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Trading Spacecraft Propellant Use and Mission Performance to Determine the Optimal Collision Probability in Emergency Collision Avoidance Scenarios

Jason A. Reiter[a]*, David B. Spencer[b]

[a] *Graduate Student, Department of Aerospace Engineering, The Pennsylvania State University, 229 Hammond Building, University Park, PA 16802*, jar577@psu.edu
[b] *Professor, Department of Aerospace Engineering, The Pennsylvania State University, 229 Hammond Building, University Park, PA 16802*, dbs9@psu.edu
* Corresponding Author

Abstract

For many spacecraft missions, even the slightest change in the orbit of the spacecraft may significantly affect its ability to perform to the desired specifications. With the high volume of debris in orbit, debris-creating events could occur with no advanced notice, making emergency collision avoidance scenarios a real possibility. Care must be taken to ensure that any potential collision is avoided while minimizing the effect of the maneuver on the spacecraft's mission performance. Assuming that the possible collisions occur at high relative velocities and perfect knowledge of the states of all objects, the required thrusting time to achieve a desired maximum collision probability can be found. Varying the desired collision probability, the resulting changes in the required thrust duration time (and, thus, propellant use) can be observed, providing options for trading the propellant use and likelihood of a collision. Additionally, both of these variables contribute directly to the ability of the spacecraft to perform to the desired mission specifications. As the maximum collision probability and required maneuver time increase, the mission performance decreases with it. The level of robustness necessary in the mission specifications can be used to limit the desired maximum collision probability. This is accomplished by determining the time and propellant required to perform the collision avoidance maneuver to the desired probability level and analyzing the effect of the time spent away from the mission orbit and the quantity of propellant required to perform the maneuver on the mission performance. Such analysis would prove significant in determining an optimal maximum collision probability (typically a subjective variable) for short-notice collision scenarios.

Keywords: Spacecraft. Collision Avoidance, Optimization, Mission Performance, Astrodynamics, Trade Studies

Nomenclature

P_{max}	Maximum Probability
d	Separation Distance [km]
r_A	Radius of Collision Cross-Sectional Integration [km]
α	In-Plane Thrusting Angle [radians]
β	Out-of-Plane Thrusting Angle [radians]
σ	Position Standard Deviation [km]
σ^*	Companion Standard Deviation [km]
σ_x	Standard Deviation, X-Axis [km]
σ_z	Standard Deviation, Z-Axis [km]

1. Introduction

In typical debris collision avoidance scenarios (COLA), the optimal maneuver is first determined by analyzing the encounter region where the two objects are predicted to collide. In situations where both the primary and secondary bodies can be tracked, their positions and velocities are determined to within their associated errors in the corresponding position covariance ellipsoids. Though not necessary for most cases with near-circular orbits, [1] demonstrates that the covariance ellipsoids may be oriented for which the velocities are not perfectly aligned with their major principal axes. At the predicted time of collision, the covariance data is used to determine a maximum probability of collision and miss distance for the encounter. For any collision avoidance scenario, the goal is to reach a desired separation distance between the two objects at the predicted time of collision by performing a propellant-optimal maneuver, as explained in [2] in detail.

One main factor driving the planning of the maneuver is the starting and desired orientation of the primary object. Depending on the current operating conditions, the spacecraft may be pointed in a certain direction to best utilize its solar panels, scientific equipment, or other directional hardware. Time must be allotted to re-orient the spacecraft so that the main thrusters are pointed in the desired direction to perform the maneuver. Additional time will be allotted to ensure that the thrust is applied at apogee or perigee, the most efficient locations in orbit to change the energy of the object. With these in mind, a maneuver is planned (assuming an impulsive maneuver) to reach the desired separation distance at the predicted time of collision with the least amount of propellant. These maneuvers can take hours, if not days, to plan and optimize.

Unfortunately, with the uncertainty in the position of any two objects and the inability to track objects smaller than 10 cm in diameter, [3] shows that not all collisions can be avoided. Debris from such a collision or other debris-creating events could put other satellites on collision courses with the newly-created debris with minimal time until the collision (possibly even less than half-an-orbit in duration). Currently, minimal previous work exists in avoiding collisions with such a small notification time.

When planning avoidance maneuvers for collisions between two catalogued objects, covariance data is ideally known for both objects. However, when the secondary object is newly-created, less opportunity exists to determine similar characteristics. Therefore, in scenarios with minimal time-to-collision, the secondary object must be considered as only a point mass and the probability must be determined with the information known about the primary object (its covariance assumed to be oriented in the direction of the velocity vector), as demonstrated in [3]. In addition, it is assumed that no time is available to rotate the spacecraft to a more desirable orientation, so the thrust is assumed to be applied in the original orientation, and the thrust magnitude must be chosen as such.

