



A Simulated Annealing Algorithm with a Dynamic Temperature Schedule for the Cyclic Facility Layout Problem

Sadan Kulturel-Konak and Abdullah Konak

Management Information Systems, Penn State Berks, Reading, PA 19610, sadan@psu.edu

Information Sciences and Technology, Penn State Berks, Reading, PA 19610, konak@psu.edu

Abstract In this paper, an unequal area Cyclic Facility Layout Problem (CFLP) is studied. Dynamic and seasonal nature of the product demands results in the necessity for considering the CFLP where product demands as well as the departmental area requirements are changing from one period to the next one. Since the CFLP is NP-hard, we propose a Simulated Annealing (SA) metaheuristic with a dynamic temperature schedule to solve the CFLP. In the SA algorithm, both relative department locations and dimensions of departments are simultaneously determined. We benchmark the performance of the proposed SA algorithm with earlier approaches on different test problems from the literature and find out that the SA algorithm is promising.

Keywords Cyclic Facility Layout Problem, Simulated Annealing

1. Introduction

In this paper, we introduce a Simulated Annealing (SA) Algorithm to solve the Cyclic Facility Layout Problem (CFLP), which arises in manufacturing systems where product types and demands are seasonal. Generally, the facility layout problem (FLP) involves with determining the locations and shapes of a set of rectangular departments within a facility in order to optimize a performance measure. Because non-value-added material handling activities account for 30-40 percent of the total operational expenses in a typical manufacturing system [24], minimizing the material handling cost has been the primary objective of the FLP. In order to reduce the material handling cost, the departments that have high levels of inter-departmental material flows between them should be located as close to one another as possible. The CFLP can be considered as a special case of the Dynamic Facility Layout Problem (DFLP) which is concerned with planning the layout of a facility over multiple time periods. The DFLP is applicable in cases where material flows among departments change over time because of new products, product demand changes, and product life cycles, etc. In such cases, a layout with a low material handling cost in a time period may lead to high material handling costs in the following periods. Significant changes in inter-departmental material flows may require adding new departments, removing, repurposing, or rearrangement of the existing departments in order to reduce the material handling cost. In addition, department area requirements may change depending on the product portfolio and demand changes. However, rearranging departments within the facility is also a costly operation. In addition to the material handling cost, the cost of rearranging departments is also considered in the DFLP.

Since its introduction by Rosenblatt [22], many variants of the DFLP and solution approaches have been proposed in the literature as summarized in [1] and [7]. In

the literature, the DFLP is most frequently studied with equal area departments (e.g., [26, 25, 27, 3, 2, 18, 16, 14]). In these approaches, the DFLP is formulated as a quadratic assignment problem. Only a limited number of papers focus on the DFLP with unequal area departments [5, 6, 13, 12, 17, 20, 29]. In the mathematical formulations of the DFLP with unequal area departments, it is generally assumed that department dimensions are predetermined problem parameters. Although the assumption of fixed department dimensions leads to linear mathematical models and makes the DFLP with unequal area departments computationally more tractable, the problem is still very difficult to solve using exact approaches. Therefore, heuristic approaches are frequently used to solve the problem in the literature.

As stated earlier, the CFLP is a version of the DFLP such that the planning horizon is partitioned into T time periods ($t = 1, \dots, T$), and after time period T , the inter-departmental flows return back to its initial state in the first time period. In other words, the product portfolio and demands significantly change from one time period to another within the planning horizon, but they remain relatively stable in each time period over multiple planning horizons. However, this cyclic nature of the inter-departmental flows is not the main motivation for the mathematical formulation and the solution approach introduced in this paper. As a result of the cyclic nature of the product portfolio and demands, departments may also have significantly different area requirements in each time period. Additionally, due to the limited size of the facility, it is impossible to fit all departments with their maximum area requirements into the facility at the same time. Therefore, some of the departments should be relocated or their sizes must be adjusted to ensure that all minimum department area requirements are satisfied as demonstrated in Figure (1). In other words, the dynamic area requirements of departments is another justification for rearranging the departments in the CFLP in addition to the the dynamic nature of inter-departmental flows. In the literature, on the other hand, the existing formulations of the DFLP with unequal area departments assume that the department dimensions are predetermined. Furthermore, the size of the facility is assumed to be much larger than the total area requirement of departments in many test instances of the DFLP with unequal area departments. This implicit assumption is widely accepted in the literature mainly because of the difficulty of finding a feasible arrangement of departments with predetermined dimensions within a facility. On the other hand, this implicit assumption is not valid in real-life scenarios where a facility with limited space is repurposed for a different set of products in each production cycle. If the facility has limited empty space, then the departments with predetermined dimensions cannot be freely arranged within the facility, and thus the department shapes and dimensions become important decision variables that determine the performance of the layout. Because of these reasons, the previous DFLP formulations and solution approaches are not well suited to the CFLP. Therefore, we have recently introduced a new formulation for the CFLP where department dimensions are considered as decision variables in addition to their locations [11].

In this paper, we develop a SA algorithm with a dynamic temperature schedule based on a recent formulation of the CFLP [11] to solve large-sized problems. Note that Lacksonen [13, 12] also proposed a two stage approach to the DFLP with unequal department areas such that relative department locations are determined in the first stage, and after fixing the locations of the departments, their exact dimensions are determined in the second stage using an area approximation technique. In the SA algorithm presented in this paper, department dimensions and locations are simultaneously determined.

