AERSP 309 Astronautics Final Exam Review

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Fall 2013

Office Hours

- Dr. Melton
  - Available by email (rgmelton@psu.edu)
- Brad
  - Possibly available by appointment (email to inquire) and by email (bsottile@psu.edu)

We're here to help you!

The Final Exam

- Look over everything you know and have ever done in this class – quizzes, exams, homework, sample problems, readings, etc.
  - The syllabus is a good place to start if you're trying to remember everything we've covered.
  - Just because I did or did not put something in this PowerPoint doesn't mean it is or is not fair game. Try as I may, I can't read Dr. Melton's mind!
- Be ready to work open-ended problems (like the exams so far), but be ready for conceptional problems (like the weekly quizzes).

The Final Exam (Cont'd)

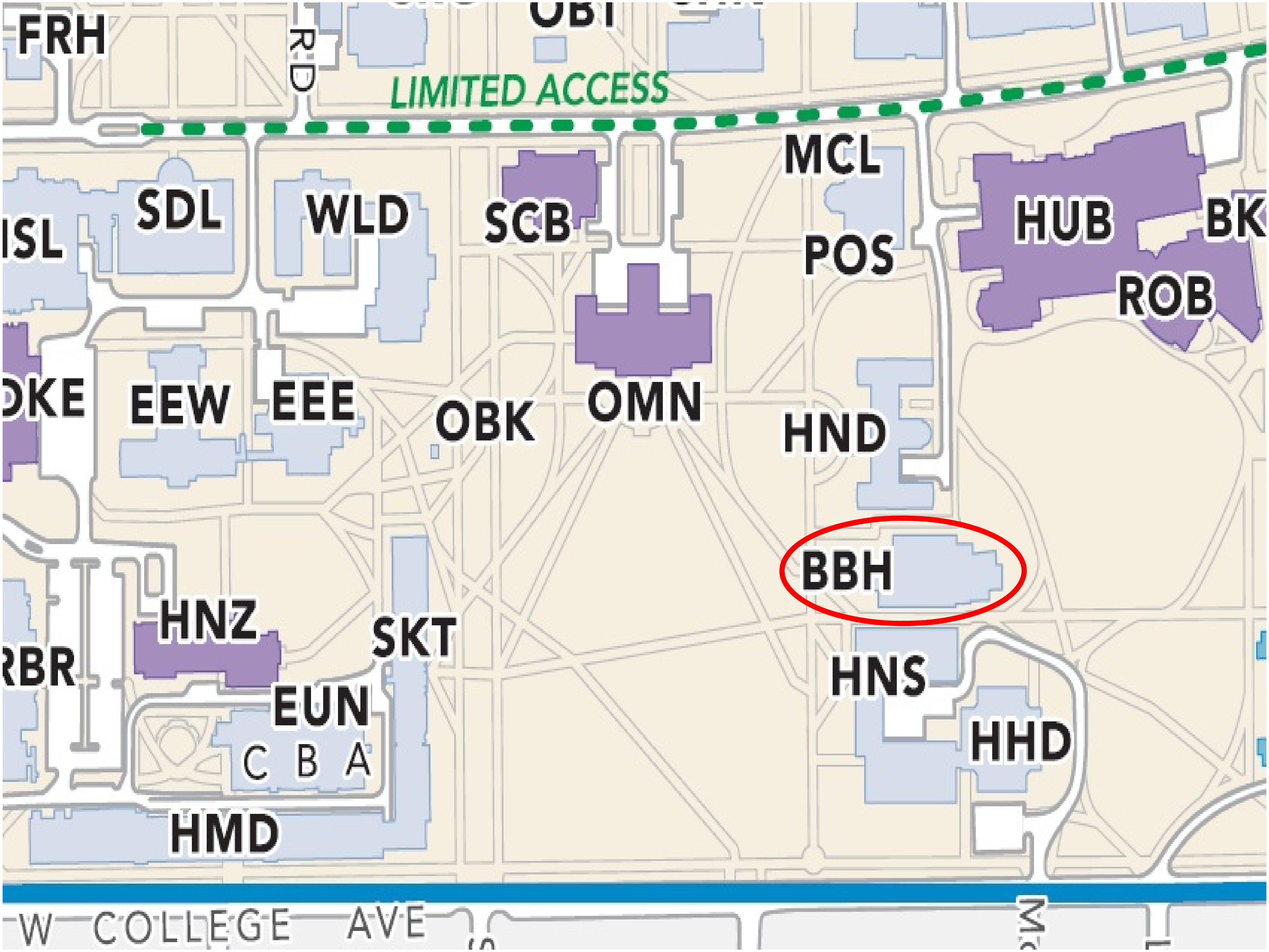
- Reminders:
  - We drop your lowest quiz score
  - We don't drop any of the homework
  - We do not curve either the exams or the course

- Ask questions if you need help! It's never too late* to ask a question about course material!

- Time and Location: Monday, December 16th, 8:00 – 9:50 a.m. in 22 BBH Building

- Exam covers everything in the course!

*Until we hand you the final exam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1. 3D Kinematics

- Direction Cosine Matrices (DCMs)
  - You need to absolutely know how to manipulate matrices (or be able to reference your equation sheet). Be able to do them symbolically and numerically!
  - Be careful with multiplying matrices – students usually make mistakes with this.
  - Know what an inverse is, how to take transposes, etc.
  - Know if your result is suppose to be a scalar, vector, matrix, etc.

Matrix multiplication

Defined

$(3 \times 3)(3 \times 3) = (3 \times 3)$ result

size

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} =$$

$$\begin{bmatrix} a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31} & a_{11}b_{12}+a_{12}b_{22}+a_{13}b_{32} & a_{11}b_{13}+a_{12}b_{23}+a_{13}b_{33} \\ a_{21}b_{11}+a_{22}b_{21}+a_{23}b_{31} & a_{21}b_{12}+a_{22}b_{22}+a_{23}b_{32} & a_{21}b_{13}+a_{22}b_{23}+a_{23}b_{33} \\ a_{31}b_{11}+a_{32}b_{21}+a_{33}b_{31} & a_{31}b_{12}+a_{32}b_{22}+a_{33}b_{32} & a_{31}b_{13}+a_{32}b_{23}+a_{33}b_{33} \end{bmatrix}$$

$$\underset{(3\times 3)}{\begin{bmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix}} \underset{(3\times 1)}{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}} = \underset{(3\times 1)}{\begin{bmatrix} a_{11}b_1+a_{12}b_2+a_{13}b_3 \\ a_{21}b_1+a_{22}b_2+a_{23}b_3 \\ a_{31}b_1+a_{32}b_2+a_{33}b_3 \end{bmatrix}}$$

Defined

Result

Result

1. 3D Kinematics

- DCMs Cont'd
  - Some vocabulary
    - Dextral: Right Handed
    - Orthogonal: Perpendicular
    - Normal: Unit Length
  - Two ways to describe the orientation of Frame B with respect to Frame A
    - Specify the angles between everything – this gives you 9 angles
    - Write the $\hat{b}$ vectors in terms of the $\hat{a}$ vectors

1. 3D Kinematics

- DCMs Cont'd
  - Vector Projection – The dot products are all cos( ). These are known as direction cosines!

$$e.g. \qquad \hat{b}_1 \cdot \hat{a}_1 = |\hat{b}_1||\hat{a}_1| \cos\alpha = \cos\alpha$$

$$\begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \hat{b}_3 \end{bmatrix} = \begin{pmatrix} \hat{b}_1 \cdot \hat{a}_1 & \hat{b}_1 \cdot \hat{a}_2 & \hat{b}_1 \cdot \hat{a}_3 \\ \hat{b}_2 \cdot \hat{a}_1 & \hat{b}_2 \cdot \hat{a}_2 & \hat{b}_2 \cdot \hat{a}_3 \\ \hat{b}_3 \cdot \hat{a}_1 & \hat{b}_3 \cdot \hat{a}_2 & \hat{b}_3 \cdot \hat{a}_3 \end{pmatrix} \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \end{bmatrix}$$

1. 3D Kinematics

- DCMs Cont'd
  - Remember our notation:

$$\begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \hat{b}_3 \end{bmatrix} = \underline{C}^{AB} \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \end{bmatrix} \quad or \quad \underline{b} = \underline{C}^{AB} \underline{a}$$

  - $C_{ij}^{AB} = \hat{b}_i \cdot \hat{a}_j$ is the direction cosine of $\hat{b}_i$ with respect to $\hat{a}_j$

1. 3D Kinematics

- DCMs Cont'd
  - Properties
    - $\underline{C}^{-1} = \underline{C}^{T}$
    - $\underline{C}\,\underline{C}^{-1} = \underline{C}\,\underline{C}^{T} = \underline{C}^{-1}\,\underline{C} = \underline{C}^{T}\underline{C} = \underline{1}$
    - $\det(\underline{C}) = +1$ iff (if and only if) both coordinate systems have the same handedness.
    - To build a DCM using multiple rotations, you must multiply in reverse order.
    - Order matters! (Unless the rotation angles are "small.")

