Cooperative Workforce Planning Heuristic with Worker Learning and Forgetting, and Demands Constraints

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“People are still the greatest assets”

W. Vanderbloemen [2016]
FLOW TO LEARN

Decades of successes and failures show that proper lean leverages human ingenuity, not automation

By Michael Ballé and Daniel Jones
Workers Learning and Forgetting effects can impact productivity estimations.

Shafer et al. [2001]

“The time needed to produce a single unit continuously decreases with the processing of additional units”*

Yang & Wang [2011], *Biskup [2008]
Learning and Forgetting increases the complexity of the scheduling problem

- Nembhard and Bentefouet (2012): *Reformulation to MIP*
- Hewitt et al. (2015): *Reformulation, Scaling Algorithm*
- Jin et al. (2016): *Integer Programming Techniques*
- Wang et al. (2016): *Branch-and-bound, Meta-heuristics*
Let’s take advantage of the optimal solution structure.

“Within parallel systems, when the demand is set for production during the entire time horizon and considering a differentiable non-decreasing model of performance, the maximum number of stints per worker per task is one”

Theorem 2 Nembhard and Bentefouet (2012)
Multi-period two parallel station system scheduling problem with L/F and demand constraints

1. A worker’s performance is a function of their skills and experience.
2. A one-to-one task-station relation.
3. No starvation or blocking.
4. The time horizon can be decomposed into same length periods.
5. At each period, a worker can only be assigned to a single task.
6. A one-to-one worker-task relation.

Nembhard & Bentefouet [2012]
Mathematical formulation: Two workers, Two station systems

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J$</td>
<td>Set of tasks, $j=1-2$</td>
</tr>
<tr>
<td>$I$</td>
<td>Set of workers, $i=1-2$</td>
</tr>
<tr>
<td>$T$</td>
<td>Set of time periods in the horizon, $t=1,..., T$</td>
</tr>
<tr>
<td>$x_{i,j,t}$</td>
<td>Binary variable that indicates whether task $j$ is performed by worker $i$ during time $t$.</td>
</tr>
<tr>
<td>$T_{max}$</td>
<td>Completion time of the last task to be finished.</td>
</tr>
<tr>
<td>$f_{i,j}$</td>
<td>The estimated productivity of worker $i$ at task $j$.</td>
</tr>
<tr>
<td>$O_{i,j,t}$</td>
<td>The output of worker $i$ performing task $j$ during period $t$.</td>
</tr>
<tr>
<td>$D_j$</td>
<td>The demand order size required by task $j$ at time period $T$, measured in product units.</td>
</tr>
</tbody>
</table>

Min $T_{max}$

Min $T_{max}$  \hspace{3cm} (1)

$t * x_{i,j,t} \leq T_{max} \ \forall \ i, \forall \ j, \forall \ t$  \hspace{3cm} (2)

$\sum_{i=1}^{2} x_{i,j,t} \leq 1 \ \forall \ j, \forall \ t$  \hspace{3cm} (3)

$\sum_{j=1}^{2} x_{i,j,t} \leq 1 \ \forall \ i, \forall \ t$  \hspace{3cm} (4)

$O_{i,j,t} \leq f_{i,j} \left( \sum_{k=1}^{t} x_{i,j,k} \right) \times \text{length of period} \ \forall \ i, \forall \ j, \forall \ t$  \hspace{3cm} (5)

$\sum_{i=1}^{2} \sum_{t=1}^{T} x_{i,j,t} * O_{i,j,t} \geq D_j \ \forall \ j$  \hspace{3cm} (6)
Cooperative Workforce Planning Heuristic

Iteration 1...

1) Select an initial worker assignment.

2) Checks if the current schedule meets the demand of both stations.

