristics})$, in which $p_t$ can be interpreted as a general price index. This results in a pooled time-series/cross-section regression model of $\log p_n$ on characteristics and time dummies whose coefficients are interpreted as price indices. It is important to note that the dependent variable must appear in log form if the regression is to be linear in the parameters. The parameters of the function $g(\ )$ are taste parameters and hence presumed stable. This then corresponds to the second of the two versions of the hedonic technique mentioned earlier.

Reflection will show that the models so far discussed are just relatively refined ways of "judging quality by price." It is paradoxical that the inferences concerning the evaluation of quality which can be drawn from the estimation techniques implied by these models are likely to break down if too many consumers are lazy and do judge quality by price. A rational individual would not put his trust in the market mechanism in this way if his marginal rates of substitution (MRS) are different from those of others. However, if his MRS are different, there are grave problems of aggregation. This is not just a matter of taste differences; I would regard income differences as even more important. The MRS which are being implicitly measured depend, in general, on the utility level as well as on tastes. This can be formally introduced into the simple repackaging model by making the function $g(\ )$ depend on the utility level or on past consumption as well as on characteristics. This, however, causes severe problems for the kind of empirical work so far discussed, because the market then no longer forces relative prices to correspond to a unique consumer evaluation of relative characteristics.

The major implication of all this for hedonic studies is that, at the very least, careful attention should be paid to cross-sectional disaggregation. As far as possible, markets should be broken into segments based on commodity groupings which make it likely that their consumers have similar MRS and these segments should be studied separately. This means that in any sample there will be less cross-sectional variation and hence more problems of multicollinearity. This suggests that investigators will not be able to afford the luxury of intertemporal disaggregation, i.e., separate cross-section regressions for each year. Since this procedure corresponds to the household production model, it appears that the simple repackaging model (which implies pooling time-series and cross-sectional data) represents the more practical theoretical framework.

For some goods a broader research strategy may need to be pursued. In my 1974a paper, I show that in the United Kingdom at least, the cost of living indices of different income groups have increased at different rates (most for the poorest in recent years). This could well be a general problem in some markets for durables. To tackle the problem, it is necessary to combine consumer survey data on durables purchases and ownership with data on prices and characteristics. At least the relevant market segments may then be determined.

In Sections II to VI, it is assumed that consumer preferences are directly reflected in relative prices thus allowing welfare statements about consumers to be deduced. In the tradition of Houthakker, some authors (for example, Phoebus Dhrymes and Makoto Ohta) have taken an opposing view and argued that hedonic
relationships are estimates of manufacturers' pricing strategies or of cost functions. However, Houthakker was not as explicit. He assumes that the consumer is faced with what I would call a “tariff schedule.” In the example which he analyzes, the consumer is faced with a base tariff per unit of the good and a second tariff per unit of the characteristic embodied in it. This has been extended by Rosen to a more general shadow price schedule which, he argues, is an equilibrium relationship sustainable in a competitive, well-informed, decentralized market. I have argued in my 1973 paper that the well-known paper by Irma Adelman and Griliches is best interpreted in this general vein. In the present paper, I correct a technical shortcoming in Houthakker and Rosen. This is the assumption that only one variety is bought at a time by one consumer. Only the simple repackaging model yields this behavior as an outcome of the optimizing process.

The important point to notice is that if hedonic relationships in part reflect manufacturers' pricing strategies, there is a problem in identifying consumer taste parameters. Since linear indifference loci in $x$ imply corner solutions, the market shadow prices may not reflect those of consumers. These identification problems are not dealt with in this paper. However, they are very important. If the purpose of the hedonic approach is to make welfare comparisons for consumers over time, it is defeated if their marginal rates of substitution cannot be identified.

There is one context in which it may be plausible, however, to argue that consumer tastes dominate. This is in second-hand durables markets where aggregate supply is fairly stable (i.e., approximately exogenous) and particular supplies are usually held in a decentralized fashion. There the models of consumer behavior discussed in this paper are highly relevant if homogeneous market segments can be identified.

The quality problem is a fundamental one in economics. It raises in sharp form the difficulties involved in drawing welfare conclusions from market data. These are problems of interpersonal comparability, of aggregation (see my 1974b paper for a further analysis)—especially of income and taste differences—and of whether market prices represent valid information about rates of substitution and rates of transformation. These are problems which have to be faced in the entire area of aggregative index numbers. They are both interesting and important.

II. The Utility Function and the Household Production Function

I examine a household which purchases market goods in quantities $x_1, \ldots, x_m$ (which yield no direct utility) whose purpose is to jointly produce the commodities $Z_1$ and $Z_2$ which yield utility according to

$$U = U(Z_1, Z_2)$$

where $U(\ )$ is assumed to be convex. Let the joint production function be $F(x_1, \ldots, x_m; Z_1, Z_2) = 0$. We assume that $F(\ )$ is “neoclassical,” i.e., given $x_1, \ldots, x_m$, the production possibility frontier in $Z_1$ and $Z_2$ is concave, and the isoquants in $x_i$ given $Z_1$ and $Z_2$ are convex. In addition, we assume that the household faces the budget constraint

$$y = \sum p_i x_i \quad i = 1, \ldots, m$$

The solution to the utility maximization problem can be thought of in two stages.