2. Problem Description and Mathematical Formulation

In this section, we present the mathematical formulation of the CFLP where department dimensions are considered as decision variables unlike the previous formulations to the DFLP. For the brevity of the presentation, we use superscript $s = \{x, y\}$ to represent the x and y -axis directions. The CFLP is defined as follows. A set of products with seasonal

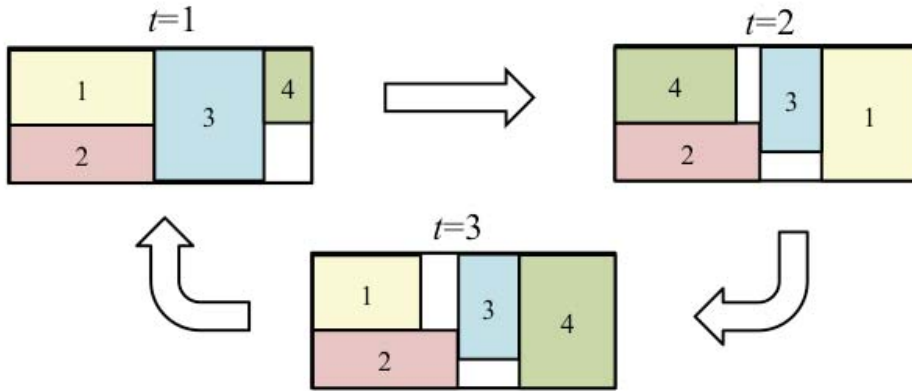


FIGURE 1. A CFLP with four departments and three time periods.

demands are manufactured within a facility with n rectangular-shaped departments. The planning horizon includes T production time periods such that the product portfolio and demands vary significantly from one period to another. Therefore, the area requirements of the departments as well as the inter-departmental material flows depend on the time period. In addition, the product portfolio and demand have a cyclic pattern, i.e., the same sequence of the production periods repeat in each planning horizon.

The input parameters of the problem are as follows:

- L^s side length of the facility along the s -axis direction
- a_{it} minimum area requirement of department i in time period t
- lb_{it}^s minimum side length of department i in time period t along the s -axis direction
- ub_{it}^s maximum side length of department i in time period t along the s -axis direction
- R_{it} variable rearrangement cost per unit distance relocation of department i from time period t to $t+1$
- Q_{it} fixed rearrangement cost of department i from time period t to $t+1$
- f_{ijt} quantity of the material flow between departments i and j in time period t
- m_{ijt} material handling cost per unit distance between departments i and j in time period t

The decision variables of the problem are summarized as follows:

- c_{it}^s s -axis coordinate of the centroid of department i in time period t
- l_{it}^s side length of department i along the s -axis direction in time period t
- z_{ijt}^s binary decision variable denoting the relative location of department i with respect to department j in time period t such that $z_{ijt}^s = 1$ if department i is enforced to precede department j in the s -axis direction, $z_{ijt}^s = 0$ otherwise.

The total material handling cost (MHC) is calculated as follows:

$$MHC = \sum_{t=1}^T \sum_{i=1}^n \sum_{j=i+1}^n m_{ijt} f_{ijt} (d_{ijt}^x + d_{ijt}^y)$$

where d_{ijt}^x and d_{ijt}^y denote the distances between the centroids of departments i and j along the x - and y - axis directions in time period t , respectively. For department pair i and j , $(d_{ijt}^x + d_{ijt}^y)$ is the rectilinear distance between their centroid coordinates, (c_{it}^x, c_{it}^y) and (c_{jt}^x, c_{jt}^y) , in time period t .

In the mathematical formulation of the CFLP presented in this paper, the actual size of department i is allowed to be larger than the minimum area requirement of the department, i.e., $l_{it}^x \times l_{it}^y \geq a_{it}$. On the other hand, the area of the facility may be smaller than the sum of the maximum area requirement of the departments, i.e., $L^x \times L^y \leq \sum_{i=1}^n \max_{t=1, \dots, T} (a_{it})$. Therefore, it is important to determine the dimensions of the departments in a way that any unnecessary department rearrangements are avoided. This strategy is only possible by considering l_{it}^x and l_{it}^y as decision variables.

In the CFLP, the rearrangement cost of a department is quantified by measuring how much its northeast and southwest corners are moved from time period t to $t + 1$. Let u_{it}^x and v_{it}^s denote how much the northeast and southwest corners of department i are moved in the s -axis direction from time period t to $t + 1$. Then, the total rearrangement cost (RC) is calculated as:

$$RC = 0.5 \sum_{t=1}^T \sum_{i=1}^n R_{it}(u_{it}^x + u_{it}^y + v_{it}^x + v_{it}^y) + \sum_{t=1}^T \sum_{i=1}^n Q_{it}r_{it}$$

where r_{it} is a binary variable such that $r_{it} = 1$ if department i is relocated from time period t to $t + 1$, and $r_{it} = 0$ otherwise. Note that the variables and cost parameters for time period T represent the changes from time period T to 1. In the DFLP literature, the rearrangement cost is usually calculated based on how much department centroids are moved between consecutive time periods. However, this approach is not appropriate for the CFLP because department sizes are implicitly considered as decision variables. It is possible that the size of a department changes between two consecutive periods, but its centroid remains the same. In fact, maintaining the same size, shape, and location for departments with high rearrangement costs throughout the planning horizon can be an economical strategy. In this strategy, departments with high rearrangement costs may have larger areas than their minimum area requirements in some time periods in order to minimize their rearrangement costs. If the facility has limited area, deciding the optimal sizes of departments and which departments will maintain their sizes over multiple time periods sets the CFLP apart from the other variations of the DFLP. However, this strategy could not be implemented using department centroids to quantify the rearrangement cost. Therefore, we calculate the rearrangement cost based on the movement of the diagonal corners of departments. The mathematical formulation of the CFLP is given in Problem CFLP as follows:

