A Concept for Simultaneous High-Frequency Actuation of Liquid Spray Characteristics

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Herein, we propose the Synchronously Actuated Response Atomizer (SARA) concept to address three target needs in the area of thermo-acoustic research in liquid-fueled systems; sufficiently broad frequency response to span the flame’s bandwidth without encountering significant actuator dynamics, independent actuation of various spray characteristics thought to be relevant to the flame response, reduced size and cooling requirements making it practical to implement the device in a research combustor. We propose physical mechanisms by which a device might be able to control three characteristics at high frequency (ideally as high as 1kHz); mass flow, droplet distribution, and cone angle. These mechanisms are described by simple models and validated in low frequency experiments. The details of the high-frequency design are also provided.

I. Introduction

Combustion instabilities present a persistent challenge for the design an operation of gas turbine engines. In the past decade, a rich variety of approaches for suppressing these destructive phenomena have been suggested, including the use of active control. The progress of these technologies has been tracked by review articles over the past two decades.

There are three strategies that best characterize the fuel actuation technologies available.

1. The use of “slow” control of spray properties to stabilize the flame,
2. Variation of the fuel pressure via an upstream actuator, and
3. The design of nozzles with internal high-frequency actuation.

Slow actuation relaxes demands on the actuator, and reduces the likelihood of fatigue failure by decreasing cyclic loading of parts. However, this scheme relies on identifying a method for impacting an inherently dynamic process with only low-frequency actuation. The approach, in effect, flaunts the system’s nonlinearities to inadvertently affect high-frequency phenomena with low-frequency actuation.

While it requires more costly high frequency actuation, upstream actuation can be quite simple to implement since it can be installed outside of the combustor where the actuator is easy to cool and space constraints are more relaxed. However, this approach is plagued by undesirable dynamics from the fluid acoustics, finite line stiffness, and damping from trapped air. Despite these challenges, examples of successful approaches include actuated valves and phase-controlled spinning obstruction.

Internal actuation minimizes the dynamic problems encountered in upstream actuation, but places sensitive actuators in close proximity to the combustor, where special considerations have to be given to cooling, and spatial constraints require small, precisely machined components. A design studied by Neumeier et. al. used an actuated plain-orifice atomizer to vary fuel flow. However, researchers noted that the actuation also had affects on droplet placement and size. A design investigated by Chang et. al. used actuated spill-return flow to vary fuel flow while minimizing the actuator’s impact on other spray characteristics.

Regardless of the approach, efforts to improve the quality of a system’s control and any attempt to generalize techniques to a broader family of systems rely on a principled understanding of the dynamics involved. It is known, for example, that spray quantities such as the droplet placement, droplet size, and total mass flow rate, impact the flame dynamically. However, because it is extremely difficult to affect one of these quantities without affecting the others (statically or dynamically), it is very challenging to make definitive statements regarding their relative importance.

It is for these reasons that we propose the construction of a nozzle capable of varying mass flow rate, droplet size distribution, and droplet placement in the combustor via the spray cone angle independently at frequencies ideally as high as 1kHz. Such a nozzle would facilitate the simultaneous study of both low-frequency and high-frequency control strategies.

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II. SARA Nozzle Concept

The Synchronously Actuated Response Atomizer (SARA) concept uses three simultaneously actuated constrictions to independently control three critical spray parameters; mass flow rate, Sauter-mean diameter (SMD), and cone angle.

The SARA concept generates a spray cone with a simplex-style swirl geometry. In static simplex nozzles, the intensity of the swirl upstream of the exit orifice determines the cone angle \(^9\), and the pressure is the primary predictor of the droplet size \(^9,10\). However, the mass flow rate is also dictated by the pressure. Further complicating matters, the swirl intensity also impacts the liquid pressure.

Figure 1 depicts the SARA concept for decoupling these dependencies so that the spray parameters can be varied independently. Actuator 3 controls the exit orifice area via a small pintle (not shown), allowing independent actuation of the swirl chamber pressure and mass flow rate. The remaining two actuators control valve openings on two parallel lines; one entering the swirl chamber with an intense swirl, and the other entering the swirl chamber with little or no swirl.