Stage 1: Minimize $C = \sum p_i x_i$ of producing any given bundle $Z_1, Z_2$. Let the

1 The assumption that only two goods are produced and that no other goods enter the utility function is made purely for the sake of simplicity of exposition. The general case when other goods enter $U(\ )$ is briefly discussed below.
Lagrangian be (where $x$ is the vector $(x_1, \ldots, x_m)$)

(3) \[ L_1 = \sum \pi_i x_i + \pi(F(x; Z_1, Z_2)) \]

Let the solution be given by the cost function $C = C(p; Z_1, Z_2)$ (see R. W. Shepard and Hirofumi Uzawa on the duality between $F(\cdot)$ and $C(\cdot)$) where $p$ is the vector $(p_1, \ldots, p_m)$. Define the shadow marginal cost of producing $Z_j$

(4) \[ \pi_j \equiv \frac{\partial C}{\partial Z_j} \quad j = 1, 2 \]

Differentiating (3) with respect to $Z_j$ gives

(5) \[ \pi_j = \pi \frac{\partial F}{\partial Z_j} \quad j = 1, 2 \]

**Stage 2:** Maximize $U = U(Z_1, Z_2)$ subject to the constraint

(6) \[ y = C(p; Z_1, Z_2) \]

Let the Lagrangian problem be

(7) \[ \max \mathcal{L}_2 = U(Z_1, Z_2) + \lambda(y - C(p; Z_1, Z_2)) \]

This is illustrated in Figure 1. The frontier $AB$ (at time 0) is obtained by solving $y_0 = C(p_0; Z_1, Z_2)$ for $Z_1$ in terms of $Z_2$. Thus $AB$ is concave when $F(\cdot)$ is neo-classical. The utility maximizing point on $AB$ is $Z^*$.

The constant utility price index is easily defined in the absence of quality change. Then the form of $C(\cdot)$ and of $F(\cdot)$ is unchanged. If expenditure is $y_t$ and prices $p_t$, the new frontier is given by $y_t = C(p_t; Z_t)$. The optimal point on it is $Z^*$. Define $y^*_t$ as the minimum expenditure to attain $U_0$ at prices $p_t$. Then the constant utility price index (with $U_0$ constant) is given by $y^*_t/y_0$. This can be explicitly written in terms of the expenditure function $y = \bar{C}(p; U)$ whose functional form depends on those both of the cost function $C(\cdot)$ and the utility function $U(\cdot)$. Thus

(8) \[ \frac{y^*}{y_0} = \frac{\bar{C}(p_t; U_0)}{\bar{C}(p_0; U_0)} \]

### III. Quality Change

When the quality of one or more of the $\{x_i\}$ changes, the household technology changes. The only formal alterations in (8) are the time subscripts $t$ and 0 that must now be given $\bar{C}(\cdot)$. It is even less plausible that the parameters of $\bar{C}(\cdot)$ could be discovered.

My brief discussion above of the first version of the hedonic technique as used in practice may have suggested that knowledge of $\bar{C}(\cdot)$ is not necessary. If shadow prices $\pi_{1t}$, $\pi_{2t}$ can be estimated, $\bar{C}(\cdot)$ is known to a local linear approximation. Then one might argue that $(\pi_{1t}Z_{10} + \pi_{2t}Z_{20})/y_0$ would at least provide an approximation with a known relationship to the true index. However, the well-known result of the conventional theory of market goods that a Laspeyres index is an upper bound on the true index does not in general hold here. The reasons are twofold. First, it may not be true that $\pi_1Z_1 + \pi_2Z_2$ has a stable relationship with market costs. Since
our index is given in ratio form, what we need is for the consistency condition
\begin{equation}
y_t = \frac{\pi_{1t}Z_{1t} + \pi_{2t}Z_{2t}}{\pi_{1a}Z_{1a} + \pi_{2a}Z_{2a}} \tag{9}
\end{equation}
to be fulfilled. This condition is stated for a given technology where s and t refer to different time periods. Define a Laspeyres index:
\begin{equation}
\frac{\pi_{11}Z_{10} + \pi_{21}Z_{20}}{\pi_{10}Z_{10} + \pi_{20}Z_{20}} \tag{10}
\end{equation}
The consistency requirement is clearly a minimum requirement for (10) to make much sense.

Now I shall prove the following:

**THEOREM**: The consistency condition for the price index is met if \( F(x; Z_1, Z_2) \) is homogeneous in both inputs and outputs.

**PROOF**: We have to show that \( \sum_{i=1}^{m} p_i x_i = k \sum_{j} \pi_j Z_j \) where \( k \) is independent of \( Z_j \) and homogeneity holds.

