STAT 250: Homework Assignment #4 Key

Question 1

The American Community Survey is an ongoing survey that provides data every year to give communities the current information they need to plan investments and services. The 2010 American Community Survey estimates that 14.6% of Americans live below the poverty line, 20.7% speak a language other than English at home, and 4.2% fall into both categories.

1. Are living below the poverty line and speaking a language other than English at home mutually exclusive?
   No, they are not as they can happen at the same time.
2. What percent of Americans live below the poverty line and only speak English at home?
   \[ .146 - .042 = 10.4\% \]
3. What percent of Americans live below the poverty line or speak a language other than English at home?
   \[ .146 + .207 - .042 = 31.1\% \]
4. What percent of Americans live above the poverty line and only speak English at home?
   \[ 1 - .311 = 68.9\% \]

Question 2

In a marine sanctuary, 35% of coral reefs are getting smaller each year. Assume the reefs are independent. A random sample of six reefs is taken.

1. What is the probability that all six reefs are getting smaller? ** \[ P(\text{All Smaller}) = .35^6 = 0.18\% \]**
2. What is the probability that none of the six reefs are getting smaller? $ \[ P(\text{None Smaller}) = .65^6 = 7.5\% \]**
3. What is the probability that at least one of the six reefs is getting smaller? ** \[ P(\text{At least 1 smaller}) = 1 - P(\text{None Smaller}) = 1-.075 = 92.5\% \]**

Question 3

The National Vaccine Information Center estimates that 90% of Americans have had chickenpox by the time they reach adulthood. Assume that the people in the problem are independent.

1. Suppose we take a random sample of 100 American adults. Is the use of the binomial distribution appropriate for calculating the probability that exactly 97 had chickenpox before they reached adulthood? Explain. (Hint: what are the four conditions needed for the binomial distribution?)
   Independent samples, only two outcomes, and the probability is constant.
2. Calculate the probability that exactly 97 out of 100 randomly sampled American adults had chickenpox during childhood.
   \[ \text{dbinom}(97,100,.90) = 0.0059 \]
3. What is the probability that exactly 3 out of a new sample of 100 American adults have not had chickenpox in their childhood?
   \( \text{dbinom}(3, 100, .10) = 0.0059 \)

4. What is the probability that at least 1 out of 10 randomly sampled American adults have had chickenpox?
   \( 1 - \text{pbinom}(0, 10, .90) = 1 \)

5. What is the probability that at most 3 out of 10 randomly sampled American adults have not had chickenpox?
   \( \text{pbinom}(3, 10, .10) = 0.9872 \)

**Question 4**

A study has shown that 51% of patients use herbal medications before surgery, against their doctor’s advice. A random sample of 20 patients was selected. Assume that the patients in the problem are independent.

1. What is the probability that exactly 8 patients have used herbal medications?
   \( \text{dbinom}(8, 20, .51) = 0.1105 \)

2. What is the probability that exactly 15 patients have used herbal medications?
   \( \text{dbinom}(15, 20, .51) = 0.018 \)

3. What is the probability that at least 8 patients have used herbal medications?
   \( 1 - \text{pbinom}(7, 20, .51) = 0.8867 \)

4. What is the probability that at most 15 patients have used herbal medications?
   \( \text{pbinom}(15, 20, .51) = 0.9924 \)

**Question 5**

You are planning a study on female wombats. You need 20 wombats in order to conduct the study. You assume that 50% of wombats are female. What is the minimum number of wombats you need to collect in order to have at least a 90% chance of having 20 female wombats?

\( 1 - \text{pbinom}(19, 48, .50) = 0.9033 \) So 48 wombats are needed.