The limiting factors are how fast the actuators can move the mechanical surfaces involved and how quickly the fluid will respond to the actuation. Though the forces involved in actuating sealed surfaces against pressurized liquid can be quite large, the displacements necessary in practical nozzles are fortuitously small. Piezoelectric actuators are uniquely suited to the task.

The fluid response is inhibited by two physical processes; acceleration of the fluid, and compression of the fluid and containing walls. The combined effects result in an acoustic response that must be carefully tuned. Prior efforts have focused on tuning the acoustic response to resonate at frequencies of interest to give elevated amplitude. However, the added phase dynamics severely complicate and can even inhibit control. It is preferable that the nozzle’s response be quasi-static—lacking any frequency-dependent behavior—in the operating band.

III. Model

Included here are models predicting the SARA nozzle’s Sauter-Mean-Diameter (SMD), mass flow rate, and cone angle. These models demonstrate the effectiveness of the design concept and will eventually be used to help design experiments to test a prototype.

For the purpose of these models, let the three actuators shown in Figure 1 have positions described in the three-element vector, \(\vec{x} = (x_1, x_2, x_3)^T\).

A. Mass Flow Rate

The entire nozzle can be modeled with three restrictions; one corresponding to each actuator. The flow through each of the restrictions can be described by a Bernoulli model,

\[
\dot{m}_i = C_i(x_i) \sqrt{\Delta P_i}.
\]
Note that the flow coefficient is a function of the corresponding actuator position. Actuators 1 and 2 are in parallel so that \( \Delta P_1 = \Delta P_2 \). The third restriction is situated downstream such that

\[
\Delta P_s = \Delta P_1 + \Delta P_3
\]

where \( \Delta P_s \) is the gage pressure in the fuel supply line and \( \dot{m} \) is the total mass flow rate through the nozzle. Combining with (1),

\[
\dot{m} = C_1 \sqrt{\Delta P_1} + C_2 \sqrt{\Delta P_2} = (C_1 + C_2) \sqrt{\Delta P_1} = C_3 \sqrt{\Delta P_3}.
\]

Finally, asserting that the sum of the pressure drops is equal to the supply pressure yields

\[
\dot{m} = C(\vec{x}) \sqrt{\Delta P_s},
\]

where

\[
C(\vec{x}) = \left\{ \left[ C_1(x_1) + C_2(x_2) \right]^2 + C_3(x_3)^2 \right\}^{-1/2}
\]

The term, \( C_1 + C_2 \) is one that will reappear in later sections due to its extreme physical significance to the operation of the nozzle. It represents the total combined flow coefficient for valves 1 and 2. It becomes convenient to write

\[
C_{12} = C_1 + C_2,
\]

so that

\[
C = \frac{1}{\sqrt{C_{12}^{-2} + C_3^{-2}}}
\]

### B. Droplet Size

The droplet diameter can be characterized by a number of statistical quantities. The one most relevant to combustion is the SMD. Defined in terms of the global probability density function, \( p(D) \),

\[
SMD = D_{32} = \frac{\int_0^{\infty} D^3 p(D) \, dD}{\int_0^{\infty} D^2 p(D) \, dD}.
\]

Sauter-Mean-Diameter is a droplet diameter weighted to indicate the average fuel mass per droplet surface area. For a spray with a finite population of droplets, it can be estimated as

\[
SMD \approx \frac{\sum_i D_i^3}{\sum_i D_i^2}.
\]

Studies on pressure-swirl or “simplex” nozzles have revealed that the SMD follows an inverse power rule with the pressure drop across the nozzle. Thus, in terms of the pressure drop across the nozzle’s exit orifice (orifice controlled by Actuator 3),

\[
SMD = \frac{K}{(\Delta P_3)^n}.
\]

where \( K \) and \( n \) are found empirically and depend on the nozzle-specific geometry and the fluid being used. Typical correlations place the values on the order \( n \approx O(0.5) \) and \( K \approx O(15 \times 10^3) \mu_m \cdot \text{Pa}^n \) for water.

Naturally, this empirical model ignores the fact that the nozzle geometry is not constant, but it reflects the notion that atomization improves as the liquid pressure (and subsequently the exit velocity) increases. Despite the unpredictable impacts variable swirl and exit geometry are likely to have on atomization quality, because this effect can be relied upon to exert a strong influence, it is an excellent starting point for a concept argument.