Problem CFLP:

$$\text{Minimize } TC = MHC + RC$$

Subject to:

$$z_{ijt}^x + z_{ijt}^y + z_{jit}^x + z_{jit}^y = 1 \quad \forall i < j, t \quad (1)$$

$$c_{jt}^s - 0.5l_{jt}^s \geq c_{it}^s + 0.5l_{it}^s - M(1 - z_{ijt}^s) \quad \forall i \neq j, t, s \quad (2)$$

$$c_{it}^s + 0.5l_{it}^s \leq L^s \quad \forall i, t, s \quad (3)$$

$$c_{it}^s - 0.5l_{it}^s \geq 0 \quad \forall i, t, s \quad (4)$$

$$d_{ijt}^s \geq c_{it}^s - c_{jt}^s \quad \forall i < j, t, s \quad (5)$$

$$d_{ijt}^s \geq c_{jt}^s - c_{it}^s \quad \forall i < j, t, s \quad (6)$$

$$u_{it}^s \geq (c_{i(t+1)}^s + 0.5l_{i(t+1)}^s) - (c_{it}^s + 0.5l_{it}^s) \quad \forall i, t < T, s \quad (7)$$

$$u_{it}^s \geq (c_{it}^s + 0.5l_{it}^s) - (c_{i(t+1)}^s + 0.5l_{i(t+1)}^s) \quad \forall i, t < T, s \quad (8)$$

$$u_{iT}^s \geq (c_{iT}^s + 0.5l_{iT}^s) - (c_{i1}^s + 0.5l_{i1}^s) \quad \forall i, s \quad (9)$$

$$u_{iT}^s \geq (c_{i1}^s + 0.5l_{i1}^s) - (c_{iT}^s + 0.5l_{iT}^s) \quad \forall i, s \quad (10)$$

$$v_{it}^s \geq (c_{i(t+1)}^s - 0.5l_{i(t+1)}^s) - (c_{it}^s - 0.5l_{it}^s) \quad \forall i, t < T, s \quad (11)$$

$$v_{it}^s \geq (c_{it}^s - 0.5l_{it}^s) - (c_{i(t+1)}^s - 0.5l_{i(t+1)}^s) \quad \forall i, t < T, s \quad (12)$$

$$v_{iT}^s \geq (c_{iT}^s - 0.5l_{iT}^s) - (c_{i1}^s - 0.5l_{i1}^s) \quad \forall i, s \quad (13)$$

$$v_{iT}^s \geq (c_{i1}^s - 0.5l_{i1}^s) - (c_{iT}^s - 0.5l_{iT}^s) \quad \forall i, s \quad (14)$$

$$l_{it}^s \leq ub_{it}^s \quad \forall i, t, s \quad (15)$$

$$l_{it}^s \geq lb_{it}^s \quad \forall i, t, s \quad (16)$$

$$r_{it}(L^x + L^y) \geq u_{it}^x + u_{it}^y + v_{it}^x + v_{it}^y \quad \forall i, t \quad (17)$$

$$a_{it} \geq l_{it}^x l_{it}^y \quad \forall i, t, s \quad (18)$$

$$d_{ijt}^s, c_{it}^s, u_{it}, v_{it} \geq 0$$

$$r_{it}, z_{ijt}^s \in \{0, 1\}$$

Constraint (1) ensures that each department pair i and j are enforced to precede one another along either the x -axis or y -axis direction in each time period t . Constraint (2) ensures that departments i and j will not overlap along the s -axis direction if $z_{ijt}^s = 1$ in time period t . Note that if $z_{ijt}^s = 0$, departments i and j may or may not overlap along the s -axis direction. Constraints (3) and (4) make sure that the departments are located only within the boundaries of the facility. Constraint (5) and (6) are used to calculate the rectilinear distance between the centroids of departments i and j in time period t . Constraints (7)-(10) are used to calculate the rearrangement of the northeast corner of each department from time period t to $t + 1$. In the similar fashion, Constraints (12)-(14) are used to calculate the rearrangement of the southwest corner of each department. At the end of period T as stated in Constraints (9), (10), (13), and (14), the departments are expected to be located at their starting locations in time period 1. Constraints (15) and (16) control the shapes and dimensions of the departments in order to prevent impractical department shapes.

Constraint (18) is the minimum area requirement of department i in time period t . Instead of this non-linear constraint, polyhedral outer approximation constraints suggested by Sherali et al. [23] are used as follows:

$$a_{it}l_{it}^x + 4\bar{x}_{ipt}^2 l_{it}^y \geq 4a_{it}\bar{x}_{ipt} \quad \forall i, p, t \quad (19)$$

where \bar{x}_{ipt} is called a tangential support point and calculated as follows:

$$\bar{x}_{ipt} = lb_{it}^x + \frac{p}{\Delta - 1}(ub_{it}^x - lb_{it}^x) \quad p = 0, \dots, \Delta - 1$$

where parameter Δ is the number of tangential support points to enforce the minimum area requirement of the departments. Parameter Δ determines accuracy of the area approximation. In the FLP with a single time period, the inequality in (19) provides a linear approximation to the non-linear relationship between the department area and side lengths, i.e., $a_i = l_i^x \times l_i^y$, without using any binary variables because department areas are forced to be as small as possible to minimize the distances between their centroids [23].