Collision avoidance maneuvers with less than half-an-orbit to both plan and execute involve additional complexities not found in typical advanced notification avoidance maneuvers due to the limited time available. For instance, finite maneuver analysis must be applied instead of assuming an impulsive maneuver. The limited time available also leads to complexity in choosing both an optimal direction and maneuver location, which will both be covered in detail later. Working under these assumptions, a solution can be determined for a propellant-optimal avoidance maneuver. An analytical solution to the problem was previously published by the authors and can be found in [4].

Varying the desired collision probability, the resulting changes in the required thrust duration time (and, thus, propellant use) can be observed, providing options for trading the propellant use and likelihood of a collision. Additionally, both the probability of collision and the propellant use contribute directly to the ability of the spacecraft to adhere to the desired mission specifications. As the maximum collision probability and required maneuver time increase, the mission performance decreases with it. The level of robustness necessary in the mission specifications can then be used to limit the desired maximum collision probability. This is accomplished by determining the time and propellant required to perform the collision avoidance maneuver to the desired probability level and analyzing the effect of the time spent away from the mission orbit and the quantity of propellant required to perform the maneuver on the mission performance. It was found that, for notification times less than around 20 minutes, it is best to decrease the collision probability as much as the available propellant will allow without regard for the time duration of the maneuver. As the notification time increases past 20 minutes, more emphasis can be placed on the time required to perform the entire maneuver and it was found that simultaneously minimizing the maneuver time and collision probability outweighed the slight extra propellant required for such a maneuver. These observations allow us to determine an optimal maximum collision probability (typically a subjective variable) for short-notice collision scenarios.

2. Optimizing the Maneuver

A numerical solution relating the desired collision probability and the required thrust duration can be found by optimizing the thrusting direction and location that results in the lowest amount of propellant to reach the desired separation distance (or maximum collision probability). The optimization of the rapid collision avoidance maneuver relies on the complexities that arise from applying finite maneuver analysis while choosing maneuver locations and directions uncharacteristic of long-duration maneuvers.

2.1 Thrust Location

A primary complexity that arises from these minimal time-to-collision maneuvers is the optimal location of the maneuver. When a longer time is allotted for the avoidance maneuver, the maneuver will always be placed at perigee or apogee, the most efficient locations in an elliptical orbit to perform a maneuver [5]. However, when less time than half-an-orbit is available to perform the maneuver, thrusting at perigee or apogee may not be an option, so a less optimal thrust location may be necessary. This results, unfortunately, in a greater ΔV and, thus, a longer maneuver time to complete the maneuver.

To determine an optimal location to perform the maneuver, a "wait time" parameter is applied. If the required maneuver time is less than the available time to perform the maneuver, then some flexibility is allowed in where thrust is applied. To lower the time spent thrusting, the "wait time" is placed at the beginning of the maneuver to ensure that the majority of the thrusting occurs at the most efficient point in the remaining orbit arc before the collision was predicted to occur. If perigee or apogee are between the notification epoch and the predicted collision location, then the "wait time" is applied such that the maneuver is centered around this optimal location.

2.2 Coast Location

Once the beginning of the thrust location is chosen based on the "wait time," a second non-thrusting coast period can be added after the maneuver occurs. If, after applying the "wait time", thrusting the remaining time-to-collision causes the spacecraft to exceed the desired minimum separation distance, then a coast phase can be applied after the maneuver to decrease the separation distance at the collision location. The coast location is chosen such that the combined time of the thrust duration and the two coast periods results in the spacecraft reaching exactly the desired separation distance at the predicted collision location.

2.3 Thrust Direction

The third optimization parameter is the direction in which the thrust is applied. Luckily, the complexity of optimizing the thrust direction becomes a simplification in this case. The angles, α and β, are the in-plane (α) and out-of-plane (β) thrusting angles in the body-fixed RCN (radial-circumferential-normal) reference frame. This reference is such that the radial component is aligned with the radial unit vector positive in the zenith direction, the normal component is aligned with the osculating angular momentum vector positive and the circumferential component is normal to the radius vector in the orbital plane and completes the right-handed triad of unit vectors, as seen in Figure 1.