Equation 8 also indicates that as pressure increases, subsequent affects on the droplet diameter diminish, and the curve levels to some practical limit determined by the fuel pressure available. That implies that in order to retain any command over SMD, the nozzle will have to operate in at a relatively low pressure.

Using the valve model from Section A., we may relate \( \Delta P_3 \) to the actuator positions

\[
\Delta P_3 = \frac{\dot{m}_2^2}{C_3^2} = \Delta P_s \left[ \frac{C_3^2}{(C_1 + C_2)^2} + 1 \right]^{-1}
\]
C. Cone Angle

To develop a physical argument for a cone angle model, it is easiest to begin with an abstraction and then apply the result to the SARA concept. First, there are several definitions useful for the purpose of this discussion.

For a velocity passing through a surface, the average of the \( k \) velocity component may be defined as

\[
\overline{v_k} = \frac{\iint_S v_k \, dS}{\iint_S dS}.
\] (10)

Furthermore, we may characterize the flow profile of an arbitrary velocity component, \( k \), relative to the mean velocity as

\[
v_k(r) = \overline{v_k} \lambda_k(r) \lambda_z(r, z).
\] (11)

In this sense, \( \lambda \) is a dimensionless function that can be used to determine a flow profile at a surface, but still allow the magnitude of the velocity to be scaled by the average. By restricting \( \lambda \) from being a function on \( \theta \), we are asserting that the flow is axisymmetric. It is worth noting, but not of prime importance to this discussion that substituting Equation 11 into Equation 10 yields a condition on \( \lambda \) that

\[
1 = \frac{\iint_S \lambda_k \, dS}{\iint_S dS},
\]

which asserts that \( \lambda \) may vary widely in shape and may possess peaks, troughs, and may even be negative, but must always have an average value of 1.

Given any point along a surface, it will become convenient to write the velocity convecting fluid across it to be divided into three components: normal, \( v_n \), tangential, \( v_t \), and the remainder, \( v_r \), all of which are depicted in Figure 2. The tangential component can be defined as the only component lying in both the plane normal to the \( z \)-axis and the surface. The normal component is intuitively the component normal to the surface. The remainder is velocity component normal to the other two.

Consider a volume through which an incompressible fluid is flowing. There may be any number of inlets, but only a single outlet. Specifically, let us consider the case that the flow through the exit orifice has a tangential (swirling) velocity that causes the fluid issuing from the opening to form a cone. If swirl and not radial velocity is the predominant cause for the effect, then the cone half-angle, \( \phi \) may be estimated by

\[
\tan \phi = \frac{\overline{v_{3,t}}}{\overline{v_{3,n}}}.
\] (12)

Also, it is convenient to think of fluid entering through a surface, \( i \), at some angle

\[
\tan \theta_i = \frac{\overline{v_{i,t}}}{\overline{v_{i,n}}}.
\] (13)

If the net viscous torque on the fluid in the volume is negligible, the conservation of angular momentum states that

\[
\iint_S \rho (\vec{r} \times \vec{u}) (\vec{u} \cdot d\vec{S}) = 0.
\] (14)

We may reduce this equation to a useful form by applying the following:
1. That the exit opening is axi-symmetric and exists in a single plane, normal to that axis; and
2. That the exit opening shares its axis with the coordinate system axis.

The series of surfaces through which fluid enter the volume may be indexed by \( i \), so that 14 can be expanded to yield

\[
\sum_i \int \int_{A_i} r v_{i,t} v_{i,n} \, dS = \int \int_{A_3} r v_{3,t} v_{3,n} \, dS.
\]

or

\[
\sum_i v_{i,t} v_{i,n} \int \int_{A_i} r \lambda_{i,t} \lambda_{i,n} \, dS = v_{3,t} v_{3,n} \int \int_{A_3} r \lambda_{3,t} \lambda_{3,n} \, dS.
\]

(15)

It is now useful to consider parameters,

\[
\beta_i = \frac{\int \int_{A_i} r \lambda_{i,t} \lambda_{i,n} \, dS}{\int \int_{A_i} r \lambda_{i,n} \, dS},
\]

(16)

and

\[
\hat{R}_i = \frac{\int \int_{A_i} r \lambda_{i,n} \, dS}{\int \int_{A_i} \lambda_{i,n} \, dS}.
\]