1. 3D Kinematics

- Rotations about the principal axes – Any new orientation can be generated by at most 3 rotations

$$\underline{C}_1(\psi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\psi) & \sin(\psi) \\ 0 & -\sin(\psi) & \cos(\psi) \end{pmatrix}$$

$$\underline{C}_2(\theta) = \begin{pmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{pmatrix}$$

$$\underline{C}_3(\phi) = \begin{pmatrix} \cos(\phi) & \sin(\phi) & 0 \\ -\sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

1. 3D Kinematics

- Vector Components in Different Coordinate Systems
  - You have two options for expressing a vector in terms of another coordinate system: Both require you to use a DCM.
    - Convert the new unit vectors into your old frame
    - Convert the components into the other frame

But ***do not*** do both! You'll get junk answers.

1. 3D Kinematics

- Angular Velocity is the time rate of change (*i.e.* derivative) of an angle.
  - As you saw in E MCH 212 and in this class, you can add these to get relative angular velocities.

- Everything in life is relative:
  - You stand on a merry-go-round/carousel/spinning disk. You select your favorite pony and get ready for the ride to start. Your friend suddenly realizes the ride is too scary for him or her and he or she decides to wait by the fence. The ride starts and your pony goes up and down while the merry-go-round rotates. How does your perception of your movement differ from your friend's perception of your movement?

1. 3D Kinematics

- Derivatives of vectors in other frames of reference
  - First Derivative

$$\frac{{}^{I}d\vec{r}}{dt} = \frac{{}^{B}d\vec{r}}{dt} + \vec{\omega}^{B/I} \times \vec{r}$$

  - Second Derivative

$$\frac{{}^{I}d^2\vec{r}}{dt^2} = \frac{{}^{B}d^2\vec{r}}{dt^2} + \frac{d\vec{\omega}^{B/I}}{dt} \times \vec{r} + 2\vec{\omega}^{B/I} \times \frac{{}^{B}d\vec{r}}{dt} + \vec{\omega}^{B/I} \times (\vec{\omega}^{B/I} \times \vec{r})$$

Strategy: Never compute anything twice!

1. 3D Kinematics

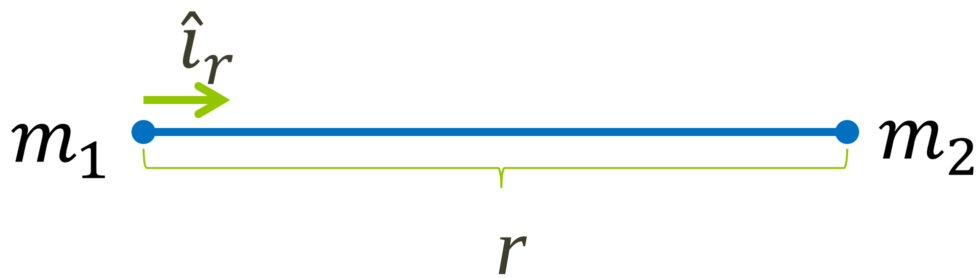
$$\frac{{}^{B}d^{2}\vec{r}}{dt^{2}} = \frac{{}^{I}d^{2}\vec{r}}{dt^{2}} - \frac{d\vec{\omega}^{B/I}}{dt} \times \vec{r} - 2\vec{\omega}^{B/I} \times \frac{{}^{B}d\vec{r}}{dt} - \vec{\omega}^{B/I} \times (\vec{\omega}^{B/I} \times \vec{r})$$

1 2 3 4 5

1. Acceleration of object in frame B
2. Inertial Acceleration = $\frac{\vec{F}}{m}$ (Remember: Newton's Laws only apply in inertial frames of reference).
3. Euler Acceleration
4. Coriolis Acceleration
5. Centrifugal Acceleration ("Center Fleeing")

Note the signs: The book is wrong about this!
Remember concepts such as "deep space," etc.

2. 3D Particle Dynamics

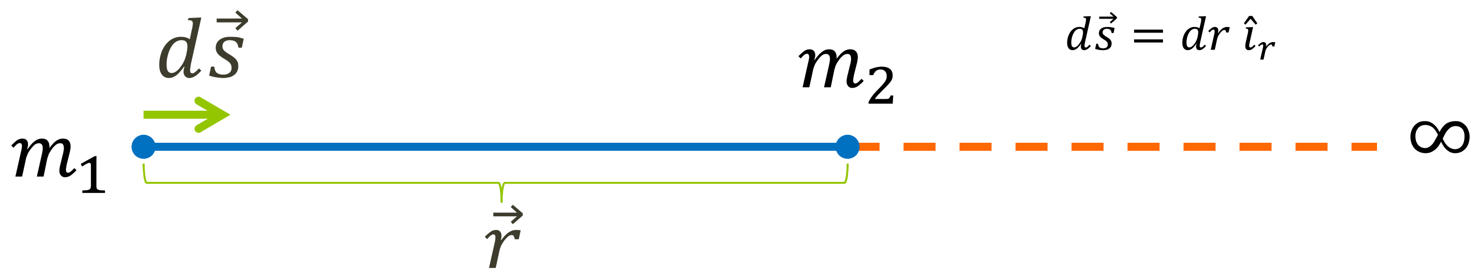


$$\vec{r} = r\hat{\imath}_r$$

- Gravitational Force acting on m_2

$$\vec{F}_2 = \frac{-Gm_1m_2}{r^2}\hat{\imath}_r = \frac{-Gm_1m_2\vec{r}}{r^3}$$

2. 3D Particle Dynamics



- Gravitational Potential Energy
  - Work done *by* an external force to move m_2 out to ∞ is $W = \int \vec{F} \cdot d\vec{s}$ where $\vec{F}$ is the force needed to cancel the gravitational attraction to m_1.

2. 3D Particle Dynamics

- Gravitational Potential Energy Cont'd
  - That force is

$$\vec{F} = \frac{Gm_1 m_2 \vec{r}}{r^3}$$

  - Doing the integration, you get

$$W = \frac{Gm_1 m_2}{r}$$

  - Gravitational Potential energy is defined as work done to move m_2 from ∞ to distance r from m_1, therefore we get

$$\mathcal{V} = -W = -\frac{Gm_1 m_2}{r} \approx mgh \text{ (only close to the earth!)}$$

2. 3D Particle Dynamics

- Vis-Viva Integral

$$\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$$

- Angular Momentum of a particle

$$\vec{h} = \vec{r} \times \vec{v} \quad or \quad h = |\vec{r} \times \vec{v}| = rvcos(\phi) = r^2\dot{\theta}$$

Note the quadrant correction:
If $\dot{r} > 0, \phi$ is positive. If $\dot{r} < 0, \phi$ is negative

- Orbital Coordinates

$$\hat{\imath}_r = \frac{\vec{r}}{r} \qquad \hat{\imath}_\theta = \hat{\imath}_z \times \hat{\imath}_r \qquad \hat{\imath}_z = \frac{\vec{h}}{h}$$

  - $\hat{\imath}_r$ - radial direction, $\hat{\imath}_\theta$ - transverse direction