Giora Hanoch has shown that \( F( ) \) is homogeneous of degree \( a \) in inputs and outputs.

Thus, substituting (12) and (13) into (11), we see that homogeneity is equivalent to \( \sum_{i} p_i x_i = a \sum_{j} \pi_j Z_j \) where \( a \) is independent of \( Z_j, p, \) and \( y \), which is the desired result. Note that \( k = a \).

The theorem requires both homogeneity and that the degree of homogeneity not be affected by quality change. If these conditions are fulfilled, then the constant utility index would be exactly known if the shadow prices at \( Z^0 \) and \( Z^* \) were known.

The second reason why the relationship of (10) to \( y_t^*/y_o \) is in general unknown is that, even if observable, the shadow prices \( \pi_{1t}, \pi_{2t} \) correspond to the actually bought bundle \( Z_t \) rather than the hypothetical bundle \( Z^* \). Figure 2 suggests that (10) could lie above or below \( y_t^*/y_o \). The tangent at \( Z^* \) and the dotted lines through \( Z^0 \) and \( Z^0^* \) have slope \(-\pi_{2t}/\pi_{1t}\). The actual shadow costs at \( Z^* \) may be such that \( \pi_{1t}Z^*_1 + \pi_{2t}Z^*_2 \) could be greater than or less than \( y^* \). Even if \( y^* = \pi_{1t}Z^*_1 + \pi_{2t}Z^*_2 \), then if \( Z^0 \) were the period 0 constraint, (10) exceeds \( y_t^*/y_o \), while if \( Z^0^* \) were the period 0 constraint, (10) lies below \( y_t^*/y_o \).

The next section shows that there are two sets of circumstances under which this problem is resolved. Also to be faced are three further practical difficulties: the
measurement of $Z_1$ and $Z_2$, the observation of the shadow costs $\pi_1$ and $\pi_2$, and aggregation across consumers.

IV. When Do $\pi_{1t}$, $\pi_{2t}$ Correspond to Current Prices and Base Utility?

I shall show that there are two alternative conditions which guarantee this. As we have seen, the problem is that the shadow costs appropriate to $Z^*$ may not be those actually observable at $Z^t$. This problem would not arise if either (a) relative shadow costs are independent of $Z$ or (b) given constant returns, the optimum points at given prices $p$ and given technology, all lie on one expansion path. The discussion above relating to Figure 2 suggests that one requirement for the resolution of the above problem is that the transformation locus be a straight line.

It is possible to show the following:

**Theorem:** A necessary and sufficient condition on the technology for $w_1/\pi_2$ to be independent of $Z_1$ and $Z_2$ (and of $U$ and $C$) is that the cost function has the form

$$C = C(\alpha_1 Z_1 + \alpha_2 Z_2, p)$$

where $\alpha_j$ depends only on $p_1, \ldots, p_m$.

**Proof:**

Sufficiency is obvious; we concentrate on necessity. The condition,

$$\frac{\partial C}{\partial Z_1} = \frac{\pi_1}{\pi_2}$$

implies a straight-line transformation locus. For a fixed value $C = C'$, the equation of this locus is

$$C' = \alpha_1 Z_1 + \alpha_2 Z_2$$

where

$$\frac{\alpha_1}{\alpha_2} = \frac{\pi_1}{\pi_2}$$

Since the same is true of any monotonic function, $C(C')$ where $G(\cdot)$ can depend on $p$, the general solution to equation (14) and $C = C(p; Z_1, Z_2)$ is

$$C = C = G(\alpha_1 Z_1 + \alpha_2 Z_2, p)$$

If $F(\cdot)$ is homogeneous of degree $a$, $C(p; Z_1, Z_2)$ is homogeneous of degree $1/a$. Then (16) takes the form

$$C = h(\alpha_1 Z_1 + \alpha_2 Z_2)^{1/a}$$

and

$$\frac{\alpha_1}{\alpha_2} = \frac{\pi_1}{\pi_2}$$

By homogeneity, Euler's equation implies

$$h(\alpha_1 Z_1 + \alpha_2 Z_2)^{1/a} \equiv a(\pi_1 Z_1 + \pi_2 Z_2)$$

Thus

$$\pi_j = \frac{h}{\alpha_j(\alpha_1 Z_1 + \alpha_2 Z_2)^{1/a-1}}, \quad j = 1, 2$$

At first sight it appears as if in the absence of restrictions on tastes, homogeneity together with a straight line transformation locus, i.e., a cost function of the form (17), ought to be enough to guarantee that the Laspeyres index (10) is an upper bound on the true index (8). It is certainly clear that irrespective of technology,

$$\frac{C_t(p_t; Z_{10}, Z_{20})}{C_0} \geq \frac{C_t(p_t; U_0)}{C_0}$$

since $(Z_{10}, Z_{20})$ yields utility $U_0$, though not necessarily in the cheapest way. However, given the technology of (17), the following reverse inequality may be true:

$$\frac{\pi_{1t} Z_{10} + \pi_{2t} Z_{20}}{\pi_{10} Z_{10} + \pi_{20} Z_{20}} < \frac{h(\alpha_1 Z_{10} + \alpha_2 Z_{20})^{1/a}}{C_0}$$