(17)

The \( \beta \) parameters describe the integral quantities in ratio with their values if the flow profiles were all entirely uniform (all \( \lambda = 1 \) everywhere on every surface). Qualitatively, \( \beta \) describes the increased angular momentum if the flow profile were skewed to favor the larger radius. The \( \hat{R} \) parameters describe a characteristic area-weighted average radius of each opening. Substituting into Equation 15, we obtain

\[
\sum_i v_{i,t} v_{i,n} \beta_i \hat{R}_i \int \int_{A_i} \, dS = v_{3,t} v_{3,n} \beta_3 \hat{R}_3 \int \int_{A_3} \, dS.
\]

or simply

\[
\sum_i v_{i,t} v_{i,n} \beta_i \hat{R}_i A_i = v_{3,t} v_{3,n} \beta_3 \hat{R}_3 A_3.
\]

(18)

Since the mass flow through any given orifice can be expressed as \( \dot{m}_i = \rho v_{i,n} A_i \), the fraction of flow through the orifice can be written as

\[
X_i = \frac{\dot{m}_i}{\dot{m}_3} = \frac{v_{i,n} A_i}{v_{3,n} A_3}
\]

Finally, substituting into Equation 18 yields

\[
\sum_i \tan \theta_i \frac{\beta_i}{\beta_3} \frac{\hat{R}_i}{\hat{R}_3} \frac{A_i}{A_3} X_i^2 = \tan \phi.
\]

(19)

The radii, areas, \( \beta \) constants, and entrance angles (\( \theta_i \)) are all constrained by the geometry. Therefore, by varying the flow through each of the inlet channels, the flow fractions, \( X_i \), alone influence the cone angle.

In the case of the SARA nozzle, there are two sets of entrance ports. One with none-swirling flow (\( \theta_1 = 0 \)) and one with swirling flow (\( \theta_2 = 45^\circ \)). Consequently, the cone angle is predicted by

\[
\frac{\beta_2 \hat{R}_2 A_3}{\beta_3 \hat{R}_3 A_2} X_2^2 = \tan \phi.
\]

(20)

and can be controlled by varying \( X_2 \) anywhere from 0 to 1.

Using the same valve model from Section A., the flow fraction through valve 2 may be written as

\[
X_2 = \frac{\dot{m}_2}{\dot{m}_1 + \dot{m}_2} = \frac{C_2(x_2)}{C_1(x_1) + C_2(x_2)}
\]

(21)

It should be noted that \( A_2 \) does not change with \( x_2 \). In this case, \( A_2 \) describes the area of the channel that guides the liquid into the swirl chamber, while \( x_2 \) actuation occurs upstream of that.
Having modeled the three physical parameters of interest in terms of the valve flow coefficients, it is important to understand how the entire nozzle behaves as a system and develop a control strategy. It is important that the three parameters’ relationships to the valve flow coefficients be sufficiently linearly independent over the entire operating range to allow practical independent control of the three parameters. We demonstrate that this is the case by proving that the Jacobian relating the input and output vectors is strictly non-singular.

If the three controlled parameters are contained in a vector, 
\[ \vec{Z} = \begin{bmatrix} \dot{m} \\ \text{SMD} \\ \phi \end{bmatrix} \]

and the valve flow coefficients are similarly contained,
\[ \vec{C} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} \]

then they are related by the three above models and can be expressed as
\[ \vec{Z} = f(\vec{C}). \]  
(22)

The linear independence of the system local to any given operating point can be determined by the eigenvalues of the linearized system,
\[ \Delta \vec{Z} = J \cdot \Delta \vec{C}, \]  
(23)

where \( J \) is the Jacobian of \( f \), and \( \Delta \vec{Z} \) and \( \Delta \vec{C} \) are small changes in \( \vec{Z} \) and \( \vec{C} \) respectively. We compute \( J \) by differentiating the models from Sections A., B., and C. to obtain
\[
J = \begin{bmatrix}
\frac{a}{C_{12}^2} & \frac{a}{C_{12}^2} & \frac{a}{C_{3}^2} \\
-b/C_{12} & -b/C_{12} & b/C_{3} \\
-c/C_{12} & -c/C_{12} + c/C_{2} & 0
\end{bmatrix}. 
\]  
(24)