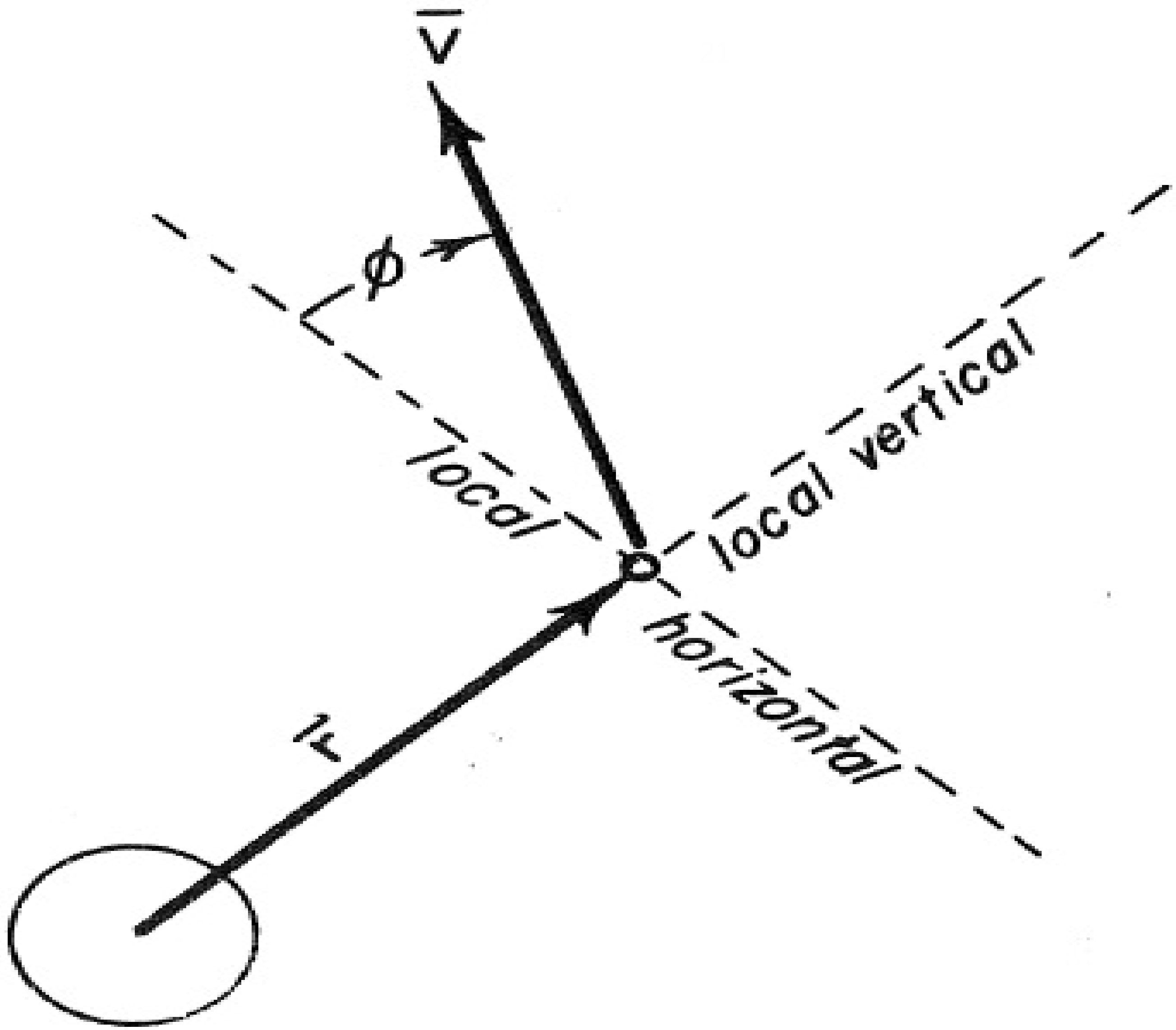


Figure 1.4-1 Flight-path angle, ϕ

Source: [1] on p. 17

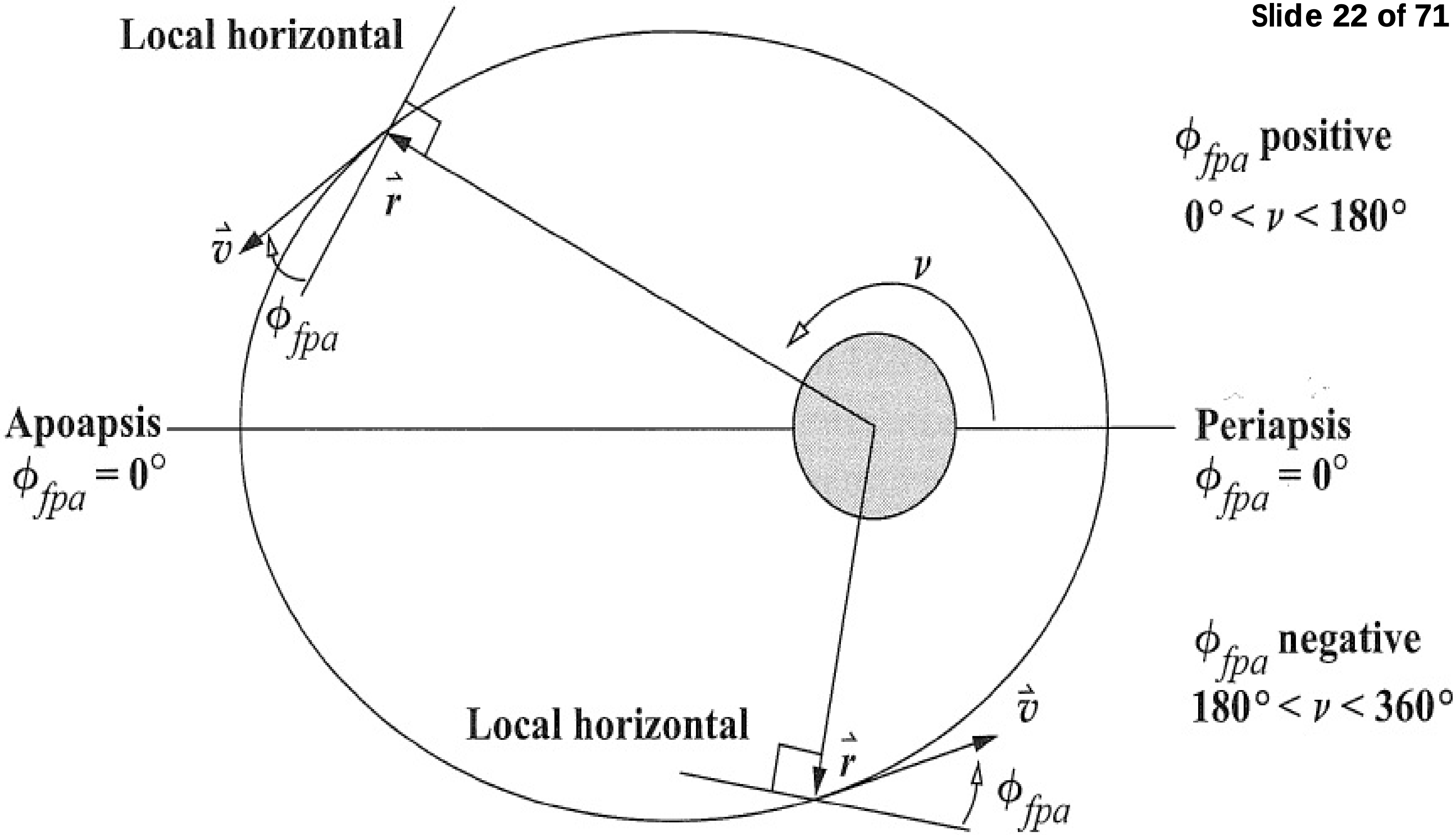


Figure 1-10. Geometry for the Flight-path Angle. The flight-path angle is always measured from the local horizontal to the velocity vector. It's always positive while the satellite travels from periapsis to apoapsis and negative for travel from apoapsis to periapsis. I've exaggerated the diagram for clarity.

Source: [2] on p. 19

2. 3D Particle Dynamics

Hey, speaking of Quadrant Checks…

You should really do them.

(See the general tutorial and the Quadrant Corrections for Orbital Mechanics handouts)

2. 3D Particle Dynamics

- Inertial Velocity of m_2

$$\frac{{}^{I}d\vec{r}}{dt} = \dot{r}\,\hat{\imath}_r + r\dot{\theta}\,\hat{\imath}_\theta$$

- Eccentricity Vector

$$\vec{e} = \frac{\vec{v} \times \vec{h}}{\mu} - \frac{\vec{r}}{r}$$

2. Particle Dynamics

- Be careful with notational issues!

$$\dot{r} = \frac{dr}{dt} \neq \left|\frac{d\bar{r}}{dt}\right|$$

$$\frac{d\bar{r}}{dt} = \bar{v} = \dot{r}\,\hat{\imath}_r + r\dot{\theta}\,\hat{\imath}_\theta$$

$$\left|\frac{d\bar{r}}{dt}\right| = \sqrt{(\dot{r}\,\hat{\imath}_r)^2 + \left(r\dot{\theta}\,\hat{\imath}_\theta\right)^2} \neq \dot{r}$$