Substituting from (18) and using $C_0 = h_0 \cdot (\alpha_1 Z_{10} + \alpha_2 Z_{20})^{1/a}$, it is easy to show that the inequality in (20) is satisfied with increasing returns to scale (i.e., $a > 1$) if $Z_{1t} > Z_{10}$ and $Z_{2t} > Z_{20}$, and with diminishing

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I am very grateful to Pollak for saving me from an error on this point.
returns to scale if \( Z_{1t} < Z_{10} \) and \( Z_{2t} < Z_{20} \).

The economic interpretation of this is that the shadow costs fall with output expansion under increasing returns and output contraction under diminishing returns. Therefore it is possible for the Laspeyres index (10) to lie below the true index (8).

It is clear therefore that the further restriction of homogeneity of degree one, i.e., constant returns, must hold if this problem is to be resolved. Then the cost function can be written in the form, say,

\[
\begin{align*}
C &= h\alpha_1 Z_1 + h\alpha_2 Z_2 \\
&= \tilde{C}_1(p) Z_1 + \tilde{C}_2(p) Z_2
\end{align*}
\]

Robert Hall (1973) drawing on Samuelson (1966) has shown that (21) is equivalent to constant returns and nonjointness. Nonjointness in the inputs \( x_1, \ldots, x_m \) means that the joint production function \( F(x_1, \ldots, x_m; Z_1, Z_2) = 0 \) can be written as

\[
\begin{align*}
Z_1 &= f_1(x_{11}, \ldots, x_{1m}) \\
Z_2 &= f_2(x_{21}, \ldots, x_{2m})
\end{align*}
\]

where \( x_i = x_{i1} + x_{i2} \) for \( i = 1, \ldots, m \)

Under constant returns and nonjointness, the shadow costs are independent of \( Z_1, Z_2 \).\(^3\) This is essentially the nonsubstitution theorem of Paul Samuelson (1951). In the current context, apart from providing a sufficient condition for the Laspeyres index of shadow costs to be an upper bound on the true index, it has the additional advantage of solving the problem of aggregation across consumers: as long as each faces the same technology, each will face the same shadow costs irrespective of tastes. The interest of form (16) is that for relative shadow costs, the aggregation problem is solved without nonjointness having to be assumed. However, neither (16) nor (21) are at all general. Both, for example, are inconsistent with the characteristics approach of Lancaster (see below) in which jointness is at the core.

Although the gain from being able to assume nonjointness is thus considerable, it is often implausible for household production and rather demanding in terms of information on internal allocation.\(^4\) The problem for hedonic work is not only in measuring the shadow costs but in identifying \( Z \). It is implausible to regard \( Z \) as being embodied in the goods. Nonjointness implies that the level of each input can essentially be split into separate parts, each of which is used in only one branch of production. Thus if \( x_i = x_{i1} + x_{i2} \), the production of \( Z_1 \) depends on \( x_{i1} \) and that of \( Z_2 \) on \( x_{i2} \). Many types of consumption are not, however, of this type. For example, the use of an automobile provides travel services and psychic rewards jointly.

Fortunately, there is a second, less restrictive condition which although it does not solve the aggregation problem is sufficient for the Laspeyres index always to be an upper bound on the constant utility index. Given constant returns of the household production function, this is simply that the utility function be homothetic.

\(^3\) The same conclusions follow if instead of market good \( x_1 \) we have labor in the following model:

\[
\begin{align*}
Z_1 &= f_1(L_1, x_{11}, \ldots, x_{1m}) \\
Z_2 &= f_2(L_2, x_{21}, \ldots, x_{2m})
\end{align*}
\]

and the budget constraint is replaced by

\[
\sum_{i=1}^{m} p_i x_i = w(L - L_1 - L_2)
\]

where \( L \) is the total time which can be allocated either to household production or paid employment and where \( w \) is the wage rate.

\(^4\) A point about allocative Pareto efficiency can also be made: although household production is a non-market activity, given nonjointness and constant returns, the same shadow prices rule in different households. This is a result rather like the factor price equalization theorem. Much of the alleged usefulness in terms of potential empirical predictions of the household production approach rests on this result. Drawing on Hall’s result, see (21), Pollak and Michael Wachter point out this approach has little content if the shadow prices vary across households. Thus much of the literature based on this approach rests on very shaky empirical foundations.
This guarantees that given prices and given technology, the relative shadow costs are the same whatever the level of consumption. The reason is obvious: constant returns in production and homothetic preferences together ensure that the optimal points will always be on the same expansion paths and hence correspond to the same marginal rate of transformation. A glance at Figure 3 will convince the reader that then the problem revealed in Figure 2 disappears: if \( Z^0 \) can be bought at \( \pi_{1t}, \pi_{2t} \) then \( Z^* \) can always also be bought. Note that again we need constant returns rather than merely homogeneity so that the total shadow cost just equals the market cost. Homotheticity, i.e., unitary income elasticities of demand for the \( Z \) is also a highly restrictive requirement which is usually refuted by empirical work.