It should be noted that constraint (15) can be dropped from the formulation, or ub_{it}^s should be modified to ensure that departments can be larger than their minimum area requirements. This is particularly important when department area requirements change significantly during the planning horizon.

3. Simulated Annealing Algorithm

The CFLP defined above is a very challenging problem to solve using exact approaches. Developing a heuristic algorithm to solve the CFLP is also challenging because of the variety of the decision variables and constraints involved in the problem. In particular, binary

decision variables z_{ijt}^s should be encoded in a consistent manner to avoid infeasible layouts [10, 19]. For example, the assignment of $z_{ijt}^x = 1$, $z_{jkt}^x = 1$, $z_{kit}^x = 1$ is illogical because this assignment states that department i precedes department k in the x -axis due to the relationship $z_{ijt}^x = 1$ and $z_{jkt}^x = 1$, but then department k precedes department i due to $z_{kit}^x = 1$. In addition, the CFLP has several different types of constraints which are challenging to handle in a heuristic algorithm [9].

3.1. Move Definition

The SA algorithm is based on the observation that a solution to the problem can be represented by binary decision variables z_{ijt}^s and the value of all other decision variables can be determined by solving a linear MIP model. It should be noted that binary decision variable r_{it} is an auxiliary variable to calculate the fixed rearrangement cost. Therefore, the optimal value of variable r_{it} can be effectively determined for a given settings of binary decision variables z_{ijt}^s . Let $z_{ijt}^s(\mu)$ denote the value of binary decision variable z_{ijt}^s for a solution μ to Problem CFLP, and $TC(\mu)$ be its objective function value. In the SA algorithm, a neighbor $\mu_{i't'}$ of solution μ is obtained by fixing $z_{ijt}^s \leftarrow z_{ijt}^s(\mu) \forall i \neq j, i \neq i', j \neq i', t \neq t', s$, and then solving the problem for the remaining binary and continuous decision variables. Thereby, the SA algorithm operates directly on the decision variables of the problem without requiring any special problem encoding scheme. In addition, the optimal dimensions of the departments can be determined for a given settings of the binary decision variables z_{ijt}^s . The pair (i', t') for $i' = 1, \dots, n$ and $t' = 1, \dots, T$ represent the move operator in the SA algorithm. If the current solution μ is a local optimum with respect to decision variables z_{ijt}^s and $z_{i't'}^s$, then $move(i', t')$ will lead to the same solution, i.e., $\mu = \mu_{i't'}$. Therefore, it is necessary to ensure that $move(i', t')$ will result in a different solution than the current solution μ . In order to achieve this goal, the following constraints are added to Problem CFLP before applying $move(i', t')$ on the current solution μ .

$$\sum_{j=i'} \sum_{s=\{x,y\}} \{z_{ijt'}^s(\mu)(1 - z_{i't'}^s(\mu)) + z_{ji't'}^s(\mu)(1 - z_{i't'}^s(\mu))\} \geq \sigma \quad (20)$$

$$\sum_{j=i'} \sum_{s=\{x,y\}} \{(1 - z_{ijt'}^s(\mu))z_{i't'}^s(\mu) + (1 - z_{ji't'}^s(\mu))z_{i't'}^s(\mu)\} \geq \sigma \quad (21)$$

Constraints (20) and (21) enforce the neighbor solution $\mu_{i't'}$ to be different from the current solution μ in at least 2σ binary decision variables. Constraint (20) enforces at least one of variables $z_{ijt'}^s$ to be zero such that $z_{i't'}^s(\mu) = 1$. On the contrary, Constraint (21) enforces at least one of variables $z_{ji't'}^s$ to be one such that $z_{i't'}^s(\mu) = 0$. Constraints (20) and (21) are also used as a diversification technique in the SA algorithm as explained in the following section.

3.2. Solution Initialization

The sequence-pair representation [15] is used to find an initial solution which is feasible with respect to the department overlapping constraints, i.e., constraint (1) in Problem CFLP. Let π_1 and π_2 be two arrays of n random numbers, and let $\pi_1(i)$ and $\pi_2(i)$ denote the random numbers in the i th position of the respective arrays. Based on the results of Murata et al. [21], we can state that each sequence pair (π_1, π_2) corresponds to a feasible setting of binary variables z_{ijt}^s of Problem CFLP if they are assigned according to Algorithm 1.