3) Generate new Schedule and test.
Design of Experiment

Learning function:

*Three-parameter Hyperbolic Learning*

- $p_{i,j}$ represented the initial productivity level of worker $i$ at task $j$
- $r_{i,j}$ the learning rate parameter of worker $i$ at task $j$
- $k_{i,j}$ the steady state production rate of worker $i$ at task $j$.

$$f_{i,j}(t) = k_{i,j} \frac{t + p_{i,j}}{t + p_{i,j} + r_{i,j}}$$

Experimental conditions:

1) Time horizon periods ($T$) = 5, 10, 15, 20, 25
2) Demand order factor ($\delta$) = 0.2, 0.4, 0.6

Demand order size sampled from Uniform Dist. :\([(20T\delta/2), (20T\delta)]\)

*Nembhard and Uzumeri [2000]*
Experimental Results:
Optimal solutions are affected by $T$ and $\delta$
74% of solution ≤1 time period differences from Optimal.
Computational Time does not increase exponentially with the problem size.

**Summary of ANOVA tests**

<table>
<thead>
<tr>
<th></th>
<th>$\delta$</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportion of Optimal Solution</td>
<td>5.70 (0.004)</td>
<td>4.02 (0.003)</td>
</tr>
<tr>
<td>Absolute Difference</td>
<td>5.87 (0.003)</td>
<td>4.66 (0.001)</td>
</tr>
<tr>
<td>CPU Time [sec]</td>
<td>0.30 (0.741)</td>
<td>4.35 (0.002)</td>
</tr>
</tbody>
</table>

*F-statistic (P-value)*
Questions
References


Additional Slides: Empirical Data Used

Three-parameter Hyperbolic Learning model

Empirical data presented by Shafer et al. (2001) and implemented by Nembhard & Bentefouet (2012).

The mean vector and variance-covariance matrix of \([ \ln(k), \ln(p), \ln(r) ]\) are approximated by \(\mu\) and \(\Sigma\) as:

\[
\mu = \begin{bmatrix} 3.36 & 4.66 & 4.83 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 0.054 & 0.397 & 0.374 \\ 0.397 & 7.821 & 5.430 \\ 0.374 & 5.430 & 4.923 \end{bmatrix}
\]

- \(p_{i,j}\) represented the initial productivity level of worker \(i\) at task \(j\)
- \(r_{i,j}\) the learning rate parameter of worker \(i\) at task \(j\)
- \(k_{i,j}\) the steady state production rate of worker \(i\) at task \(j\).

\[ f_{i,j}(t) = k_{i,j} \frac{t + p_{i,j}}{t + p_{i,j} + r_{i,j}} \]
Problem parameters:
• Production time horizon \((T)\)
• Demand order size of both tasks required at the end of the time horizon \((D_j \forall j)\)
• Output of worker \(i\) performing task \(j\) during period \(t\) \((O_{i,j,t})\).

Heuristics parameters:
• Arrival time controller law gain \((\alpha)\),
• Switching time controller law gain \((\gamma)\),
• Upper bound for the discrete event simulation iterations \((\theta)\).

The values of these parameters can have a direct impact on the stability of the heuristic and its performance on generating good schedules, as shown by Prabhu (2000).

Based on initial experimental results these parameter were set: \(\alpha = 0.2, \gamma = 0.4, \text{ and } \theta = 1,000\).
Input: \( T, D_j, O_{i,j,t}, \alpha, \gamma, \theta \) \( \forall i \ \forall j, \forall t \);

1. **Step 1**: Define:
   Simulation iteration \( r = 0 \), Arrival vector \( a(\theta) = \emptyset \) with it first entry \( a(1) = 1 \), Switching vector \( s(\theta) = \emptyset \) with it first entry \( s(1) = 1 \), Task assignment \( \omega_1 = 1, \omega_2 = 2 \), Tasks production output vector \( P_1(\theta) = \emptyset \) and \( P_2(\theta) = \emptyset \);

