AERSP 309

Note Title 9/7/2012

Help Session
6pm mondays
203 EE West

$$ {}^{I}\frac{d\bar{r}}{dt} = {}^{B}\frac{d\bar{r}}{dt} + \bar{\omega}^{B/I} \times \bar{r} $$

Today, Acceleration

$$ {}^{I}\frac{d^2\bar{r}}{dt^2} = {}^{I}\frac{d}{dt}\left({}^{I}\frac{d\bar{r}}{dt}\right) $$

Apply The Rule

$$ {}^{I}\frac{d}{dt} = {}^{B}\frac{d}{dt} + \bar{\omega}^{B/I} \times \_\_\_ $$

Twice

$$ {}^{I}\frac{d\bar{r}}{dt} = {}^{B}\frac{d\bar{r}}{dt} + \bar{\omega}^{B/I} \times \bar{r} $$

$$ {}^{I}\frac{d^2\bar{r}}{dt^2} = {}^{I}\frac{d}{dt}\left[{}^{B}\frac{d\bar{r}}{dt} + \bar{\omega}^{B/I} \times \bar{r}\right] $$

$$^{I}\frac{d}{dt}[\quad] = {}^{B}\frac{d}{dt}[\quad] + \bar{\omega}^{B/I} \times [\quad]$$

$$^{I}\frac{d^2\bar{r}}{dt^2} = {}^{B}\frac{d}{dt}\left[{}^{B}\frac{d\bar{r}}{dt} + \bar{\omega}^{B/I} \times \bar{r}\right] + \bar{\omega}^{B/I} \times \left[{}^{B}\frac{d\bar{r}}{dt} + \bar{\omega}^{B/I} \times \bar{r}\right]$$

$$= {}^{B}\frac{d^2\bar{r}}{dt^2} + \left({}^{B}\frac{d\bar{\omega}^{B/I}}{dt}\right) \times \bar{r} + \bar{\omega}^{B/I} \times {}^{B}\frac{d\bar{r}}{dt}$$

$$+ \bar{\omega}^{B/I} \times {}^{B}\frac{d\bar{r}}{dt} + \bar{\omega}^{B/I} \times \left(\bar{\omega}^{B/I} \times \bar{r}\right)$$

$$\boxed{^{I}\frac{d^2\bar{r}}{dt^2} = {}^{B}\frac{d^2\bar{r}}{dt^2} + \left(\frac{d\bar{\omega}^{B/I}}{dt}\right) \times \bar{r} + 2\bar{\omega}^{B/I} \times {}^{B}\frac{d\bar{r}}{dt} + \bar{\omega} \times \left(\bar{\omega}^{B/I} \times \bar{r}\right)}$$

$$^{I}\frac{d^2\bar{r}}{dt^2} = \text{Inertial Acceleration} = \frac{\bar{F}}{m}$$

Note: $\bar{F} = m\bar{a}$ applies only in inertial or Newtonian frames.

Look at $\frac{d\bar{\omega}^{B/I}}{dt}$

$$^{I}\frac{d\bar{\omega}^{B/I}}{dt} = {}^{B}\frac{d\bar{\omega}}{dt} + \cancel{\bar{\omega}^{B/I} \times \bar{\omega}^{B/I}} = 0$$