Introducing other market goods does not create serious problems. There are two cases: separability and nonseparability.

**Separability:** We write the utility function

\[
U = U(X_0(Z_1, Z_2), X_1, \ldots, X_n)
\]

Then the consumer chooses in two stages: first how to allocate his budget among \( X_0, X_1, \ldots, X_n \) and given the budget for \( X_0 \), how to choose the optimal values of \( Z_1 \) and \( Z_2 \). Then a category price index can be defined as the expenditure necessary to reach a given level of category utility \( X_0 \). Such a category price index is easily incorporated into an overall price index.

**Nonseparability:** a proper subindex cannot in general be defined when

\[
(23) \quad U = U(Z_1, Z_2, X_1, \ldots, X_n)
\]

However, with nonjointness and constant returns, an upper bound as before is given by:

\[
(24) \quad a \sum_{j=1}^{2} \pi_{jt} Z_{jt} + \sum_{i=1}^{n} P_{it} X_{i0}
\]

Increasing the number of \( Z \) beyond two does not affect any of the substantive results of this paper.

**V. Some Special Cases**

The best known model of household production is that of Lancaster. Here we have

\[
(25) \quad Z_1 = \sum b_{1i} x_i, \quad Z_2 = \sum b_{2i} x_i
\]

Each unit of good \( i \) is made up of a fixed amount \( b_{1i} \) of \( Z_1 \) and \( b_{2i} \) of \( Z_2 \).

The transformation locus corresponding to a given income and market prices can be represented as in Figure 4.

For any given bundle \( Z_1, Z_2 \), it can be assumed that the consumer will have minimized the cost of purchasing that bundle subject to \( x_i \geq 0 \) at the given prices \( p_i \).

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1. This is a well-known point, see Gorman (1961). For a particularly lucid explanation, see Pollak.
2. We do not make Lancaster's distinction, here inessential, between market goods and "activities." Note by the way that the technology reveals constant returns.
Given divisibility, shadow prices \( \pi_1 \) and \( \pi_2 \) corresponding to this linear programming problem can be calculated. As Richard Lipsey and Gideon Rosenbluth show, for purchased goods

\[
\pi_i = \pi_1 b_{1i} + \pi_2 b_{2i}
\]

In principle, linear programming methods could be used to compute the \( \pi_j \). However, a serious difficulty is likely to be that the solution will be sensitive to the exact way the problem is specified: information on the \( b_j \) will never be available in enough detail and with enough accuracy. The form of (26) also suggests a regression approach. Because some sort of average relationship, though ideally disaggregated according to market segments, is then estimated and the \( b_j \) are typically somewhat collinear, the omitted or misspecified variable problem is likely to be less serious. Thus two problems mentioned above, i.e., measuring \( Z_j \) and estimating \( \pi_j \) appear to be solved.\(^7\)

There remains the problem of aggregating across consumers. Clearly, if consumers’ indifference curves are similar enough\(^8\) in shape, they will all choose points corresponding to the same \( \pi_1/\pi_2 \). Then, abstracting from information problems, for the sellers of a good to sell anything, they must price according to the relationship \( p_i = \pi_1 b_{1i} + \pi_2 b_{2i} \). In a way, the identity of tastes enforces a strong form of perfect competition.

If the indifference curves are not similar enough, different people will choose points corresponding to different values of \( \pi_1/\pi_2 \). In this case if one nevertheless wants to pick up some sort of median relationship, there is an argument for weighting the observations by value shares. However, for the poorer members of the community, the price quality relationships may well be substantially different and it is difficult to know what meaning to attach to the estimated relationships.

Suppose now that the production function is additive but not linear. For example, suppose\(^9\)

\[
\begin{align*}
Z_1 &= \sum b_{1i} \log x_i \\
Z_2 &= \sum b_{2i} \log x_i
\end{align*}
\]

Here

\[
x_i p_i = \pi_1 b_{1i} + \pi_2 b_{2i}
\]

It is no longer so plausible to regard \( b_{ji} \) as the amount of the \( j \)th characteristic

---

\(^7\) Incidentally, it may be that one unit of the good, irrespective of how much \( b_1 \) and \( b_2 \) are embodied in it, has an intrinsic value. In that case, \( p_i = \pi_1 + \pi_2 b_{2i} \) which is a form often fitted. Here the utility function is \( U = U(Z_0, Z_1, Z_2) \) where \( Z_0 \) is the number of units of market goods consumed.

\(^8\) This not only implies similarity of tastes; if tastes are distinctly nonhomothetic, then incomes would have to be similar as well.