It should be noted that a feasible setting of the binary decision variables which satisfies the department overlapping constraints does not necessary suffice that the solution will be feasible with respect to the other constraints. When departments have restrictive aspect ratio constraints and the facility area is utilized close to 100 percent, it is difficult to find a feasible arrangement of the departments [15]. In the literature, this problem is addressed by

Algorithm 1 Initialization of the current solution μ

```

 $z_{ijt}^s(\mu) \leftarrow 0 \quad \forall i \neq j, t, s$ 
for  $t = 1, \dots, T$  do
   $\pi_1(i) \leftarrow \text{Rand}() \quad \forall i = 1, \dots, n$ 
   $\pi_2(i) \leftarrow \text{Rand}() \quad \forall i = 1, \dots, n$ 
  for  $i = 1, \dots, n; j = i + 1, \dots, n$  do
    if  $\pi_1(i) < \pi_1(j)$  and  $\pi_2(i) < \pi_2(j)$  then  $z_{ijt}^x(\mu) \leftarrow 1$ 
    end if
    if  $\pi_1(i) > \pi_1(j)$  and  $\pi_2(i) > \pi_2(j)$  then  $z_{jit}^x(\mu) \leftarrow 1$ 
    end if
    if  $\pi_1(i) > \pi_1(j)$  and  $\pi_2(i) < \pi_2(j)$  then  $z_{ijt}^y(\mu) \leftarrow 1$ 
    end if
    if  $\pi_1(i) < \pi_1(j)$  and  $\pi_2(i) > \pi_2(j)$  then  $z_{jit}^y(\mu) \leftarrow 1$ 
    end if
  end for
end for
for  $t = 1, \dots, T$  do
  for  $i = 1, \dots, n; j = i + 1, \dots, n; s = x, y$  do
    if  $\text{Rand}() < \gamma$  then
      Fix binary variables  $z_{ijt}^s \leftarrow z_{ijt}^s(\mu)$  and  $z_{jit}^s \leftarrow z_{jit}^s(\mu)$ 
    end if
  end for
end for
Solve Problem CFLP to determine  $TC(\mu)$ 

```

expanding the facility dimensions and/or modifying department area requirements [4, 8, 10]. To increase the likelihood of finding feasible solutions, we randomly set γ percent of the binary decision variables according to the sequence-pair representation [15] and use Problem CFLP to determine the values of the remaining decision variables in the initialization phase. Parameter γ should be selected considering the tradeoffs between the CPU time and the solution quality. For the test problems, $\gamma > 0.60$ was used based on our observations in preliminary experiments. Values of γ smaller than 0.60 resulted in long runs without improving solution quality. This strategy does not ensure that the SA algorithm starts with a feasible solution, yet it reduced the number of the iterations when the first feasible solution was found in our preliminary experiments.

As stated before, the facility size is also assumed to be much larger than the total area requirement of departments in many unequal-area DFLP test problems. In real life greenfield facility designs, the size of the facility is not a hard constraint, thus the facility dimensions can be expanded to find better arrangement of the departments. However, the CFLP in this paper is motivated by real-life cases where a facility with limited empty space is repurposed in each production season. Therefore, the SA algorithm is expected to perform well for test problems with limited empty space in the facility. In order to quantify infeasibility of a candidate solution in the SA algorithm, constraint (3) is modified as follows:

$$c_{it}^s + 0.5l_{it}^s - q_{it}^s \leq L_s \quad \forall i, t, s$$

where q_{it}^s denotes how much department i violates the north ($s = y$) and the west ($s = x$) boundaries of the facility in time period t . Variable q_{it}^s is penalized in the objective function by multiplying it with $(L^x + L^y) \sum_{t=1}^T \sum_{i=1}^n \sum_{j=i+1}^n f_{ijt}$. Note that all other constraints are enforced to obtain neighbor $\mu_{i't'}$. Therefore, if a solution has $q_{it}^s = 0 \quad \forall i, t, s$, then it is also a feasible for the original problem.

3.3. The Procedure of the SA Algorithm

The overall procedure of the SA algorithm is given in Algorithm 2. In the preliminary computational experiments with a canonical SA, we observed that the current solution converged to local optima, and the search was stagnated rather quickly. Early convergence to local optima is a typical problem in heuristic algorithms, and various strategies are recommended in the literature to address this problem. In our case, early convergence is particularly an important problem because of the way that neighbor solutions are generated from the current solution. As stated earlier, when the current solution is local optimum with respect to the move definition, all neighbor solutions yield the same objective function value with the current solution. In other words, the search become stuck in a plateau from which the current solution is not able to move away. Therefore, we modified the canonical SA [28] to address this problem.

In each iteration of the algorithm, $move(i', t')$ is selected randomly and uniformly without replacement from the set of all possible moves (denoted by set Π in Algorithm 2). When all moves in set Π are exhausted, the set is reinitiated. Thereby, all possible moves are considered once in every $n \times T$ consecutive iterations. In the beginning of the search, the reduced MIP problem is solved for an optimality gap of $\epsilon = 0.05$, which is gradually reduced to zero.

The probability of accepting a neighbor solution $\mu_{i't'}$ as the new current solution μ is calculated as follows:

$$p \leftarrow \begin{cases} 1 & \text{if } TC(\mu_{i't'}) < TC(\mu) \\ \exp((1 + \frac{TC(\mu_{i't'}) - TC(\mu)}{\tau \cdot TC(\mu)})^{-1}) & \text{if } TC(\mu_{i't'}) \geq TC(\mu) \end{cases} \quad (22)$$

where τ denotes the current temperature value. The selection probability p is between 0 and 0.5. We preferred this acceptance probability function because of two reasons. Firstly, using the percent difference instead of the absolute difference between $TC(\mu_{i't'})$ and $TC(\mu)$ is advantageous because the penalty term, which can be very large for infeasible solutions, makes it challenging to determine a proper starting value for τ (τ_0). Thereby, the initial temperature τ_0 can be selected between 0 and 1 without considering the magnitude of the objective function value. Secondly, the acceptance probability of the canonical SA becomes 1 when $TC(\mu_{i't'}) \approx TC(\mu)$, which is very likely to be observed in our case. In such cases, neighbor $\mu_{i't'}$ will be selected as the new current solution. On the other hand, the selection probability in given equation (22) is 0.5 if $TC(\mu_{i't'}) \approx TC(\mu)$.