With the thrust location known, the optimal in-plane and out-of-plane angles can be chosen to minimize the duration of the maneuver. Considering all possible thrust locations, spacecraft parameters, and orbit orientations in Low-Earth Orbit, an optimal maneuver direction was chosen for each maneuver which required the least amount of time spent maneuvering. It turns out that, when the maneuver duration is shorter than approximately 90 minutes (about the time to complete one orbit in LEO), the optimal maneuver duration is always radial (90° from the velocity vector) in the in-plane axis (α) and along the velocity vector direction in the out-of-plane axis (β).

Though the thrust direction is always assumed to be radial, the binary switch between thrusting along the direction of the position vector or in the reverse direction is determined based on the instantaneous velocity vectors of both the primary and secondary objects. If the angle between the two vectors (determined using the law of cosines) is between 0 and 180°, the thrust is performed along the direction of the position vector. Otherwise, if the angle is between 180° and 360°, the thrust is performed in the reverse direction of the position vector. This ensures that the maneuver cannot increase the probability that a collision will occur between the two objects at a different point in time before or after the predicted collision.

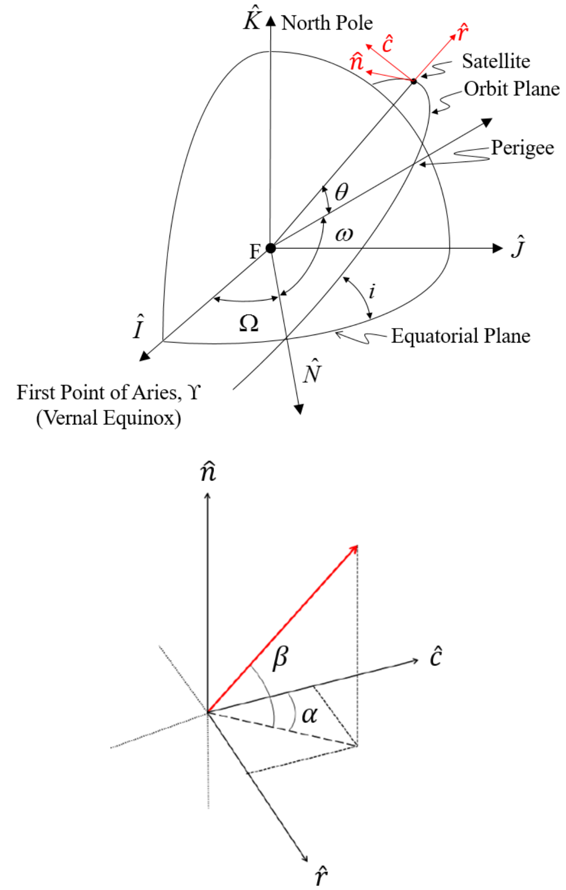

Fig. 1. Top: RCN reference frame with respect to the Central-Body-Centered inertial reference frame (IJK). Bottom: α and β thrust angles with respect to the RCN reference frame.

2.4 Numerical Method Optimization

With the three optimization parameters defined, the method in which they are chosen can be explained in further detail. This leads to the final complexity: finite maneuver modeling. When rapid collision avoidance maneuvers are necessary, the time-to-collision is not significantly greater than the time spent maneuvering to complete the maneuver. This means that the maneuver can no longer be assumed to be impulsive. Instead, finite maneuver analysis must be used, where the maneuver is integrated (including propellant mass loss) with the position and velocity of the spacecraft at every time step in the propagation. Comparing the two methods, it was found that performing short duration maneuvers with the impulsive maneuver assumption (performed at the notification epoch) resulted in separation distance values over 460% greater than if a finite maneuver were applied for the entire time-to-collision duration. The large difference between the two distances, and the fact that finite maneuver analysis is considered to be the more accurate of the two methods, means that the impulsive maneuver assumption can no longer be made.

Now that all of the assumptions are accounted for, the position and velocity of the spacecraft can be propagated forward in the arc between the notification epoch and the predicted collision location using finite maneuver modeling applied in MATLAB's ode45. Propagating the orbit with a variable time-step solver like ode45 allows for larger steps to be taken when allowed and smaller time steps to be taken when required by the solution. This ensures that the orbit propagation is as accurate as possible with relative and absolute tolerances set to $1x10^{-6}$. In collision avoidance scenarios with larger time frames, the fidelity of the orbit propagator would be an issue. However, with such minimal time-to-collision values, the orbit isn't propagated long enough for any perturbations such as drag and oblateness to affect the position and velocity of the spacecraft.