\(^9\) Gorman in a regrettablly unpublished paper from 1956 anticipates Lancaster’s work to a remarkable degree. Gorman considered the two cases, i.e., \( Z_j = \sum b_{ji} x_i \) and \( Z_j = \sum b_{ji} \log x_i \). However, he assumed that \( b_{ji} \) are unobservable. He proposed, in great detail, that factor analysis could be used to estimate simultaneously (at least up to constants of proportionality) \( b_{ji} \) and \( \pi_j \). In a sense, he can be said to have solved a problem more difficult still than that solved by the hedonic technique, well before the latter achieved any currency. His suggestion of factor analysis could also be useful in analyzing the residuals for regressions where it is thought that not all the relevant \( b_{ji} \) were included.
embodied in the $i$th market good. If $b_{ji}$ is the horsepower of the model, it is now no longer total horsepower which is relevant, but the sum of horsepower weighted by the logs of the quantities purchased. Notice that in this form, the relationships between the prices of the market goods and $b_{ji}$ involve the quantities purchased, $x_i$.\textsuperscript{10} Not only does this cause severe aggregation problems, but since the technology is not constant returns, the Laspeyres upper bound may not hold.

The most popular empirical form for hedonic regressions has been of $\log p_i$ on $\sum \pi_j b_{ji}$. It is easy to see that the following impossibility theorem holds:

**THEOREM:** The semilog form of the price/characteristics relationship is not possible in the household production framework.

**PROOF:**

The most general way of formulating models like (25) and (27) to incorporate information from the characteristics is as $Z_i = f_i(b_{j1}, \ldots, b_{jm}; x_1, \ldots, x_m)$.

$$p_i = \pi_1 \frac{\partial f_1}{\partial x_1} + \pi_2 \frac{\partial f_2}{\partial x_2}$$

Thus we need

$$\pi_1 \frac{\partial f_1}{\partial x_1} + \pi_2 \frac{\partial f_2}{\partial x_2} = e^{\pi_1 b_{j1} + \pi_2 b_{j2}}$$

This cannot be satisfied for any nontrivial version of the function $f_i(\cdot)$ because an additive form in $\pi_1$, $\pi_2$ cannot be identical to a multiplicative form in $\pi_1$, $\pi_2$, i.e., $a\pi_1 + b\pi_2 \neq A(\pi_1) \cdot B(\pi_2)$.

The nonjoint model is no more yielding in this respect. Also, the nonjoint model of household production suffers from the handicap that the allocation of $x_i$ within the household and hence $Z_i$ are typically unobservable.\textsuperscript{11}

\textbf{VI. An Alternative Approach: The Simple Repackaging Hypothesis}

Outside the household production framework, and in particular the Lancaster model, there is only one other simple theoretical model of consumer behavior in which the hedonic technique as currently practiced can be interpreted as a direct reflection of consumer behavior.

Under this hypothesis, it is assumed that each market good has a quality index which is a function of a set of physical characteristics. This relationship is the same for all market goods of a general type (say, refrigerators or some grouping of refrigerators) and being a question of tastes, is independent of market variables. Under this assumption, which is essentially the Fisher-Shell assumption of "simple repackaging" quality differences and quality change, market goods of a given type can be aggregated; the aggregate is simply the sum of the quality indices weighted by the number of units of each good purchased. Formally, the utility function can be written\textsuperscript{12}

$$U = U\left(X_0\left(\sum_{i=1}^{m} a_i x_i\right), X_1, \ldots, X_n\right)$$

where $X_0(\cdot)$ is the category utility function for the group $(x_1, \ldots, x_m)$ and $X_s$, $s=1, \ldots, n$, are other market goods or category functions for groups of other market goods.

The $a_i$ are quality indices and may be made a function of the characteristics $(b_{j1}, b_{j2})$,

10 See my 1972 paper for a similar result in a different theoretical framework.

11 However, as we shall see in Section VII, the semilog model can be interpreted in a different way. It must also be pointed out that a partial relationship of a semilog form between the price and one characteristic is quite permissible. This is because the units in which $b_{ji}$ are measured are arbitrary. It may be that the appropriate measure is $b_{ji} = \exp b_{ji}$. Then the partial relationship between $p_i$ and the originally measured characteristic $b_{ji}$ is indeed semilog. Indeed the Lancaster model is fairly general in this way; characteristics can, for example, be defined in terms of the interactions of the actually measured characteristics.

12 A similar formulation to analyze quality change is proposed in a working paper by Hugh Davies.
For empirical work, a particular form for $g(\cdot)$ has to be put forward whose choice is mainly an empirical question. If $g(\cdot)$ is exponential with a constant term, then the popular semilog regression form results. However, since the $a_i$ are parameters of the utility function, they must be permitted to be stable over the medium term at least, or else made stable functions of utility or lagged consumption or income. This is quite different from the usually quite abrupt changes from year to year which are implied by the single year regression method recommended by Griliches (1971).