1. **Step 2**: Select the initial worker assignment;
   \[
   \text{if } \sqrt{\left(\sum_{t=1}^{T} x_{2,1,t} \cdot O_{2,1,t} - D_1\right)^2 + \left(\sum_{t=1}^{T} x_{1,2,t} \cdot O_{1,1,t} - D_2\right)^2} < \sqrt{\left(\sum_{t=1}^{T} x_{1,1,t} \cdot O_{1,1,t} - D_1\right)^2 + \left(\sum_{t=1}^{T} x_{2,2,t} \cdot O_{2,2,t} - D_2\right)^2} \text{ then}
   \]
   \[\omega_1 \leftarrow 2 \text{ and } \omega_2 \leftarrow 1;\]
   end

1. **Step 3**: Applied switching and arrival controllers;
   2. \( P_1(r) \leftarrow \sum_{t=a(r)}^{S(r)-1} \omega_{2,1,t} \cdot O_{\omega_1,1,t} + \sum_{t=a(\theta)}^{T} x_{1,1,t} \cdot O_{\omega_2,1,t} \);
   3. \( P_2(r) \leftarrow \sum_{t=a(r)}^{S(r)-1} \omega_{2,1,t} \cdot O_{\omega_1,1,t} + \sum_{t=a(\theta)}^{T} x_{2,1,t} \cdot O_{\omega_2,1,t} \);
   4. if \( P_1(r) \geq D_1 \) \& \( P_2(r) \geq D_2 \) then
   5. \( a(r + 1) \leftarrow a(r) + \alpha \left[ \min \left( \frac{P_1(r) - D_1}{(0.5 \times (P_1(r) + P_2(r)))}, \frac{P_2(r) - D_2}{(0.5 \times (P_1(r) + P_2(r)))} \right) \right] \), else \( a(r + 1) \leftarrow a(r);\)
   6. if \( P_1(r) < D_1 \) or \( P_2(r) < D_2 \) then
   7. \( s(r + 1) \leftarrow s(r) + \gamma \left[ \min \left( \frac{D_1 - P_1(r)}{(0.5 \times (P_1(r) + P_2(r)))}, \frac{D_2 - P_2(r)}{(0.5 \times (P_1(r) + P_2(r)))} \right) \right] \), else \( s(r + 1) \leftarrow \min(s(r), a(r));\)
   8. **Step 4**: Repeat Step 3 and 4 with \( r \leftarrow r + 1 \) until \( r + 1 = \theta \)

9. **Step 5**: Output: \( \omega_1, \omega_2, a(m), s(m), x_{i,j,t} \) \( \forall i \ \forall j, \forall t;\)
   \[a(m) = \min(a(\theta) \mid P_1(\theta) \geq D_1 \& P_2(\theta) \geq D_2)\]
   \[x_{\omega_1,1,t} = 1 \text{ for } a(m) < t < s(m - 1), x_{\omega_1,2,t} = 1 \text{ for } s(m) < t < T, 0 \text{ otherwise}\]
   \[x_{\omega_2,1,t} = 1 \text{ for } s(m) < t < s(m - 1), x_{\omega_2,2,t} = 1 \text{ for } s(m) < t < T, 0 \text{ otherwise}\]
Example 1

**Makespan**

\[ T = 5, \text{ Demand Factor} = 0.6 \]
Additional Slides: Algorithm’s Outputs

Example 1

**Shortage**

![Graph of Shortage](image1)

**Orders RMSD**

![Graph of Orders RMSD](image2)
Additional Slides: Algorithm’s Outputs

Example 1

Operator Switching Time

Production Start Time
Additional Slides: Algorithm’s Outputs

Example 1

Station 1 (order size=50)

Output [units]

Station 2 (order size=50)

Output [units]
Example 2

MakeSpan (T=5, Demand Factor=0.6)

- Time Period
  - 1
  - 3
  - 5

- Simulation Iterations
  - 0
  - 20
  - 40
  - 60
  - 80
  - 100

Shortage

- Units
  - 0
  - 15
  - 30

Orders RMSD

- Units
  - 0
  - 200

- Simulation Iterations
  - 0
  - 20
  - 40
  - 60
  - 80
  - 100
Example 2

Station 1 (order size=44)

Station 2 (order size=46)

Operator Switching Time

Production Start Time