The optimal thrusting duration is determined by propagating the spacecraft's orbit as such while iterating through all possible "wait time" and coast time values. The first step in each iteration is to propagate the nominal trajectory forward for the "wait time" duration. Subtracting the "wait time" and coast time from the time-to-collision, the thrust duration is found to be the time remaining in the maneuver. The spacecraft's position is then propagated further for the thrust duration while applying the finite burn. Finally, the position is once again propagated (without the finite maneuver) for the coast time. The resulting position vector is then compared to the position of the secondary body at the time of the collision to determine the separation distance. After iterating through all possible wait time and coast time options, the "wait time" and coast time values can be found that result in the desired separation distance being achieved while also minimizing the necessary maneuver time to do so. This iteration process can be seen summarized in the flowchart in Figure 2.

Once the minimized thrust duration and the resulting separation distance are known, a relationship between the thrust duration and maximum collision probability can be developed. Given the nature of the collision avoidance problem, using a desired collision probability is typically preferred over the separation distance. Luckily, a relationship (assuming a high relative velocity between the two colliding objects) between the two exists as stated in [1]:

$$d = \frac{1}{1000}\sigma^* \sqrt{2 * \ln\left(1 - e^{-\frac{r_A^2}{2*\sigma^2}}\right) - \ln(P_{max})} \quad (1)$$

in which

$$\sigma = \sqrt{\sigma_x \sigma_z} \quad (2)$$

and

$$\sigma^* = \sqrt{\sigma_z^2 \left(1 + \left(\left(\frac{\sigma_z}{\sigma_x}\right)^2 - 1\right)\right)^{-1}} \quad (3)$$

which simplifies to

$$\sigma^* = \sigma_x \quad (4)$$

With the separation distance known, the corresponding maximum collision probability can be calculated from the two-dimensional covariance of the and the combined radius of the spacecraft and secondary object. This gives a relationship between the thrust direction and the maximum collision probability, which is the desired correlation.

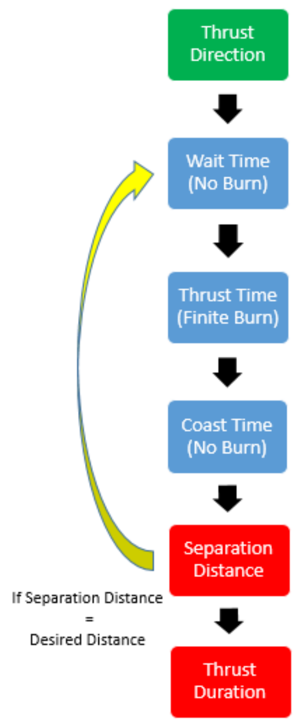

Fig. 2. Optimization Flow Chart to the Minimal Thrust Duration That Achieves the Desired Separation Distance

2.5 Effect of Perturbations

In propagating a spacecraft's trajectory, whether applying thrust or not, the acceleration due to perturbative forces must be taken into account. However, when propagating for such short periods of time, the perturbations will have less of an effect on the final position of the spacecraft at the predicted collision time. This was put to the test by propagating a spacecraft's orbit a full period prior to a predicted collision with a desired separation distance of up to 10 km. It was found that the perturbed trajectory varied from the unperturbed trajectory by only 0.3463%. This means that, for all rapid collision avoidance maneuvers in Low-Earth Orbit, the perturbing forces can be assumed to be insignificant and the trajectories can be propagated without perturbations.

3. Trading a Complete Trajectory

With the avoidance maneuver optimized, the return trajectory to the nominal orbit can be solved as well. Minimum-propellant Hohmann transfers and the minimum-time Lambert's problem were considered for the return. A trade study can be conducted comparing the minimum-propellant transfer, minimum-time transfer, and minimum-probability cases to determine the desired return trajectory given the weights provided for each metric by the mission designers.

3.1 Return Trajectories

In the case of the Hohmann transfer, the spacecraft would remain on its post-collision orbit until it reaches perigee and then would commence a Hohmann transfer to arrive on the original orbit at apogee. While this transfer will always require the least amount of propellant to perform, it also takes significantly longer than the minimum-time Lambert's problem. As is to be expected, solving for a perigee-to-apogee (180°) Lambert's problem transfer gives the same required ΔV and time-of-flight as the Hohmann transfer. This means that all possible transfers, despite the desired optimization parameter(s), can be solved for using Lambert's problem by varying the time-of-flight.

In order to explore all possible return trajectory options, Lambert's problem is solved for all possible departure locations (where after the predicted collision location the spacecraft departs from the post-collision orbit), all arrival locations on the nominal orbit, and all return maneuver durations (up to one orbital period in duration). The required ΔV for all possible transfers was found for all combinations of departure location, arrival location, and maneuver duration values. However, the impulsive maneuver assumption made for Lambert's problem is not valid for the majority of the minimum-time transfer options. This requires two adjustments to be made in order to ensure accurate ΔV values and realistic return trajectory options.