$$^{I}\frac{d\bar{\omega}^{B/I}}{dt} = \frac{d\bar{\omega}^{B/I}}{dt}$$

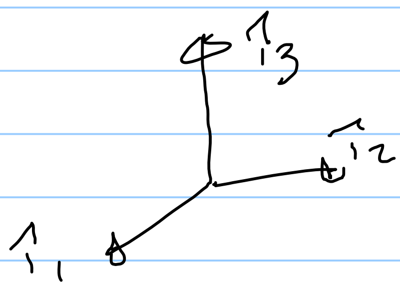
No Superscript is Needed

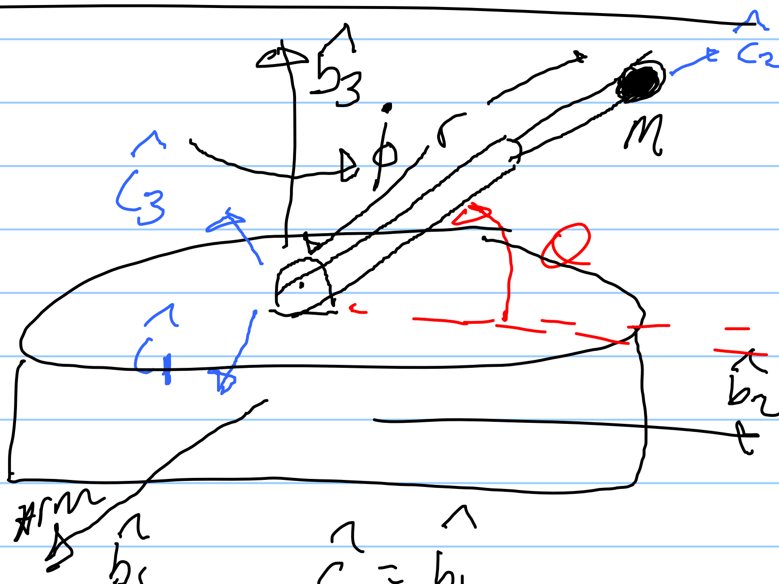
$$\underbrace{{}^B\frac{d^2\bar r}{dt^2}}_{\text{Acceleration of The object in Frame B}} = \underbrace{{}^I\frac{d^2\bar r}{dt^2}}_{\text{Inertial Accel} = F/m} - \underbrace{\frac{d\bar\omega^{B/I}}{dt}\times\bar r}_{\text{Euler Accel.}} - \underbrace{2\bar\omega^{B/I}\times{}^B\frac{d\bar r}{dt}}_{\text{Coriolis Accel}} - \underbrace{\bar\omega^{B/I}\times(\bar\omega^{B/I}\times\bar r)}_{\text{Centrifugal accel}}$$

Centrifugal

Center | Fleeing

Example





B - attached To Disk

C - Attached To The Arm

$\hat c_1 = \hat b_1$

$\hat i_3 = \hat b_3$

Disk Rotates w/ Ang Velocity

$$\bar\omega^{B/I} = \dot\phi\,\hat b_3 = \dot\phi\,\hat i_3$$

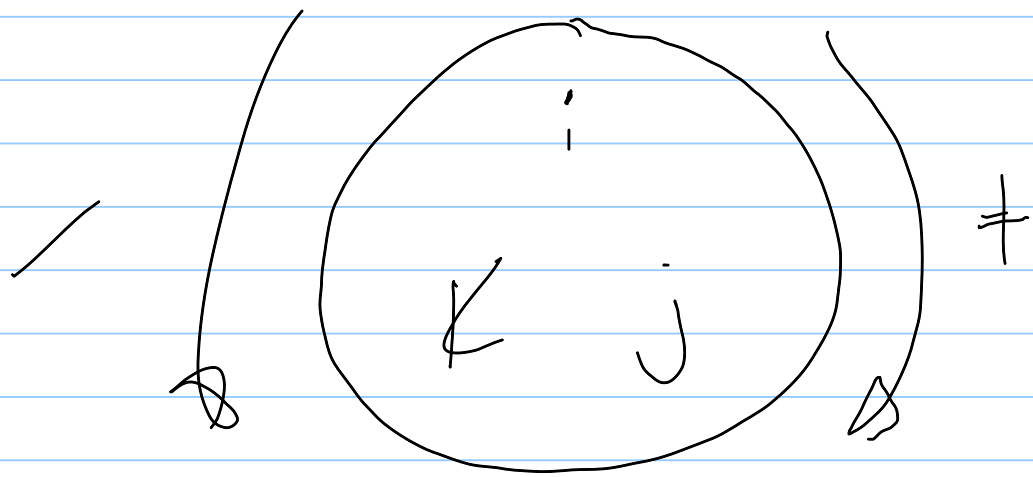
Arm Rotates w/ Ang Velocity

$$\bar\omega^{C/B} = \dot\theta\,\hat c_1 = \dot\theta\,\hat b_1$$

$\bar{r} = r\,\hat{c}_2$ ($\bar{r}$: position vector of m)

$${}^{B}\frac{d\bar{r}}{dt} = {}^{C}\frac{d\bar{r}}{dt} + \bar{\omega}^{C/B} \times \bar{r}$$

$${}^{C}\frac{d\bar{r}}{dt} = \dot{r}\,\hat{c}_2 \qquad \bar{\omega}^{C/B} \times \bar{r} = \dot{\theta}\,\hat{c}_1 \times r\,\hat{c}_2$$
$$= r\dot{\theta}\,\hat{c}_3$$



$${}^{B}\frac{d\bar{r}}{dt} = \dot{r}\,\hat{c}_2 + \underline{r\dot{\theta}\,\hat{c}_3}$$

$${}^{I}\frac{d\bar{r}}{dt} = {}^{C}\frac{d\bar{r}}{dt} + \bar{\omega}^{C/I} \times \bar{r} \qquad \bar{\omega}^{C/I} = \omega^{C/B} + \bar{\omega}^{B/I}$$
$$= \dot{\theta}\,\hat{c}_1 + \dot{\phi}\,\hat{b}_3$$

need DCM C^{CB}

Goal: $${}^{I}\frac{d\bar{r}}{dt} = {}^{C}\frac{d\bar{r}}{dt} + \bar{\omega}^{C/I} \times \bar{r}$$
$$\quad \| $$
$$\dot{r}\,\hat{c}_2$$