2. Particle Dynamics

- A few more helpful relations
  - Speed of a satellite on a circular orbit

$$v_c = \sqrt{\frac{\mu}{r}}$$

  - Escape Velocity

$$v_{esc} = \sqrt{\frac{2\mu}{r}}$$

  - Semi-latus Rectum

$$p = \frac{h^2}{\mu} = a(1 - e^2)$$

2. Particle Dynamics

- Transverse Velocity Component

$$v_\theta = r\dot{\theta} = \frac{h}{r}$$

- Another expression for eccentricity

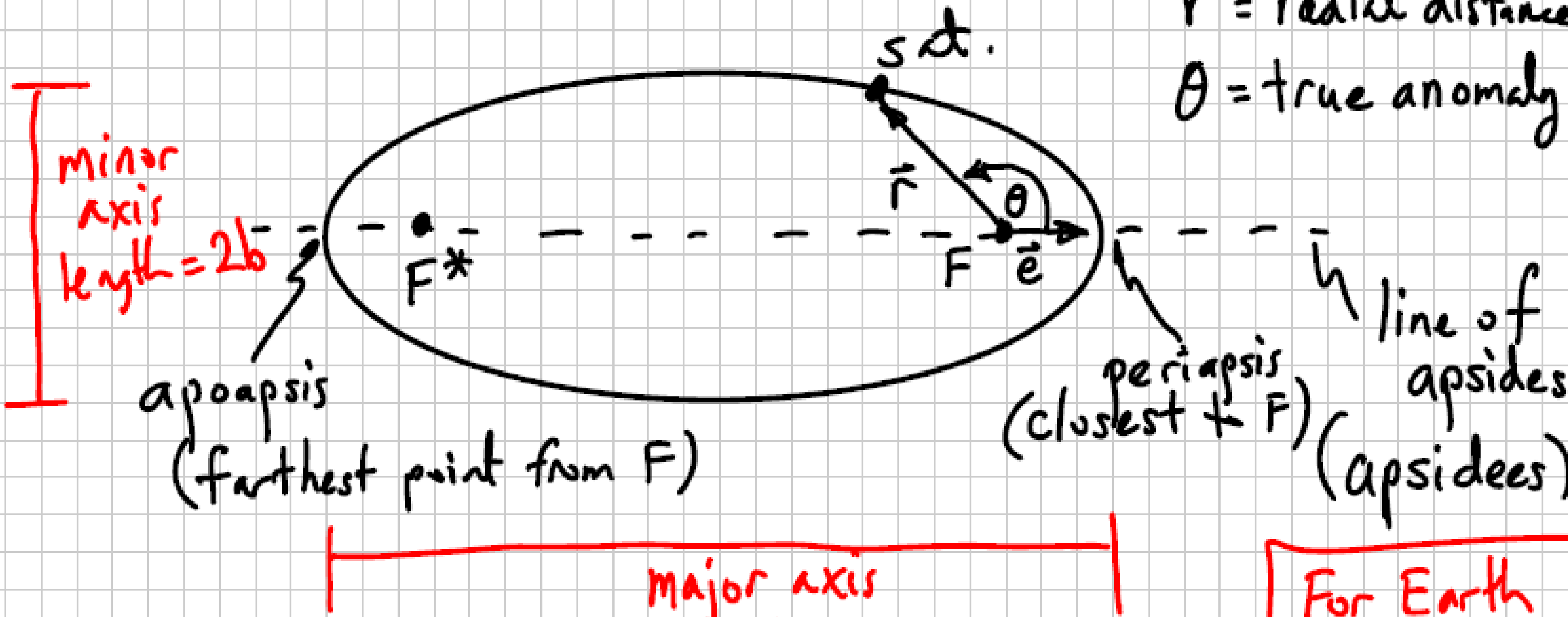
$$e = \sqrt{1 + \frac{2p\varepsilon}{\mu}}$$

Ellipse ($0 < e < 1$)

F = focus (grav. center)

r = radial distance

θ = true anomaly



b = semi-minor axis

a = semi-major axis

For Earth
periapsis = perigee
apoapsis = apogee

3. Two-Body Orbital Mechanics

- The Orbit Equation

$$r = \frac{p}{1 + e\cos\theta}$$

- Conic Sections

Orbit Type	Eccentricity	Semi-Major Axis	Energy
Circle	$e = 0$	$a > 0$	$\mathcal{E} < 0$
Ellipse	$0 < e < 1$	$a > 0$	$\mathcal{E} < 0$
Parabola	$e = 1$	$a = \infty$	$\mathcal{E} = 0$
Hyperbola	$e > 1$	$a < 0$	$\mathcal{E} > 0$

Skill to know

- Be able to generate new equations
- For example, find an expression for the eccentricity of an orbit in terms of only the apse radii.

Proof that $e = f(r_p, r_a)$

- Start from the orbit equation

$$r = \frac{p}{1 + e * \cos(\theta)}$$

- For periapsis, $\theta = 0°$

$$r_p = \frac{p}{1 + e}$$

- For apoapsis, $\theta = 180°$

$$r_a = \frac{p}{1 - e}$$

Proof that $e = f(r_p, r_a)$ Cont'd

- Solve for semi-latus rectum

$$p = r_p(1 + e)$$

$$p = r_a(1 - e)$$

- Equate the two expressions

$$r_p(1 + e) = r_a(1 - e)$$

Proof that $e = f(r_p, r_a)$ Cont'd

- Manipulate this algebraically now

$$r_p + e * r_p = r_a + e * r_a$$

- Bring all of eccentricities over to one side

$$e * r_a + e * r_p = r_a - r_p$$

$$e(r_a + r_p) = r_a - r_p$$

$$e = \frac{r_a - r_p}{r_a + r_p} \qquad \blacksquare$$

3. Two-Body Orbital Mechanics

- Kepler's Laws
  - 1. Spacecraft move on an elliptical path with gravitational source at one focus
  - 2. Radius vector sweeps out equal areas in equal amounts of time
  - 3. Period Equation: $T = 2\pi\sqrt{\frac{a^3}{\mu}}$
- Excess Hyperbolic Velocity

$$v_\infty = \sqrt{2\varepsilon} \quad (\text{at } \infty)$$

3. Two-Body Orbital Mechanics

- Kepler's Time Equation

$$M = E - e * \sin(E)$$

$$M = \sqrt{\frac{\mu}{a^3}}(t - T_o) = \sqrt{\frac{\mu}{a^3}}\left(t - T_p\right) = \sqrt{\frac{\mu}{a^3}}\Delta t$$

3. Two Body Orbital Mechanics

- e.g. Newton-Rhapson

$$f(E) = E - e * sin(E) - M = 0$$

$$f'(E) = 1 - e * \cos(E)$$

$$E_{new} = E_{old} - \frac{f(E_{old})}{f'(E_{old})}$$

3. Two Body Orbital Mechanics

- Classical Orbital Elements

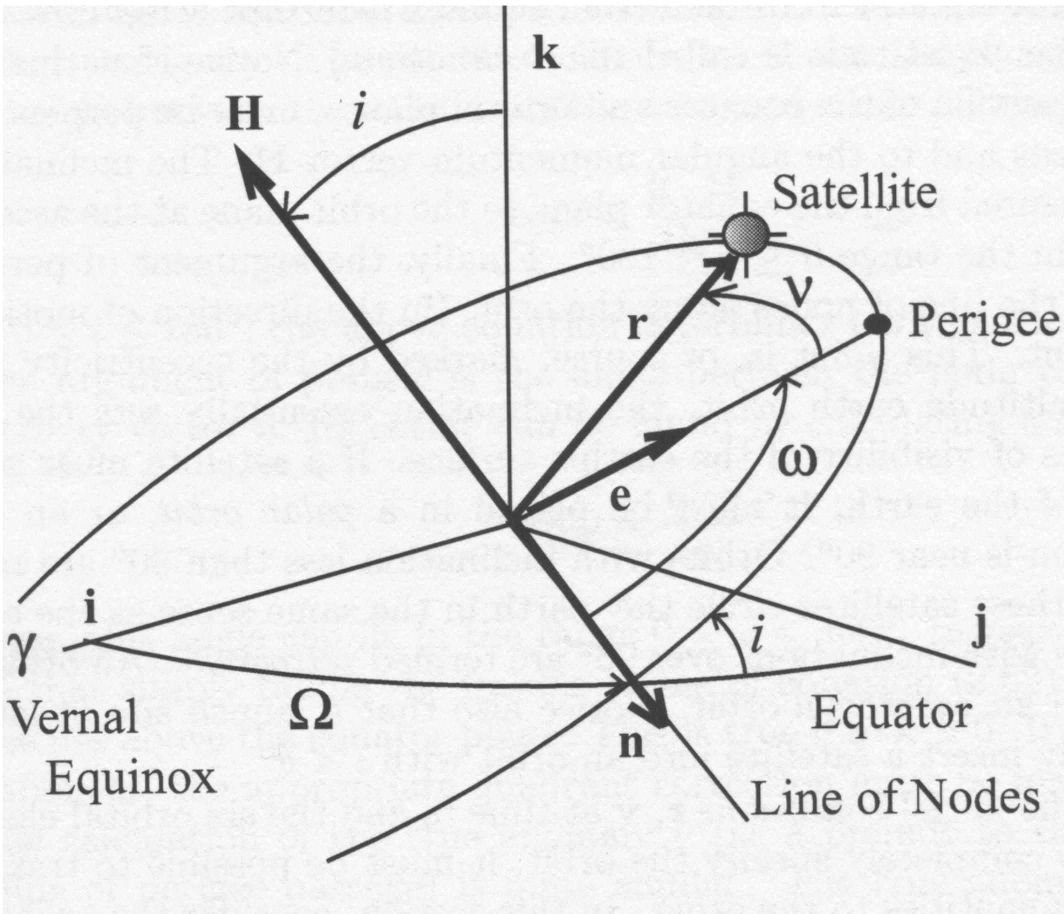


Figure 2.10: The classical orbital elements.