First, the initial avoidance maneuver is recreated assuming an impulsive Lambert's problem maneuver for all possible maneuver durations up to one orbital period in length. These values are then divided into the actual required ΔV values for the avoidance maneuver to get a relationship between the impulsive and finite maneuver values. This relationship is then used to determine the actual required ΔV for the possible return trajectories. The second adjustment is to ensure that the ΔV required can actually be accomplished in the allotted maneuver duration. Given the relationship between the maneuver duration and the ΔV of the avoidance maneuver, it can be determined how long it takes to impart the required ΔV into the return trajectory. If the calculated maneuver duration is greater than the allotted maneuver duration, then that trajectory is considered to no longer be a valid option. The remaining trajectories are then compared for their corresponding ΔV values and maneuver durations, and the process is repeated for each desired separation distance/ collision probability value. The resulting relationship between maximum allowed collision probability, ΔV, and maneuver duration are the basis for the trade study.

3.2 *Trade Study Metrics and Weights*

When designing a satellite mission, many factors are considered related to the health of the spacecraft, the cost of launching it and its propellant reserves into space, and the sensitivity of the nominal orbit to perturbations and course corrections. In Low-Earth Orbit, for example, the orientation and location of the spacecraft can have a significant impact on its ability to meet the prescribed mission requirements. In addition, in order to minimize the launch mass of the spacecraft, only a minimal amount of extra propellant is included for unplanned maneuvers. This means that any avoidance maneuver must be carefully considered given the likelihood of collision (a measure of effectiveness), the time spent away from the nominal orbit (a measure of performance), and the amount of propellant (proportional to the ΔV) required to perform the maneuver (a measure of performance). These three values are used as metrics when weighing the avoidance maneuver and return trajectory options and deciding on a preferred course of action.

With an unlimited number of options available to return the spacecraft to the nominal orbit, a trade study is necessary to narrow down and compare the possibilities. In narrowing down the options to be compared, a maximum of three options were considered at one time: the minimum propellant case, the minimum time case, and/or the minimum collision probability case. Given that the weighting applied to each of these is largely dependent on the design of the mission, the availability of propellant onboard the spacecraft, and the imminence of the predicted collision, multiple trade studies must be conducted in order to determine the best course of action under various conditions. In comparing the trade studies, patterns can be noted, such as the effect of the notification time on the feasibility of the maneuvers. Patterns such as this can be used to make suggestions to the mission designers in the preliminary design stage. As collisions are predicted to occur, engineers involved in the operation of the spacecraft can use the same trade studies with their own desired weights and limitations applied to the metrics to determine the ideal course of action to avoid – or risk colliding with – the approaching object. The results of these trade studies and subsequent recommendations for use in future collision avoidance operations are summarized in the following section.

4. Results

In solving for the avoidance maneuver, the optimum maneuver duration was determined for various ranges of separation distance and maximum collision probability values. When determining, operationally, the desired collision probability, there is rarely a "correct answer." This is where trade studies become useful. The spacecraft operators can choose weights for the three metrics (maximum collision probability, total maneuver duration, and total required ΔV) which allows them to narrow down the possible maneuver choices, if they elect to perform one at all.

For a 2000 kg spacecraft with a 180 N propulsion system in a circular 700 km orbit inclined to 50°, notification times of 8 minutes, 15 minutes, 20 minutes, and 30 minutes were considered and used for the trade studies. For this generic test case, with no real covariance data, the covariance in both the x and z directions is set to a generic value of 5 km, with a combined radii of 10 m. For all scenarios, the total ΔV available for the maneuver is assumed to be limited to less than 1 km/s in order to realistically trade the possible options. Increasing or decreasing this limitation would have a negligible effect on the results of this study.

When considering only an 8-minute notification time, this limitation on the total ΔV significantly decreases our available maneuver options. Table 1 shows the total required ΔV and maneuver time for the minimum propellant and minimum time cases for each separation distance/collision probability value. The values in red denote those that exceed the 1 km/s limitation. In this case, the minimum propellant case at a 0.1 km separation distance is being compared to the minimum propellant case at a 0.4 km separation distance.