A dynamic temperature schedule is used to update τ based on the historical distance between the current solution μ and the best solution μ^* in the objective space. Let λ represent the smoothed average distance between $TC(\mu)$ and $TC(\mu^*)$ as follows:

$$\lambda \leftarrow (1 - \alpha) \frac{TC(\mu^*) - TC(\mu)}{TC(\mu^*)} + \alpha \lambda \quad (23)$$

where α is the smoothing parameter. The current temperature τ is updated in each iteration as follows:

$$\tau \leftarrow \min(\tau_{min}, \tau_k(1 - \lambda)) \quad (24)$$

The dynamic temperature schedule approach given in equations (23) and (24) leads to a fast cooling schedule if the current solution is far away from the best solution in recent iterations and to a slow cooling schedule if the current and the best solutions are very close to one another in recent iterations. In other words, the SA exhibits a risk-taker behavior if $TC(\mu^*) \approx TC(\mu)$ or a risk-averse behavior if $TC(\mu^*)$ is significantly lower than $TC(\mu)$ in recent iterations.

The SA algorithm has also a diversification approach. Note that all possible moves are exhaustively applied in a random order. If the best solution μ^* has not been updated after

all moves in set Π are performed, τ is set to τ_0 and the value of the parameter σ is changed to 2 if it is 1 and vice versa. Changing the value of the parameter σ causes the move operator to generate neighbors that are different than the ones generated in the previous iterations. In addition, the value of parameter ϵ is reduced. Figure (2) illustrates an example for the convergence of the best solution μ^* and the current solution μ along with the current temperature τ during the search. As seen in the figure, if the SA algorithm has not been able to improve the best solution μ^* , the search moves into a diversification mode by frequently increasing and decreasing the temperature. On the other hand, when the best solution μ^* is being improved, the temperature is steadily reduced, i.e., the search becomes more deterministic. The SA algorithm adaptively decides the level of diversification (a slow cooling schedule) or intensification (a fast cooling schedule) based on equation (23).

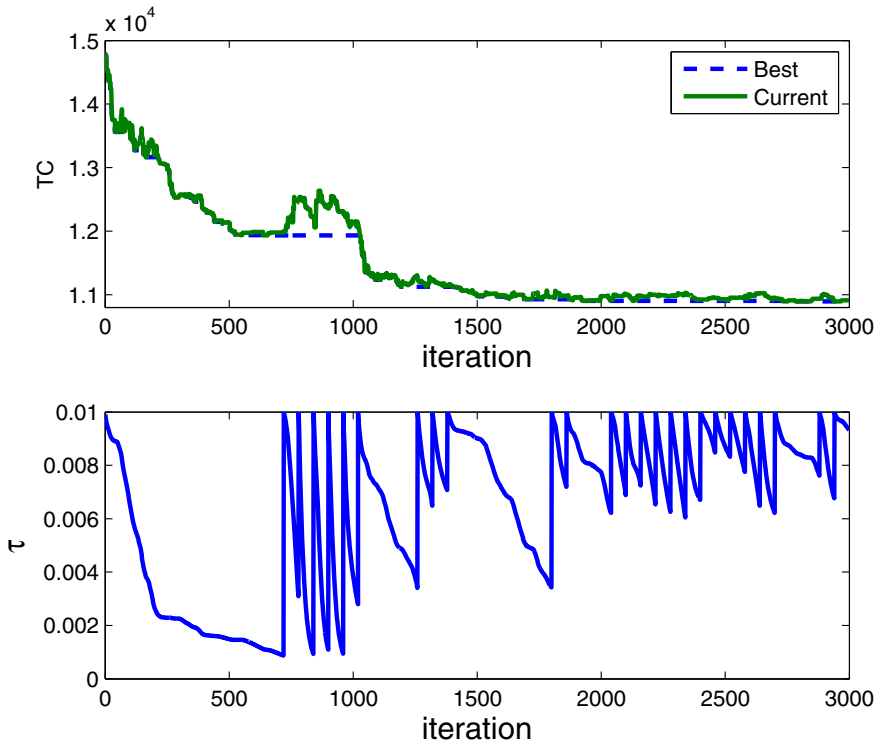


FIGURE 2. An example for the convergence of the SA algorithm and the dynamic temperature update schedule.

4. Computational Experiments

We benchmarked the performance of the SA algorithm using four test problems. In the literature, the only test problem for the CFLP is a 15-department and three-period test problem (P15), which has been recently published based on a real-life case [11]. In P15, the layout of the third period is considered as the starting layout for the first period. P15 assumes a rectangular facility with a limited empty space, and facility dimensions are $L^x = 30$ and $L^y = 20$. In addition, the minimum area requirements of the departments significantly change over the planning horizon, and the facility area is smaller than the sum of the maximum sizes of the departments. In addition, the problem has a maximum aspect ratio requirement of 2