Source: [3] on p. 65.

Various symbols are used in the literature to denote true anomaly – common ones include θ (which we've been using in class), ν ("nu," as shown in this figure but can be easily be confused for velocity v), and f.

3. Two Body Orbital Mechanics

- Perifocal Coordinates

$$\vec{r}^P = r\cos\theta\,\hat{p} + r\sin\theta\,\hat{q} + 0\,\widehat{w}$$

$$\vec{v}^P = -\sqrt{\frac{\mu}{p}}\sin\theta\,\hat{p} + \sqrt{\frac{\mu}{p}}(\mathrm{e} + \cos\theta)\hat{q} + 0\,\widehat{w}$$

- To convert from ECI to Perifocal Coordinates

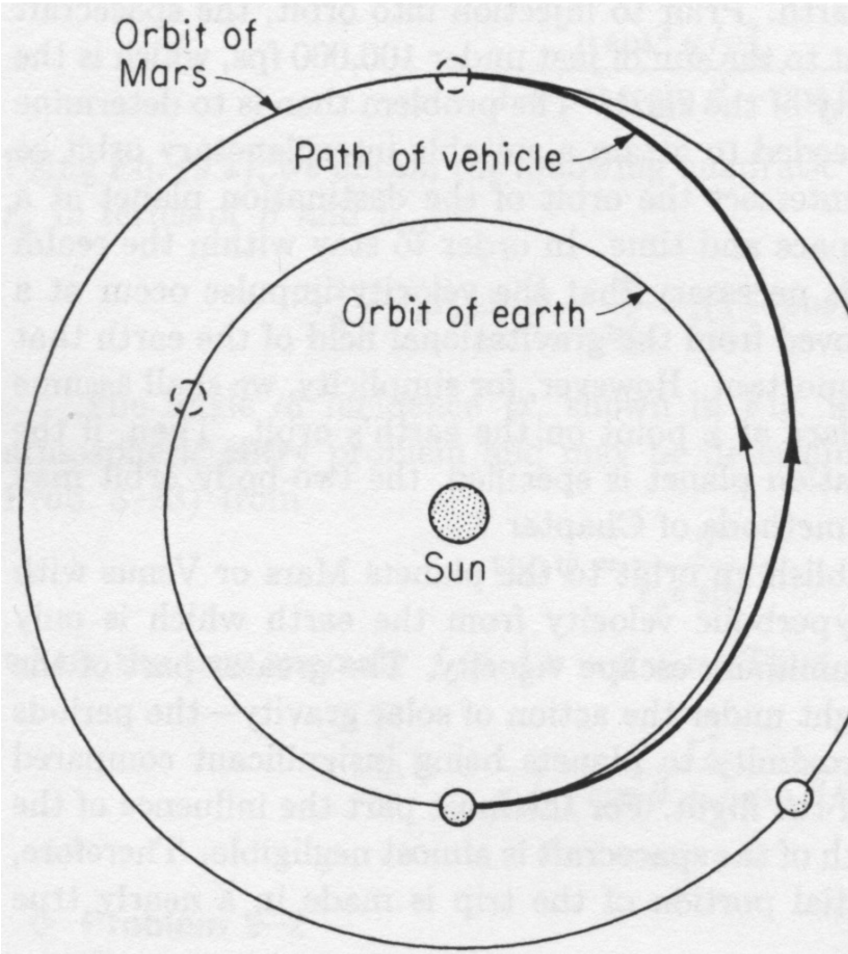
$$\underline{C}^{EP} = \underline{C}_3(\omega)\underline{C}_1(i)\underline{C}_3(\Omega)$$

- To convert from Perifocal to ECI Coordinates

$$\underline{C}^{PE} = \left(\underline{C}^{EP}\right)^T$$

4. Orbital Maneuvers and Transfers

- Hohmann Transfer



Source: [4] on p. 428

Fig. 9.3: Earth-to-Mars Hohmann orbit.

4. Orbital Maneuvers and Transfers

- Hohmann Transfer Cont'd
  - Minimizes Δv but maximizes time
  - You need to calculate four velocities
    - v_{inner} = Velocity on inner circular orbit
    - v_{outer} = Velocity on outer circular orbit
    - v_p = Velocity at periapsis of the Hohmann ellipse
    - v_a = Velocity at apoapsis of the Hohmann ellipse

$$\Delta v_{total} = \left| v_p - v_{inner} \right| + \left| v_{outer} - v_a \right|$$

$$T_{xfer} = \frac{1}{2} T_H = \pi \sqrt{\frac{a_H^3}{\mu}} = \pi \sqrt{\frac{(r_{inner} + r_{outer})^3}{8\mu}}$$

- Remember, you can't have negative Δv!

4. Orbital Maneuvers and Transfers

- Impulsive approximation
  - An object's position does not change during the short time that a force is applied to change the object's velocity
  - In reality, the position does change but not significantly
- $\Delta\vec{v}$ in general will involve:

Change in magnitude of $\vec{v}$

or

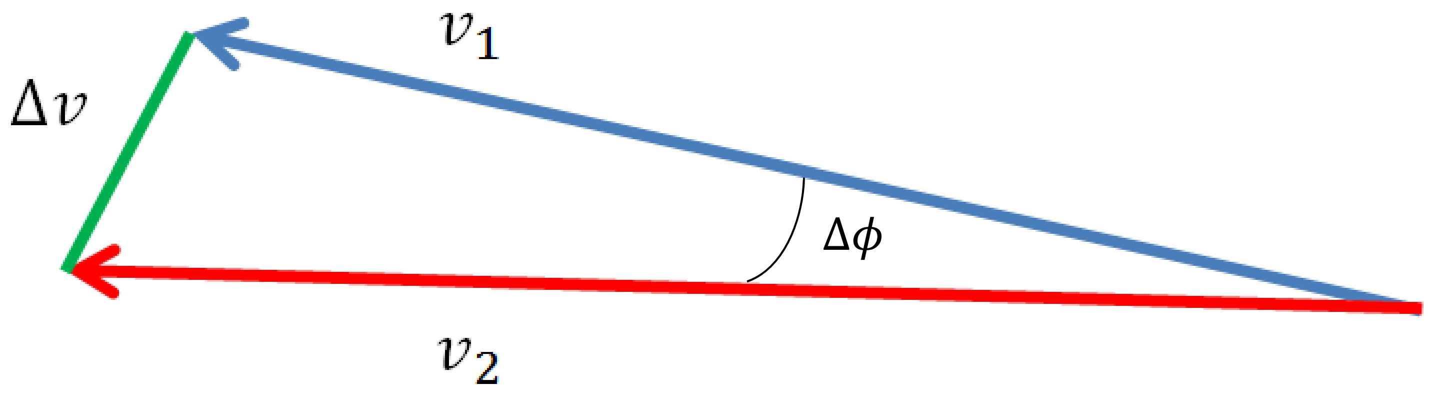
Change in the direction of $\vec{v}$

or

Change in magnitude <u>and</u> direction of $\vec{v}$

4. Orbital Maneuvers and Transfers

- Law of Cosines – collapses to the Pythagorean Theorem when the angle is 90 deg. (math is cool!)



$$\Delta v = \sqrt{v_1^2 + v_2^2 - 2v_1 v_2 \cos(\Delta\phi)}$$

- What happens if the line of apsides does not rotate? $\theta\ doesn't\ change$!

where $\Delta\gamma = \gamma_2 - \gamma_1$. Therefore, the formula for Δv ***without plane change is***

$$\boxed{\Delta v = \sqrt{v_1^2 + v_2^2 - 2 v_1 v_2 \cos\Delta\gamma}} \quad \text{(impulsive maneuver, coplanar orbits)} \tag{6.8}$$