To begin the trade study, since avoiding the collision is the main priority, the collision probability metric is given a weighting of 0.5 out of the total available 1.0. It is useful to note that the actual magnitude of the collision probability is not significant since the default covariance values used to find the maximum collision probability from the desired separation distance is just assumed to be generic values. The significance is how the probability varies with the separation distance and required maneuver duration, which is unaffected by the relative magnitudes. The weights for the propellant use and maneuver time metrics are then both set to 0.25 to begin with (see Table 2). Because all ΔV values for the minimum time cases exceed 1 km/s, no minimum time option is included in the trade. Instead, only the two minimum propellant options, one at the minimum collision probability, are compared. Because the minimum probability case also takes the least amount of time, the minimum probability case easily out-scores the minimum propellant case, and is highlighted in the trade study matrix.

Table 1. Maneuver Options for 8-Minute Notification Time

		Min Propellant		Min Time	
d (km)	P	ΔV (km/s)	Time (min)	ΔV (km/s)	Time (min)
0.1	4.00E-12	0.120	106.8	66.57	39.67
0.2	3.99E-12	0.551	106.8	66.92	39.67
0.3	3.98E-12	0.733	106.2	67.10	39.67
0.4	3.97E-12	0.917	97.50	67.27	39.67
0.5	3.96E-12	1.101	91.83	69.42	39.67
0.6	3.94E-12	1.285	85.83	69.59	39.67
0.7	3.92E-12	1.470	79.83	69.77	39.67
0.8	3.90E-12	1.654	78.83	69.95	39.67
0.9	3.87E-12	1.840	73.50	70.12	39.67

Table 2. Minimum Propellant Trade Studies for 8-Minute Notification Time, Equal Propellant and Time Weights

		Options	
Criteria	Weight	Min Propellant	Min Probability
Probability	0.50	1.00	5.00
Propellant	0.25	5.00	1.00
Time	0.25	1.00	5.00
Total		2.00	4.00

The question now is what possible weighting schemes would make the minimum propellant solution more preferable. It was found that even weighing the propellant use at 4 times more significant than the maneuver time still does not overcome the 100% difference between the two total metric values. Table 3 shows that this trade still results in the minimum probability case dominating.

It was found, actually, that only by decreasing the collision probability weight to 0.25 and increasing the propellant use weight to 0.5 do the two options become equivalent (seen in Table 4). This means that, for notification times around 8 minutes, choosing a maneuver that minimizes the collision probability would be the most logical choice unless there is a significant constraint on the propellant available for the maneuver.

Table 3. Minimum Propellant Trade Studies for 8-Minute Notification Time, Unequal Propellant and Time Weights

		Options	
Criteria	Weight	Min Propellant	Min Probability
Probability	0.50	1.00	5.00
Propellant	0.40	5.00	1.00
Time	0.10	1.00	5.00
Total		2.60	3.40

Table 4. Minimum Propellant Trade Studies for 8-Minute Notification Time, Unequal Propellant and Time Weights, Decreased Probability Weight

		Options	
Criteria	Weight	Min Propellant	Min Probability
Probability	0.25	1.00	5.00
Propellant	0.50	5.00	1.00
Time	0.25	1.00	5.00
Total		3.00	3.00

Next, the same process can be repeated for a notification time of 15 minutes. As seen in Table 5, nearly doubling the notification time decreases the total ΔV required for the minimum time cases, but not enough to fall beneath the 1 km/s limit. The available probability range widens for the minimum propellant case, however, for nearly the same propellant cost as the 8-minute notification scenario. The notable change between the two is the separation distance values of the minimum propellant cases. When increasing the notification time to 15 minutes, the lowest separation distance no longer requires the least amount of propellant. Instead, a 0.2 km separation distance now minimizes the total ΔV. This leads to a slight, but insignificant change in the trade studies compared to the 8-minute notification scenario. Once again, setting the probability weighting to 0.25, the propellant use weighting to 0.5, and the maneuver time weighting to 0.25 leads to the minimum propellant scenario only slightly beating out the minimum probability scenario. For both 8- and 15-minute notification times, choosing a maneuver that minimizes the collision probability would be the most logical choice unless there is a significant constraint on the propellant available for the maneuver.

Increasing the notification time just an additional 5 minutes changes the scenario rather significantly. Table 6 shows the same separation distance ranges as the 15-minute notification case, but the additional available time decreases the required ΔV so much so that the minimum time solution now falls beneath the 1 km/s limit. As noted in the 15-minute notification scenario, the minimum separation distance value no longer results in the lowest ΔV, and that holds true in this scenario as well. This time, the minimum ΔV occurs at a separation distance of 0.3 km. It can also be noticed that the maneuver time between each minimum propellant and each minimum time case does not vary at these separation distances. Therefore, only the minimum propellant and minimum time options were chosen to be explored for the following trade study.