In a sense, the highly restrictive implication of linear additivity of the category utility function has an attractive property. This is that if for one good $k$, $a_k/p_k > a_i/p_i$ for $i = 1, \ldots, m$, $i \neq k$, only good $k$ is bought. If $a_k/p_k = a_i/p_i$ for one or more $i \neq k$, the distribution of demand between these goods and good $k$ is indeterminate. It seems to be quite frequently observed that only one variety of a good is bought by one consumer at any one time. Thus rather plausible discontinuities in individual behavior when the prices $p_i$ or quality indices $a_i$ change, or when new goods appear, are a consequence of this model.\(^{13}\)

If everyone has similar tastes, then it is assumed that competition will force relative prices to be approximately equal to relative quality indices at each point in time, i.e., $p_i$ will be approximately proportional to $g(b_{1i}, b_{2i})$. Over time, the general level of prices changes and the constant of proportionality can be interpreted as a price index. Thus

\[(31) \quad a_i = g(b_{1i}, b_{2i})\]

This becomes a linear regression model after a log transformation. It is estimated on pooled time-series/cross-section data and corresponds to the second variant of the hedonic technique mentioned in the introduction. Thus $\log p_{it} = \log \bar{p}_t + \log g(b_{1it}, b_{2it})$. The $\log \bar{p}_t$ can be picked up as the coefficients of time dummies. This model can be generalized to used durables when in logarithmic terms there is also an additive age effect. This was done by Hall (1969) in formalizing Philip Cagan’s earlier work.

Of course, it is not plausible that everyone has the same tastes. Families have different compositions and it is unlikely that they would have similar preferences, for example with respect to roominess versus acceleration in automobiles. Perhaps even more serious is the plausible situation that $a_i$ or (which is equivalent) the function $g(\cdot)$ are nonhomothetic, i.e., depend on utility $U$ or at least category utility $X_o$.\(^{14}\) This does not invalidate the linearity in $x$ of the indifference loci or the discontinuous behavior which was discussed above. But it implies that for different income levels consumers will have different marginal rates of substitution. It therefore calls into question the assumption that competition will force relative prices into equality with relative quality indices. If quality means different things at different levels of income, we have a serious problem of how to interpret the above regression model based on aggregated market data. That this is in some respects part of the general problem of what aggregate index numbers are supposed to represent (see my 1974b paper) is no ground for complacency. At the very least, some cross-sectional disaggregation ought to be tried in order to attempt to

\[^{13}\text{Note that the parameters in the utility function corresponding to new goods are known given } g(\cdot) \text{ as soon as the specifications are known. A similar property but with non-linear indifference curves results from my extension of "simple" to "variable repackaging" in my 1972 paper. However, predicted behavior is then continuous.}\]

\[^{14}\text{Davies, in a revised version of his paper, has formulated such a nonhomothetic model with some particular functional forms.}\]
sepate out different market segments. However, this may not be enough and budget data may have to be analyzed. An attempt to do this, though in the context of the Lancaster model rather than the present one, has been made by William Alexander.

It is clear that in general, if the Lancaster model applies, quality change is not of the simple repackaging type. Suppose the market goods are refrigerators and $Z_1$ is size and $Z_2$ minimum temperature. It is quite likely that quality change, say, a lower minimum temperature, will augment the value of some other good, say ice cream. This is not equivalent to "more" of a refrigerator, though for a consumer with a given income at a given price configuration there will no doubt be a determine tradeoff between more and having a lower minimum temperature. The point is that as soon as income and prices change, this tradeoff will be different. However, the gain in generality of the Lancaster model relative to simple repackaging is not great. This is because if there are other market goods, we still need separability for the Laspeyres upper bound to hold.

Only if the utility function is separable and the category function $X_0(\cdot)$ is linear and additive both in the $x_i$, $i = 1, \ldots, m$ and the $Z_j$, $j = 1, 2$, do the Lancaster model and the simple repackaging model coincide. Then

$$\sum a_ix_i = \gamma_1Z_1 + \gamma_2Z_2$$

where $\gamma_1$, $\gamma_2$ are constants and

$$a_i = \gamma_1b_{1i} + \gamma_2b_{2i}$$

Thus the hedonic regressions have the form

$$p_{it} = \beta_i(\gamma_1b_{1it} + \gamma_2b_{2it})$$

or

$$p_{it} = \beta_i(\gamma_0 + \gamma_1b_{1it} + \gamma_2b_{2it})$$

if as raised in footnote 7 a unit of $x_i$, $i = 1, \ldots, m$, has a value independent of how much it possesses in characteristics. The $\gamma_j$ are constant over time. Incidentally (35) and (36) raise some estimation problems since an additive time dummy cannot pick up the quality corrected price index $\hat{p}$.15

Since this section has been about various sorts of separability, this seems a good place to comment on Muth. His paper is less about household production than about separability. Muth assumes

$$Z_1 = f_1(x_1, \ldots, x_k)$$
$$Z_2 = f_2(x_{k+1}, \ldots, x_m)$$

where $f_1(\cdot)$ and $f_2(\cdot)$ reveal constant returns. Clearly this is the well-known case of homothetic separability and Muth's substantive results are a development of results by Robert Strotz, Michio Morishima, and Gorman (1961). A similar point is made by Patrick Geary and Morishima.