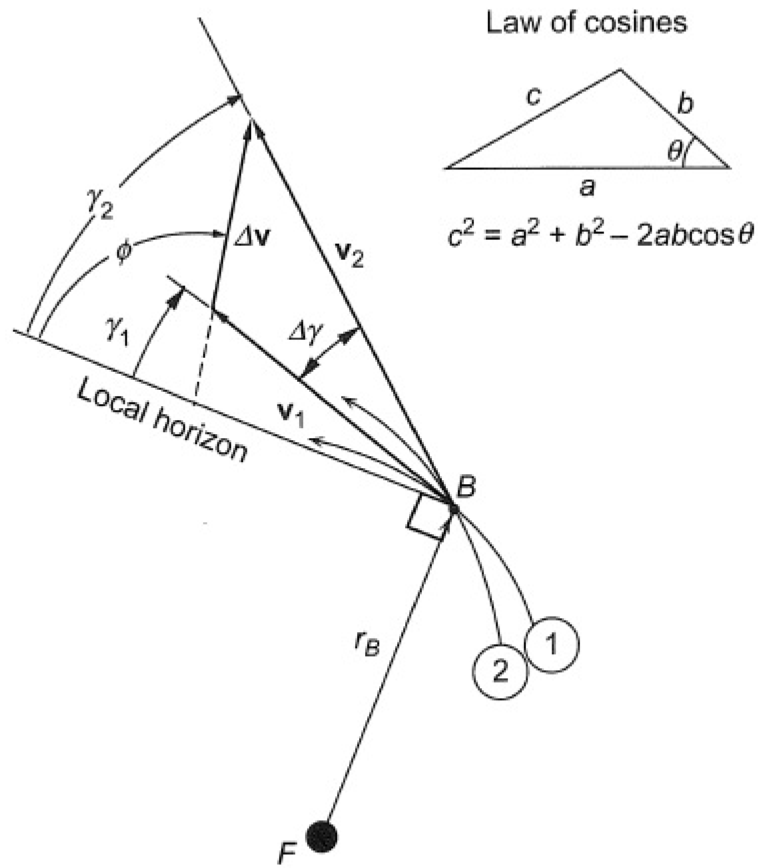


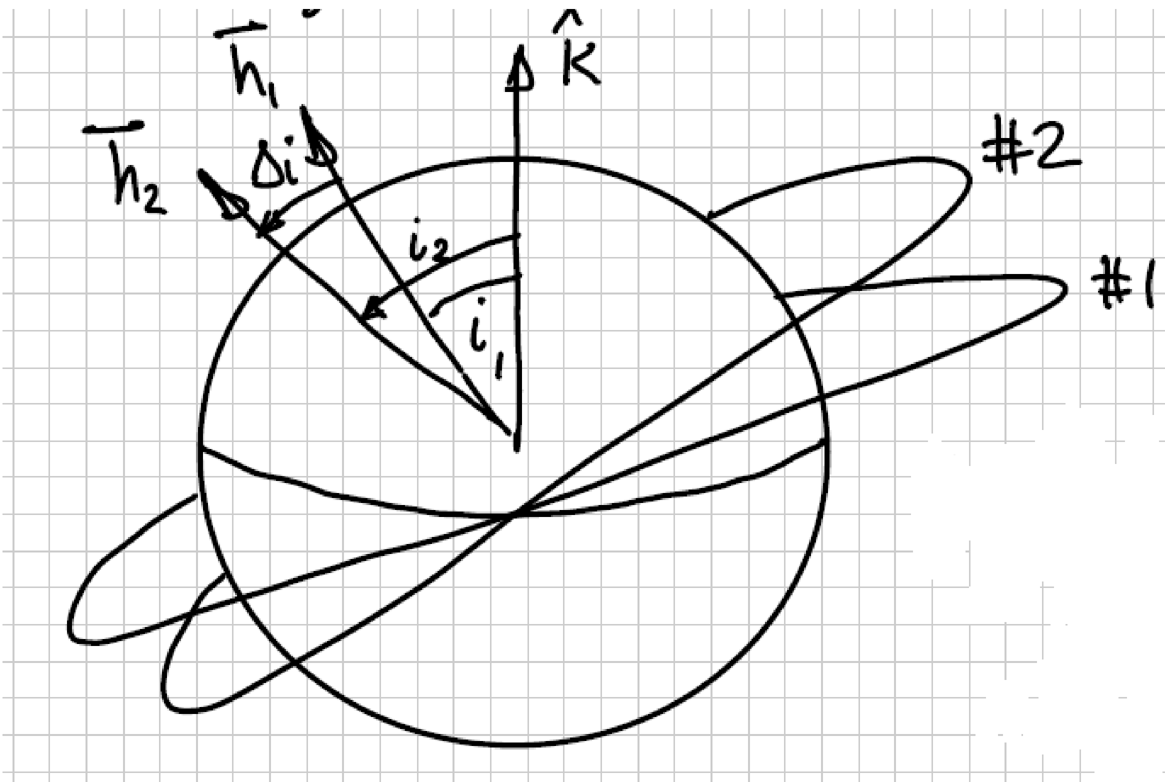
FIGURE 6.14

Vector diagram of the change in velocity and flight path angle at the intersection of two orbits (plus a reminder of the law of cosines).

Source: [5] on p. 339

4. Orbital Maneuvers and Transfers

- Pure Inclination Change



4. Orbital Maneuvers and Transfers

- Pure Inclination Change Cont'd

$$\Delta v = \sqrt{v_\theta^2 + v_\theta^2 - 2 v_\theta v_\theta \cos(\Delta i)}$$
$$= v_\theta \sqrt{2(1-\cos\Delta i)}$$
$$= 2 v_\theta \sin\left(\frac{\Delta i}{2}\right)$$

- Remember, v_θ is the transverse velocity component

$$v_\theta = \frac{h}{r}$$

4. Orbital Maneuvers and Transfers

- Non-Keplerian Orbits
  - Influenced by forces in addition to the central gravitational source
  - Examples
    - Atmospheric Drag

$$\vec{F}_D = -\frac{1}{2}\rho v^2 A_{ref} C_d \frac{\vec{v}}{v}$$

    - Spacecraft is using propulsive thrust
    - Spacecraft is experience the effects of Earth's oblateness

4. Orbital Maneuvers and Transfers

- The earth is really not a sphere.
  - It is oblate
  - We model this phenomena as an extra band of mass near the equator
  - Oblateness causes the ascending node to move westward (known as regression of the node)

$$\left(\frac{d\Omega}{dt}\right)_{avg} = \frac{-3\sqrt{\mu}J_2R_{\oplus}^2}{2a^{\frac{7}{2}}(1-e^2)^2}cosi$$

$$\left(\frac{d\omega}{dt}\right)_{avg} = \frac{-3\sqrt{\mu}J_2R_{\oplus}^2}{2a^{\frac{7}{2}}(1-e^2)^2}\left(\frac{5}{2}\sin^2 i - 2\right)$$

4. Orbital Maneuvers and Transfers

- Orbits
  - LEO – Low Earth Orbit
    - Think International Space Station (ISS) or the Space Shuttle
  - MEO – Middle Earth Orbit
  - GEO – Geostationary Equatorial Orbit
    - Inclination of about zero degrees
    - Circular
    - Period is equal to Earth's rotational period with respect to inertial frame of reference
  - Molniya Orbit
    - Russian for "lightening"
    - Period of 12 hours
- What's in a day?
  - 1 solar day = 24 hours
  - 1 sidereal day = 23 hours, 56 minutes and 4 seconds

4. Orbital Maneuvers and Transfers

- Sun-Synchronous Orbit

$$\left(\frac{d\Omega}{dt}\right)_{avg} = \frac{2\pi\, rad}{year}$$

  - The ascending node must move eastward

- Low Thrust Orbit Transfer

$$\Delta t = t - t_o \approx \frac{\sqrt{\mu}}{\frac{F_T}{m}}\left(\frac{1}{\sqrt{a_o}} - \frac{1}{\sqrt{a}}\right)$$

- Escape Condition: $\mathcal{E} = 0$

4. Orbital Maneuvers and Transfers

Hey, did you know that…

Center of Mass

DOES *NOT* EQUAL

Center of Gravity!

(generally)

Make sure you know how to calculate both!

4. Orbital Maneuvers and Transfers

- Also, while we're in the neighborhood of not doing things, don't forget that…

Potential Energy DOES *NOT* EQUAL *mgh!*

- It's just an approximation that is only valid near the earth's surface. Aircraft and objects below the altitude of aircraft can get away with using it; spacecraft cannot. You'll hear more about this in AERSP 306 Aeronautics.

5. Rigid Body Dynamics

- Rigid Body: An object with physical extent (i.e. not a particle) that has no flexibility.

- Chasles's Theorem
  - The translational motion of an object with respect to an inertial frame of reference *N* can be treated separately from the rotational motion of the object with respect to N.

5. Rigid Body Dynamics

- Inertia Matrix (Tensor)

$$\underline{I} = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix}$$

- Properties
  - The matrix is symmetric
  - Diagonal elements are always greater than or equal to 0
  - Off-diagonal elements can be positive, negative or 0
  - Units are often $kg \cdot m^2$

5. Rigid Body Dynamics

- Angular Momentum Vector

$$\underline{H} = \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix}$$

- Angular Velocity Vector

$$\underline{\omega} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

5. Rigid Body Dynamics

- To calculate $\underline{H}$, you solve

$$\underline{H} = \underline{I}\underline{\omega}$$

- Energy of Rotation

$$T_{rot} = \frac{1}{2}\underline{\omega}^T \underline{I\omega} = \frac{1}{2}\underline{\omega}^T \underline{H}$$

- “Find the angle:” Between $\underline{\text{H}}$ and $\underline{\omega}$

$$\cos(\theta) = \frac{\vec{H} \cdot \vec{\omega}}{|\vec{H}||\vec{\omega}|}$$

Are you inertially fixed?