Table 5. Maneuver Options for 15-Minute Notification Time

		Min Propellant		Min Time	
d (km)	P	ΔV (km/s)	Time (min)	ΔV (km/s)	Time (min)
0.1	4.00E-12	0.203	128.8	13.20	46.50
0.2	3.99E-12	0.120	128.8	13.18	46.50
0.3	3.98E-12	0.374	128.8	13.33	46.50
0.4	3.97E-12	0.373	128.8	13.31	46.50
0.5	3.96E-12	0.557	128.8	13.46	46.50
0.6	3.94E-12	0.558	128.8	13.44	46.50
0.7	3.92E-12	0.743	128.8	13.59	46.50
0.8	3.90E-12	0.746	128.8	13.57	46.50
0.9	3.87E-12	0.931	127.17	13.73	46.50
1	3.84E-12	0.934	122.83	13.70	46.50

Table 6. Maneuver Options for 20-Minute Notification Time

		Min Propellant		Min Time	
d (km)	P	ΔV (km/s)	Time (min)	ΔV (km/s)	Time (min)
0.1	4.00E-12	0.208	118.8	0.347	50.83
0.2	3.99E-12	0.203	118.8	0.339	50.83
0.3	3.98E-12	0.199	118.8	0.330	50.83
0.4	3.97E-12	0.376	118.8	0.502	50.83
0.5	3.96E-12	0.377	118.8	0.495	50.83
0.6	3.94E-12	0.378	118.8	0.487	50.83
0.7	3.92E-12	0.564	118.8	0.664	50.83
0.8	3.90E-12	0.566	118.8	0.660	50.83
0.9	3.87E-12	0.569	118.8	0.656	50.83
1	3.84E-12	0.757	118.8	0.840	50.83

Starting with the same initial weighting system (0.5 for probability, 0.25 for propellant use, and 0.25 for maneuver time), an identical trade study (Table 7) to that found in Table 2 was performed, except this occasion with the minimum time case instead of the minimum probability case. This time, with both cases having a higher collision probability and the propellant required for the minimum time case being only slightly higher than the minimum propellant case, the minimum time case results in the highest overall metric. This was found to hold true for all scenarios except when the propellant minimization metric is chosen to be at least four times greater than the time minimization metric (Table 8). From this, it appears that at notification times of around 20 minutes and greater, the minimum time case becomes more ideal as the required total ΔV for the maneuver is not significantly greater than that of the minimum propellant case.

Table 7. Minimum Propellant, Minimum Time Trade Studies for 20-Minute Notification Time, Equal Propellant and Time Weights

		Options	
Criteria	Weight	Min Propellant	Min Probability
Probability	0.50	1.35	1.00
Propellant	0.25	5.00	4.08
Time	0.25	1.00	5.00
Total		2.18	2.77

Table 8. Minimum Propellant, Minimum Time Trade Studies for 20-Minute Notification Time, Unequal Propellant and Time Weights

		Options	
Criteria	Weight	Min Propellant	Min Probability
Probability	0.50	1.35	1.00
Propellant	0.40	5.00	4.08
Time	0.10	1.00	5.00
Total		2.78	2.63

The difference between the total ΔV required for the minimum propellant and minimum time cases becomes even smaller for a 30-minute notification time scenario. Just as was noted in the previous scenarios, the maximum separation distance achievable increased between the two cases, as did the separation distance value corresponding to the lowest ΔV possible to complete the maneuver. In this trade study, in addition to the same minimum propellant and minimum time cases, an additional minimum time case was chosen such that the maneuver time is equivalent, the separation distance is near the maximum achievable, and the ΔV required is equivalent to that of the minimum propellant transfer for the same separation distance value.

Like in the previous trade studies, the same initial weighting system (0.5 for probability, 0.25 for propellant use, and 0.25 for maneuver time) was chosen. The three cases can be seen compared in Table 10. As the minimum probability case out performs the other cases in the collision probability metric, has an equivalent timing metric value to the minimum time case, and does not perform poorly in propellant consumption despite having the highest ΔV of the three cases, it just barely wins out over the minimum time case. Only when the propellant weighting is increased to four times the time weighting do the results change (Table 11). Despite the emphasis placed on minimizing the propellant usage, the minimum time case now stands out because, despite the slight increase in the required ΔV, it significantly out performs the minimum propellant case with regards to the maneuver time. It was found that only when increasing the propellant metric weighting to nearly 0.9 does the minimum propellant case dominate over the minimum time case.