VII. The Houthakker Approach

Having reached this point, it is now possible to discuss the classic paper by Houthakker.16 Adelman and Griliches claim this paper as an antecedent of theirs. However, as Robert Lucas17 has pointed out, their attempt to base the hedonic ap-
proach in consumer theory has some serious gaps—another reason for going back to Houthakker.

Reducing his model to essentials and giving a concrete illustration we have the utility function

\[(37) \quad U = U(x, b, X_1, X_2, \ldots, X_n)\]

where \(x\) refers to the quantity of (say) auto services consumed with (say) horsepower \(b\). Suppose that the other goods \(X_1, \ldots, X_n\) are available in only one variety each. Assume that all auto services actually consumed have the same horsepower, i.e., that only one variety is bought, but that a continuum of services with different amounts of horsepower are available.

The crucial assumption which is not justified further, is that the consumer faces a tariff schedule quite outside his control. This relates the price of the unit to horsepower and essentially is a two-part tariff of the form

\[(38) \quad p = \pi_0 + \pi_b b\]

Houthakker calls \(\pi_0\) the "quantity price" and \(\pi_b\) the "quality price."

Thus the maximization problem is subject to two constraints: the budget constraint and the tariff schedule.

Maximize \(U = U(x, b, X_1, X_2, \ldots, X_n)\) subject to

\[(39) \quad y = px + \sum_{i=1}^{n} P_i X_i \]

\(\rho = \pi_0 + \pi_b b\)

If we forget about the other goods, \(X_1, \ldots, X_n\), this can be represented in Figure 5. Note that for a unique interior solution, the optimal indifference curve must not be "less convex" than the tariff schedule.

The fact, as Houthakker states, that the arguments in the indirect utility function are \(\pi_0, \pi_b, P_1, \ldots, P_n, y\), can be given intuitive support by the following argument:

Define \(bx = Z\) and redefine the utility function to be

\[U = \bar{U}(x, Z, X_1, X_2, \ldots, X_n)\]

\(Z\) is just like a conventional good with a price \(\pi_b\). Thus, the expenditure function through which the cost of living index and its approximations can be defined has the form

\[y = m(\pi_0, \pi_b, P_1, \ldots, P_n, U)\]

This presentation of Houthakker's argument clarifies, I hope, a subtle confusion in much of the hedonic literature. My use of the term "tariff schedule" is related to Richard Stone's discussion of transport prices in a quality context. He argues that often the price can be thought of as consisting of a ton/mile component and a loading or unloading component. Here then, the so-called shadow prices are unambiguously given to the consumer and can be thought of as primarily production cost determined. It is often not clear in later literature whether shadow prices are internal or external to household decision
making. The Adelman and Griliches paper is a case in point (see my 1973 paper).

The extension of Houthakker’s analysis to several characteristics is trivial as long as the tariff schedule maintains its linear form. A further extension is contained in the recent paper by Rosen. Rosen assumes the existence of a general tariff schedule \( p_i = p(b_1, \ldots, b_k) \) where \( b_j \) is the amount per \( j \)th good of the \( j \)th characteristic. Given its existence, he tells an analytically rich story of consumers with different tastes and producers with different technologies in a competitive, well-informed market context, which explains how such a tariff schedule may be sustained.

One uncomfortable feature of the papers both of Houthakker and Rosen is the assumption that only one variety is bought. This is imposed as an extraneous assumption rather than as the outcome of the optimizing model.

Fortunately, however, the discussion of the previous section at once reveals the solution to this problem. The answer is quite simply that the utility function of the individual consumer must have the form (30). The reason is that within the category of goods with which we are concerned, if only one variety is bought, we must have a corner solution. Now the only sort of indifference curve which will always produce a corner solution when faced with a linear budget constraint is a linear one (we assume that concave indifference curves are ruled out). The form (30) exactly corresponds to the two-stage decision process which is really implied by Houthakker and Rosen. At one stage the choice of variety is made, at the other the choice of how much is to be spent on that category and hence on that variety.

However, as we saw in Section VI, there is a way of generalizing (30). This is to make the parameters \( \{a_i\} \) functions of the level of utility or category utility. Then the marginal rates of substitution differ for different levels of income. For Rosen this would have the analytical advantage of providing a natural motivation for what he calls “taste differences,” i.e., differences in the relative values of the \( \{a_i\} \) for different consumers. The existence of smooth distributions of such differences plays an important role in his analysis.

Even though I have succeeded in clarifying the consumer theory implied in the papers of Houthakker and Rosen, the point remains that their shadow prices do not directly reflect the \( \{a_i\} \). Instead their shadow prices are supply or market generated in some way. If consumers adjust fully, their shadow prices in utility terms should reflect these market shadow prices if the latter exist. A central problem of the exercise then is to which social group, if any, the measured price index corresponds.

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