5. Rigid Body Dynamics

- Diagonalizing the inertia tensor
  - Find the eigenvalues of the inertia tensor
  - Use those eigenvalues to generate eigenvectors
  - Use the similarity transformation to transform from the arbitrary to the principal axis system.
  - Don't forget to check the cross product!
  - In this class, be familiar with how this process works but absolutely be solid with how to construct the DCM from the eigenvectors.
    - In more advanced courses (e.g. AERSP 450), you'll be expected to do the diagonalization by hand.

- Similarity Transformation

$$\underline{I}^B = \underline{C}^{AB}\underline{I}^A\underline{C}^{BA}$$

5. Rigid Body Dynamics

- Properties of Principal Axes
  - Principal axes always exist for a real physical object
  - For any plane of symmetry (with respect to the mass distribution) one principal axis is perpendicular to the plane of symmetry and the other two principal axes lie in the plane of symmetry
  - The sum of any two principal moments of inertia are greater than the third.
  - $tr\ I^B = tr\ I^{B'} = I_1 + I_2 + I_3 = I_{11} + I_{22} + I_{33}$
- Don’t forget we can also use the parallel axis theorem!

5. Rigid Body Dynamics

- Euler's Equations for Rigid Body Motion

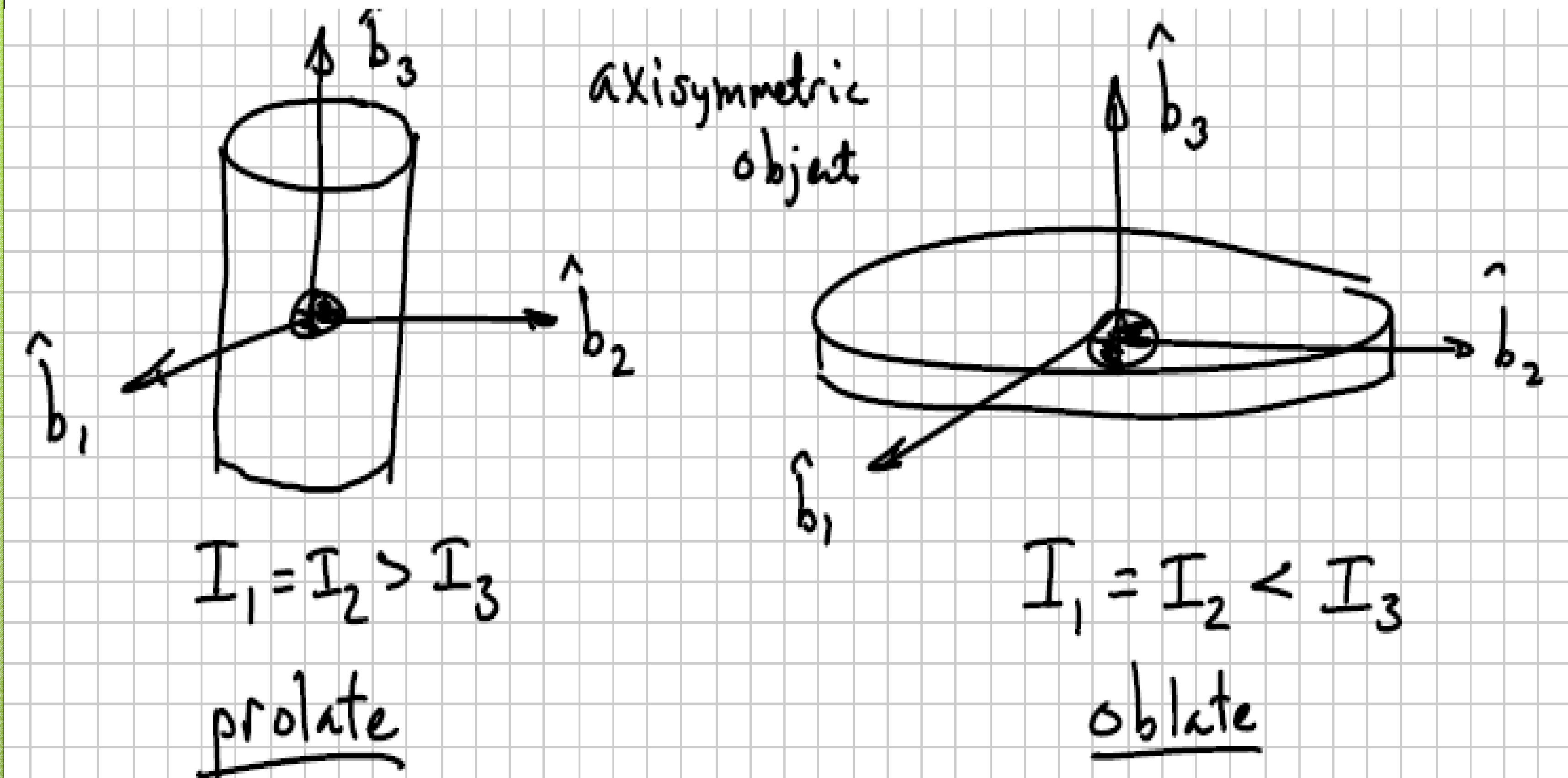
$$M_1 = I_1\dot{\omega}_1 + \omega_2\omega_3(I_3 - I_2)$$
$$M_2 = I_2\dot{\omega}_2 + \omega_3\omega_1(I_1 - I_3)$$
$$M_3 = I_3\dot{\omega}_3 + \omega_1\omega_2(I_2 - I_1)$$

- Non-linear and coupled differential equations
- No general solution exists but just like for the Navier-Stokes equations we can solve some specific cases

- This is just another way of saying

$$\vec{M} = \frac{{}^{I}d\vec{H}}{dt}$$

5. Rigid Body Dynamics

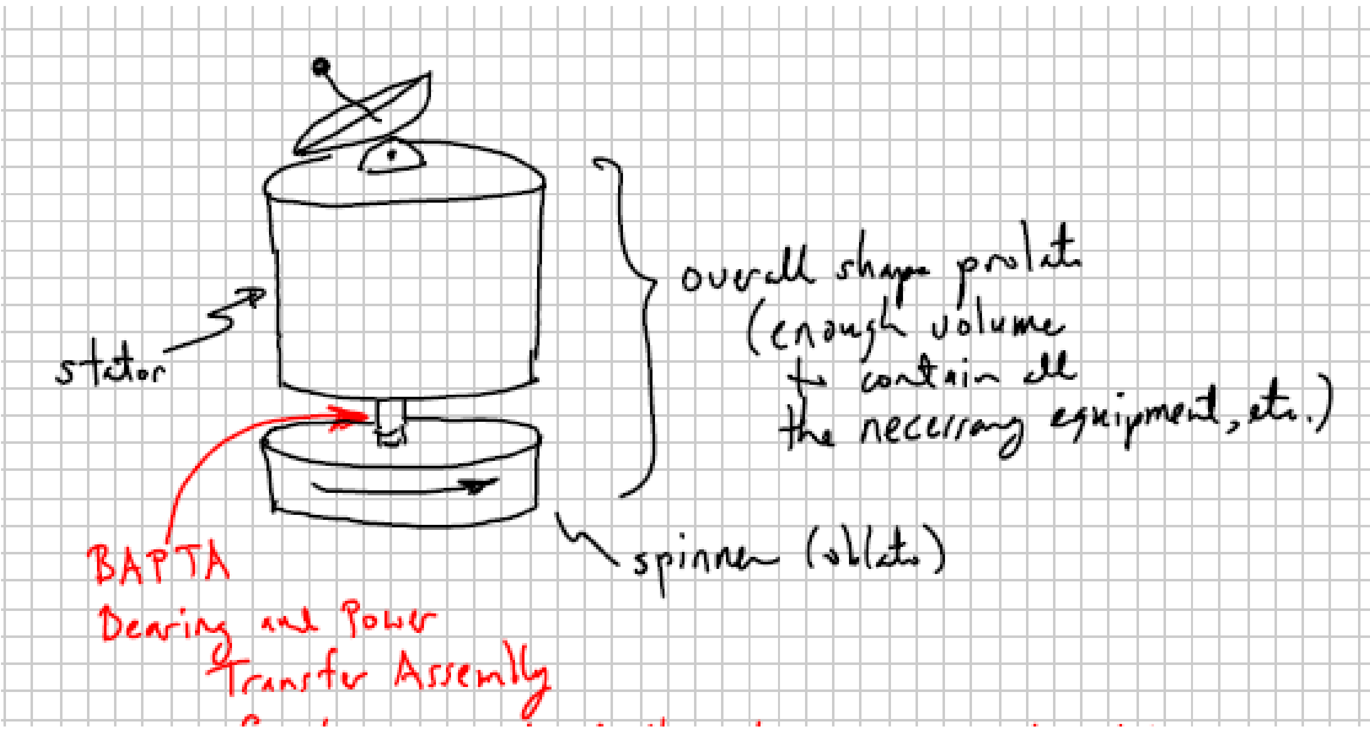


5. Rigid Body Dynamics

- Body cones
- Gyroscopic stiffness
- Stability of rotation around the principal axes
  - Make sure you know and understand what happened to Explorer 1
  - The axis of the intermediate moment of inertia is absolutely unstable
  - Minimum moment of inertia is okay… until the satellite gets perturbed by something
  - Maximum moment of inertia is preferred!
  - More on this idea of stability of a system in AERSP 304