Table 9. Maneuver Options for 30-Minute Notification Time

		Min Propellant		Min Time	
d (km)	P	ΔV (km/s)	Time (min)	ΔV (km/s)	Time (min)
0.1	4.00E-12	0.228	128.8	0.237	58.17
0.2	3.99E-12	0.220	128.8	0.234	58.17
0.3	3.98E-12	0.212	128.8	0.230	58.17
0.4	3.97E-12	0.206	128.8	0.227	58.17
0.5	3.96E-12	0.201	128.8	0.224	58.17
0.6	3.94E-12	0.390	128.8	0.402	58.17
0.7	3.92E-12	0.392	128.8	0.399	58.17
0.8	3.90E-12	0.394	128.8	0.397	58.17
0.9	3.87E-12	0.395	67.83	0.395	58.17
1	3.84E-12	0.393	67.83	0.393	58.17
2	3.41E-12	0.768	67.83	0.768	58.00

Table 10. Minimum Propellant, Minimum Time, Min Probability Trade Studies for 30-Minute Notification Time, Equal Propellant and Time Weights

		Options		
Criteria	Weight	Min Propellant	Min Time	Min Probability
Probability	0.50	1.27	1.00	1.87
Propellant	0.25	5.00	4.75	3.64
Time	0.25	1.00	4.99	4.99
Total		2.14	2.93	3.09

When comparing all four trade studies side-by-side, a few conclusions can be made for use in future satellite mission design and operations. First, and most notable, is that choosing to avoid the collision at a higher probability threshold does not always guarantee the lowest required propellant use or time for the maneuver. The higher the notification time, the more the required propellant can be minimized by choosing a lower collision probability threshold. In addition, for collision avoidance maneuvers with notification times less than about 20 minutes, the most logical choice would be the maneuver that minimizes the collision probability unless significant emphasis is placed on propellant savings. For collision avoidance maneuvers with notification times greater than 20 minutes and less than one orbital period, the minimum time solution becomes the most logical option while also minimizing the collision probability as much as the onboard propellant supply will allow. Fortunately, as the notification time increases, the required ΔV values increase less with decreasing collision probability thresholds. This means that, with only basic constraints on the propellant available for the maneuver, it is still possible to minimize both the collision probability threshold and total maneuver time with the preferred solution. These observations would allow for an optimal collision probability threshold to be chosen in future missions given the weights of each metric and the available propellant at the time of the maneuver.

Table 11. Minimum Propellant, Minimum Time, Min Probability Trade Studies for 30-Minute Notification Time, Unequal Propellant and Time Weights

		Options		
Criteria	Weight	Min Propellant	Min Time	Min Probability
Probability	0.50	1.27	1.00	1.87
Propellant	0.40	5.00	4.75	3.64
Time	0.10	1.00	4.99	4.99
Total		2.74	2.90	2.89

9. Conclusions

Currently, on-orbit assets are susceptible to unpredicted debris-creating events. As discussed, satellite collision avoidance maneuvers for such events become more complex (such as the necessity for finite maneuver analysis) since the time-to-collision is minimal. This short amount of time available to both plan and execute the maneuver means that the speed at which the maneuver time for the maneuver can be determined is significant. In the case that such a scenario is encountered, a technique was devised in [4] in order to find a solution that could be calculated at a moment's notice given only the parameters of the spacecraft and the location of the collision. This resulted in a solution for the majority of the necessary quick response collision avoidance maneuvers in Low-Earth Orbit, assuming only that the covariance of the debris cloud is oriented along its velocity vector.

In many cases, it is desirable to optimize the whole maneuver to include the return trajectory as well in order to ensure that decreasing the collision probability is worth the effect of the maneuver on the mission operations. Whether limitations are placed on the maneuver by the amount of propellant available onboard or restrictions on the orbital position of the spacecraft, all three metrics must be considered through the use of a trade study to determine the best course of action. While, in some cases, the best action may be not to move the spacecraft at all, the assumption was made that a maneuver was necessary when performing the trade studies. It was found that, for notification times less than around 20 minutes, it is best to decrease the collision probability as much as the available propellant will allow. As the notification time increases past 20 minutes, more emphasis can be placed on the time required to perform the entire maneuver and it was found that simultaneously minimizing the maneuver time and collision probability outweighed the slight extra propellant required for such a maneuver. While the mission requirements and satellite characteristics will vary widely between collision scenarios, this information can be considered by mission designers when allocating onboard propellant for such maneuvers and by satellite operators when determining the best course of action to avoid an impeding collision. In rapid collision avoidance maneuvers, not much time can be spent determining the optimal collision probability threshold. With this work, just by knowing the desired weighting for the metrics and basic covariance data, an optimal collision probability threshold can be chosen for the maneuver and used to calculate the necessary maneuver duration from the generated analytical equation.

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