Here,
\[
a = \frac{\dot{m}}{C_{12}^{-2} + C_{3}^{-2}} 
\]  
(25a)
\[
b = 2 \text{ SMD} \frac{n}{C_{3}^2/C_{12}^2 + 1} 
\]  
(25b)
\[
c = 2\phi 
\]  
(25c)

It is simple enough to prove that the system is linearly independent by taking the determinant of \( J \). After some simplification,
\[
\det(J) = -\frac{abc}{C_{12}C_{2}C_{3}} \left( \frac{1}{C_{12}^2} + \frac{1}{C_{3}^2} \right). 
\]  
(26)

Equation 26 is strictly negative, thus the Jacobian is inherently linearly independent.

IV. Static Prototype

A static prototype was constructed to demonstrate the low-frequency behavior of the three quantities and act as a basis from which to investigate the number of practical concerns that are relevant to the design. Though the exit orifice and the swirl geometry are precisely to the scale of a practical nozzle, the mechanical components are large and interchangeable to facilitate iterative modifications to the design.

Depicted in Figure 3, the Gen I prototype used a 4mm diameter conical pintile to actuate on a 0.4mm diameter orifice. Immediately upstream of the exit orifice, flow leaving the orifice is forced through 45° swirl channels 0.5mm in width into a conical swirl chamber 2mm in diameter at its base, located roughly 1mm below the orifice at its apex. In order to reach the exit orifice, the pintle passes through the swirl chamber. Seepage flow along the clearances between the pintle and the swirl chamber walls is prevented by upstream o-rings. Experiments were conducted using water in two configurations, also depicted in Figure 3.

In the first configuration (A), the pintle was position to vary the exit orifice area. In this configuration, the SMD was measured using PDA to scan a 49-point 9x9mm grid, 10mm downstream from the orifice. The data was then analyzed to compute the total planar-averaged SMD.

Figure 4 shows the results of these tests. SMD bears only a slight dependence on pintle location, encouraging the use of the pressure model from Section B. However, the perceptible increase in SMD with decreasing exit orifice size could possibly be modeled as a coupling between cone angle and the atomization quality. It stands to reason that actuating on the exit orifice depletes the cone angle (as predicted by Equation 20), which should absolutely impact the prompt atomization process in a manner that the pressure correlation fails to include. Thus, a more accurate model might include \( x_3 \) in the power fit.
Figure 3. Swirl geometry and pintle assembly in both test configurations; (A) the pintle engages the o-rings properly, 100% of the flow exits along the highlighted path through the swirl channels, and the pintle is used to reduce the exit orifice area; (B) the pintle is retracted and disengaged from the o-rings to meter a non-swirling seepage flow along the highlighted path through the center of the assembly.

Figure 4. SMD for various pressures and pintle locations in configuration A.
In the second configuration (B), the pintle was retracted to disengage from the o-ring and allow a non-swirling seepage flow into the swirl chamber. In this configuration, the mass flow and cone angle were measured with respect to the nozzle flow coefficient and the supply pressure. Mass flow was measured using a rotometer inline with the water supply system, and the cone angle was measured by processing digital images of visible light scattering in the spray field, wherein it was found that the cone exhibited a well-defined optical boundary.

The results of these tests are shown in Figure 5. Because the experimental design prohibited accurate measurement of the pintle location while in the retracted position, the bypass flow was quantified by measuring the total nozzle flow coefficient. The cone angle demonstrated nearly a 100% change over the range of the actuation.

![Figure 5](image)

Figure 5. Test results from configuration B. (a) Shows mass flow rate and flow coefficients for various pintle locations and various supply pressures. (b) Shows cone angle as a function of the flow coefficient.

V. Conclusions

In the present work, we developed simple models describing the expected mass flow rate, droplet diameter, and cone angle for the SARA concept nozzle, and substantiated them with a low-frequency experiment. From the results, there are two key conclusions that can be drawn:

1. The SARA actuation scheme is inherently linearly independent, and that
2. Models predicting the SARA concept performance are consistent with low-frequency experiment.

These conclusions are sufficient to state that the concept is successful at low frequencies. However, it remains to be shown how the response will change at high frequencies. There will be some critical frequency at which the response is no longer quasi-static, but dynamic experiments are necessary to determine that frequency precisely.

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References