5. Rigid Body Dynamics

- Duel Spin Satellite



5. Rigid Body Dynamics

- Gravity Gradient Torque
  - Relies on the fact that the center of mass is not equal to the center of gravity – the difference between the two gives you your moment arm!

$$M_{gg,1} = \frac{3\mu}{R^3} C_{21}^{OB} C_{31}^{OB} (I_3 - I_2)$$

$$M_{gg,2} = \frac{3\mu}{R^3} C_{11}^{OB} C_{31}^{OB} (I_1 - I_3)$$

$$M_{gg,3} = \frac{3\mu}{R^3} C_{11}^{OB} C_{21}^{OB} (I_2 - I_1)$$

5. Rigid Body Dynamics

- Attitude Sensing and Control
  - Thrusters
  - Gyro devices
  - Gravity Gradient
  - Magnetic
  - Be familiar with all of them (both the mathematics and the concepts)!

6. Rocket Performance

- The Rocket Equation
  - Assumption: Deep Space

$$\Delta v = v_{ex} \ln\left(\frac{m_1}{m_2}\right) = v_{ex} \ln\left(\frac{m_1}{m_1 - m_p}\right)$$

$$m_p = m_1\left(1 - e^{\frac{-\Delta v}{v_{ex}}}\right)$$

$$T = |v_{ex}\dot{m}|$$

$$I_{sp} = \frac{T}{\dot{w}} = \frac{v_{ex}}{g_{SL}}$$

6. Rocket Performance

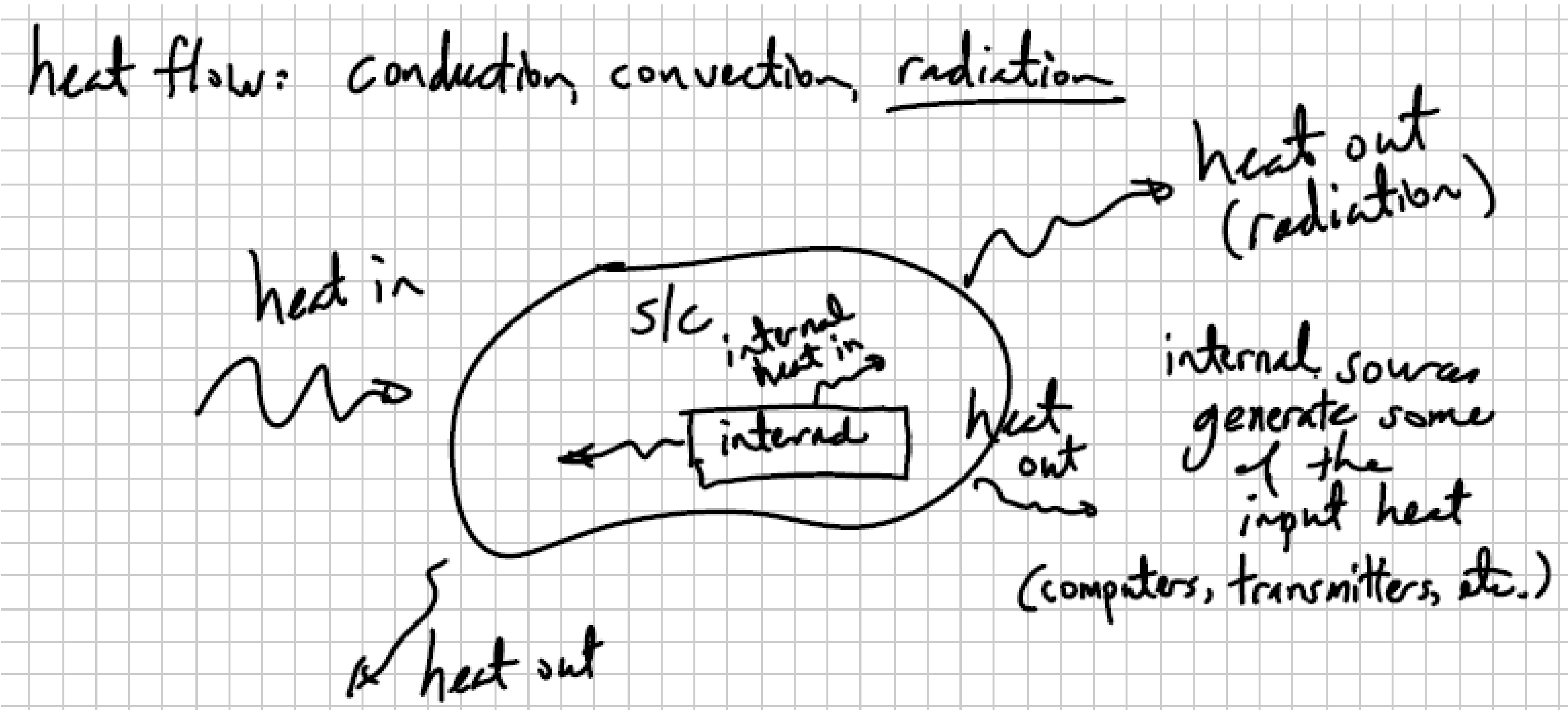
- Propulsion Technology
  - Propellant: Fuel + Oxidizer
    - Liquid Propellant
    - Solid Propellant
    - Know advantages and disadvantages!
  - Electric Propulsion
  - Microwave Propulsion
  - Micropropulsion (currently being researched – beyond the scope of this course but is interesting)
  - Nuclear Propulsion
- Rocket staging
  - Higher Δv but far more complicated!
    - Understand the mathematics of how this works

7. Space Environment

- If you are interested in the space environment, the Department offers a course in Spacecraft-Environment Interactions (AERSP 497I/597I)
  - I took this course as an undergraduate student – come talk to me if you're interested in it.
- Other courses are available with similar foci – e.g. plasma interactions with spacecraft in the Ionosphere, rarified gas dynamics, etc.
- We very briefly touched on some high points in this course

7. Space Environment

- Thermal Environment in Space



7. Space Environment

- Wien’s Law: The wavelength distribution of thermal radiation from a black body at any temperature has basically the same shape at any other temperature.

$$\lambda^*(cm) = \frac{0.2897}{T(K)}$$

7. Space Environment

- Stefan-Boltzmann Law

$$\phi_{out} = \sigma\varepsilon T^4 \equiv \left[\frac{energy}{area \cdot time}\right] = output\ flux$$

$$\sigma = 5.672 \times 10^{-8} \frac{W}{m^2 \cdot K^4}$$

$$\phi_{in} = 1.371 \times 10^3 \frac{W}{m^2} \left(\frac{R_\oplus}{R}\right)^2$$

$$P_i = \phi_i A_{proj}$$

That's all, folks!

I have old homework and exams if you want pick them up.

You've been a great class. Thank you for a good semester and good luck on your final exams!

Don't forget our office hours if you have any last minute questions before the exam!

References

- [1] Bate, R.R., Mueller, D.D., and White, J.E., *Fundamentals of Astrodynamics*, Dover Publications, NY, 1971.

- [2] Vallado, D.A. with McClain, W.D., *Fundamentals of Astrodynamics and Applications*, 3rd Ed, Microcosm Press, Hawthorne, CA and Springer, New York City, 2007.

- [3] Wiesel, W.E., *Spaceflight Dynamics*, 3rd Ed., Aphelion Press, Beavercreek, OH, 2010.

- [4] Battin, R.H., *An Introduction to the Mathematics and Methods of Astrodynamics*, Revised Ed., AIAA Education Series, AIAA, New York, 1999.

- [5] Curtis, H.D., *Orbital Mechanics for Engineering Students*, 2nd Ed., Elsevier, New York City, NY, 2010.