Algorithm 2 Procedure of the SA algorithm

```

Set  $update \leftarrow 0$ 
Set  $\epsilon \leftarrow 0.05$ 
Set  $\tau \leftarrow \tau_0$ 
Randomly initiate  $\mu$  and  $\mu^* \leftarrow \mu$ 
for  $g = 1, \dots, g_{max}$  do
     $update \leftarrow update + 1$ 
    Randomly initialize set  $\Pi$ 
    while  $\Pi \neq \emptyset$  do
        Select randomly and uniformly  $move(i', t')$  from  $\Pi$  and set  $\Pi \leftarrow \Pi \setminus (i', t')$ 
        Fix binary variables  $z_{ijt}^s \leftarrow z_{ijt}^s(\mu) \quad \forall i \neq j, t, s$ 
        Unfix binary variables  $z_{i'jt'}^s$  and  $z_{ji't'}^s \quad \forall i' \neq j, t, s$ 
        Add constraints (20) and (21) to the problem
        Solve the reduced problem up to  $\epsilon$  optimality and determine  $TC(\mu_{i't'})$ 
        Calculate  $p$  using equation (22)
        if  $p \geq Rand()$  then
             $\mu \leftarrow \mu_{i't'}$ 
        end if
        if  $TC(\mu) \leq TC(\mu^*)$  then
            Set  $\mu^* \leftarrow \mu$ 
            Set  $update \leftarrow 0$ 
            Set  $\sigma \leftarrow 1$ 
        end if
        Remove constraints (20) and (21) from the problem
        Calculate  $\lambda$  based on Equation (23)
        Calculate  $\tau$  based on Equation (24)
    end while
    if  $update > 0$  then
        Set  $\tau \leftarrow \tau_0$ 
        Set  $\sigma \leftarrow (\sigma \bmod 2) + 1$ 
        Set  $\epsilon \leftarrow \min(0, \epsilon - 0.005)$ 
    end if
end for

```

for each department, which restricts the number of possible layout configurations. Because of these reasons, finding feasible solutions for P15 is challenging. For this problem, $\Delta = 30$ was used to ensure a very small area approximation error.

In addition to P15, three test problems, DFLP 12-3, 12-5, and 20-3 from Lacksonen [13] were studied. In these problems, the dimensions of the departments are not predefined unlike the majority of DFLP test cases in the literature. A maximum aspect ratio of 2 is used to constraint department shapes. Although department area approximation errors were not provided by Lacksonen [13], the approximation approach was expected to result in maximum 3 percent error according to an earlier work [12]. The SA algorithm was run for five random replications with $\Delta = 5$ to compare the results with the earlier results with respect to the same level of area approximation error. In these problems, the fixed rearrangement cost depends on the size of the departments, and there is no variable rearrangement cost. In DFLP 12-3 and DFLP 12-5, two new departments replace existing ones in each planning period. Similarly, three new departments replace the existing ones in DFLP 20-3. In addition to the rearrangement cost of existing departments, there is a cost for replacing an existing department with a new one. Therefore, instead of Problem CFLP, the DFLP model given in [13] was used. In all experiments, the parameters of the SA algorithm were set as $g_{max} = 50$,

$\tau_0 = 0.01$, $\tau_{min} = 0.0001$, $\alpha = 0.1$, $\gamma = 0.60$ for P15, and $\gamma = 0.85$ for the other test problems. The SA algorithm was coded in the AMPL modeling language, and reduced MIP problems were solved using CPLEX 12.4 with a single thread. All experiments were performed in a Linux-based computer with a Intel Xeon E5450 Quad-Core 3.0 GHz processor and 32 GB memory.

The percent area approximation error of department i in time period t was calculated for the best-feasible solution as follows:

$$e_{it} = 100 \max(a_{it} - l_{it}^x l_{it}^y, 0) / a_{it} \tag{25}$$

Table 1 summarizes the results found by the SA algorithm in five replications and the previous best-known solutions of the test problems. In Table 1, $e_{max} = \max(e_{it} : \forall i, t)$ denotes the maximum area approximation error. As seen in the table, the SA algorithm significantly improved upon the best solutions of DFLP 12-3, 12-5, and 20-3 reported by Lacksonen [13]. Compared to the Large-Scale SA algorithm [11], the SA algorithm improved upon the best-known results for DFLP 12-5 and P15. In the other two test problems, the SA algorithm found very close solutions to the best-known solutions, but could not improve them. A major advantage of the SA algorithm is its robustness without requiring many parameters to tune. The solutions found in five random replications of the SA algorithm were very close to one another. As a result, the SA algorithm outperformed the Large-Scale SA [11] on the average.

TABLE 1. Comparison of the SA algorithm with previous approaches

Problem	Best [13]	Best [11]	Avg. [11]	SA Best	SA Avg.	Avg. CPU (s)	%Imp. Best	%Imp. Avg.	e_{max}
DFLP 12.3	7094	6622.8	6728.3	6707.5	6715.0	14007	-1.279	0.197	0.010
DFLP 12.5	12271	11412.4	11711.5	11090.1	11154.2	21132	2.824	4.758	0.012
DFLP 20.3	12903	12148.6	12325.8	12171.4	12235.8	12235	-0.188	0.730	0.010
P15	–	8376.6	9034.8	8295.8	8431.6	18658	0.965	6.676	0.027

5. Conclusions

In this paper, a simulated annealing algorithm with a dynamic temperature schedule is introduced for the Cyclic Facility Layout Problem which arises in manufacturing systems with seasonal product portfolio and demands. In addition, the proposed formulation relaxes the assumption of predetermined and fixed department dimensions, which is widely used in the DFLP literature. The proposed formulation is particularly applicable to real-life cases where a set of departments with seasonal area requirements are to be located in a facility with limited size. The SA can be applied to other types of facility layout problems because it operates directly on the decision variables of the mathematical formulation of the problem. The proposed dynamic temperature schedule is also shown to be effective.

References

[1] J. Balakrishnan and C.H. Cheng. Dynamic layout algorithms: A state of the art survey. *Omega*, 26(4):507–521, 1998.

[2] J. Balakrishnan and C.H. Cheng. The dynamic plant layout problem: incorporating rolling horizons and forecast uncertainty. *Omega*, 37(1):165–177, 2009.

[3] J. Balakrishnan, C.H. Cheng, and D.G. Conway. An improved pair-wise exchange heuristic for the dynamic plant layout proble. *International Journal of Production Research*, 38(13):3067–3077, 2000.

[4] Y.A. Bozer and C.T. Wang. A graph-pair representation and mip-model-based heuristic for the unequal-area facility layout problem. *European Journal of Operational Research*, 218(2):382–391, 2011.

- [5] T. Dunker, G. Radons, and E. Westkamper. Combining evolutionary computation and dynamic programming for solving a dynamic facility layout problem. *European Journal of Operational Research*, 165(1):55–69, 2005.
- [6] F. Jolai, R. Tavakkoli-Moghaddam, and M. Taghipour. A multi-objective particle swarm optimisation algorithm for unequal sized dynamic facility layout problem with pickup/drop-off locations. *International Journal of Production Research*, 50(15):4279–4293, 2012.
- [7] S. Kulturel-Konak. Approaches to uncertainties in facility layout problems: Perspectives at the beginning of the 21st century. *Journal of Intelligent Manufacturing*, 18:273–284, 2007.
- [8] S. Kulturel-Konak. A linear programming embedded probabilistic tabu search for the unequal-area facility layout problem with flexible bays. *European Journal of Operational Research*, 223:614–625, 2012.
- [9] S. Kulturel-Konak, B.A. Norman, D.W. Coit, and A.E. Smith. Exploiting tabu search memory in constrained problems. *INFORMS Journal on Computing*, 16(3):241–254, 2004.
- [10] S. Kulturel-Konak and A. Konak. Linear programming based genetic algorithm for the unequal area facility layout problem. *International Journal of Production Research*, 51(14):4302–4324, 2013.
- [11] S. Kulturel-Konak and A. Konak. A large-scale hybrid simulated annealing algorithm for cyclic facility layout problems. *Engineering Optimization*, in-print:1–16, 2014.
- [12] T.A. Lacksonen. Static and dynamic layout problems with varying areas. *Journal of the Operational Research Society*, 45(1):59–69, 1994.
- [13] T.A. Lacksonen. Preprocessing for static and dynamic facility layout problems. *International Journal of Production Research*, 35(4):1095–1106, 1997.
- [14] T.A. Lacksonen and E.E. Enscore. Quadratic assignment algorithms for the dynamic layout problem. *The International Journal of Production Research*, 31(3):503–517, 1993.
- [15] Q. Liu and R.D. Meller. A sequence-pair representation and mip-model-based heuristic for the facility layout problem with rectangular departments. *IIE Transactions*, 39(4):377–394, 2007.
- [16] V. M. Pillai, I.B. Hunagund, and K.K. Krishnan. Design of robust layout for dynamic plant layout problems. *Computers and Industrial Engineering*, 61(3):813–823, 2011.
- [17] A.R. McKendall and A. Hakobyan. Heuristics for the dynamic facility layout problem with unequal-area departments. *European Journal of Operational Research*, 201(1):171–182, 2010.
- [18] A.R. McKendall and W-H. Liu. New tabu search heuristics for the dynamic facility layout problem. *International Journal of Production Research*, 50(3):867–878, 2012.
- [19] R.D. Meller, W. Chen, and H.D. Sherali. Applying the sequence-pair representation to optimal facility layout designs. *Operations Research Letters*, 35(5):651–659, 2007.
- [20] B. Montreuil and A. Laforge. Dynamic layout design given a scenario tree of probable futures. *European Journal of Operational Research*, 63:271–286, 1992.
- [21] H. Murata, K. Fujiyoshi, S. Nakatake, and Y. Kajitani. Vlsi module placement based on rectangle-packing by the sequence-pair. *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, 15(12):1518–1524, 1996.
- [22] M.J. Rosenblatt. The dynamics of plant layout. *Management Science*, 32:76–86, 1986.
- [23] H.D. Sherali, B.M.P. Fraticelli, and R.D. Meller. Enhanced model formulations for optimal facility layout. *Operations Research*, 51(4):629–644, 2003.
- [24] J.A. Tompkins. *Facilities planning*. John Wiley and Sons, 2010.
- [25] T.L. Urban. Computational performance and efficiency of lower-bound procedures for the dynamic facility layout problem. *European Journal of Operational Research*, 57:271–279, 1992.
- [26] T.L. Urban. A heuristic for the dynamic facility layout problem. *IIE Transactions*, 25(4):57–63, 1993.
- [27] T.L. Urban. Solution procedures for the dynamic facility layout problem. *Annals of Operations Research*, 76:323–342, 1998.
- [28] P.J.M. Van Laarhoven and E.H.L. Aarts. *Simulated annealing*. Springer, 1987.
- [29] T. Yang and B.A. Peters. Flexible machine layout design for dynamic and uncertain production environments. *European Journal of Operational Research*, 108(1):49–64, 1998.