ELECTRICAL SIGNATURES FOR CHEMICAL ACTION AT THE WORK SURFACE IN AN OXYFUEL FLAME

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ABSTRACT
An oxyfuel cutting torch was biased with a positive DC voltage relative to steel and copper work pieces, and ion currents were seen to flow due to ions generated chemically in the flame and at the work surface. A repeatable 150% rise in these currents occurs over steel at a surface temperature around 1050°C. Equally strong intermittent spikes appear over clean and corroded steel samples alike. They are easily removed by a median value filter, and are believed to be due to small reacting particulates. Tests in which salt was deliberately deposited on the steel surface produces a signal similar in shape, but exaggerated in magnitude to those currents supposed to be from other surface impurities.

1 Introduction

1.1 Oxyfuel cutting
In the oxyfuel cutting process, a high-temperature flame is used to heat the work piece, but the actual material removal process is accomplished by chemical action between the work and a jet of pure oxygen. Like laser and plasma cutting, the mechanized oxyfuel process begins with a pierce operation, which forms a small hole in the work piece from which the cutting path can begin. Unlike the other thermal processes, the plate must be brought up to a specific temperature (the kindling temperature) before a pierce can begin. If the work surface is not sufficiently hot, the pierce will fail, and the part will likely be ruined.

Despite its importance to industries that use thick steel, mechanized oxyfuel cutting has never been satisfactorily automated. Oxyfuel cutting occupies an economic niche emphasizing thickness, quality, speed, and reliability, but market expectations for low system costs make establishing an appropriately robust sensing suite extremely challenging. The problem is worsened by the severe environment in which sensors need to operate; rich with infrared radiation, intense acoustic and electrical noise, and a barrage of molten and frozen metal particulates.

To viably automate the process, a sensing suite should be able to:
- Verify the torch ignition and flame stability;
- Verify that the plate is ready to pierce;
- Verify that the pierce is through the plate;
- Measure the standoff distance from the work piece;
- Measure fuel/oxygen ratio of the flame; and
- Diagnose a loss-of cut.

The most advanced commercially available systems measure standoff distance using either a capacitive or a magnetic sensor. Gas flow rates are usually balanced using on-torch valves, but some systems have active control to compensate for drift in the flow rates. Preheat operations prior to the pierce are usually simply executed for a pre-determined time, but some systems detect infrared (IR) light intensity.

These sub-systems can cost as much as the torch itself, and often suffer their own complications. For example, capacitive height sensors are notoriously vulnerable to damage from slag and heat, and they fail completely at the plate’s edge. The IR
sensors can lead to false ready-to-pierce signals due to rust and surface particulates that quickly heat to temperatures well above that of the core of the plate below. Magnetic and IR sensors also require active electronics to be mounted on the torch, which can drastically complicate the torch design, and rules out the possibility of a retrofit for existing equipment.

1.2 Ion current sensing

Ion currents are electrical currents that pass through a fluid due to the motion of charged particles (ions). Their prediction involves a rich set of physics including fluid transport and electrical mobility of particles of drastically different sizes (i.e. charged molecules and free electrons).

The earliest attempt to apply ion currents to sensing in oxy-fuel torches appears to be recorded by Mott, Chouinard, and Hardings 1944 patent [1], wherein a DC voltage is applied between the torch and work piece, and the current is taken as an indication of the standoff distance. Later inventors would propose improvements like using constant current sensing and the transition to semiconductors and digital control systems [2–4]. Despite the patent record, interviews with industry professionals and a survey of the relevant literature reveals no publication or product successfully implementing the approach. The most recent attempts some decades ago were abandoned due to the lack of processing resources at the time [5]. Despite a number of successful efforts to use ion currents for sensing in internal combustion engines [6–10] and in furnaces [11–13], the complexity of the signal found in oxyfuel cutting systems was far more demanding.

The bulk of contemporary knowledge about ion currents in flames was formed since Mott et al. were awarded their patent with the most relevant physics addressed quite recently [14]. Any list of the most important authors on the subject would have to contain H. F. Calcote, and in 1957, he wrote that “the most critical problem in this field is the actual identification of the ion [15].” A continuum model for the transport of ions in sparse plasmas came in the early 1960s [16], and a treatment of the dominant physics in atmospheric flames would not arrive for another decade [17–20]. By the end of the twentieth century, consensus had formed around \( \text{H}_3\text{O}^+ \) and \( e^- \) (the free electron) as the dominant ions [14], which has empowered more recent efforts to produce simplified models predicting their formation and transport [21–24].

In atmospheric flames, the properties of the positive and negative charge carriers (\( \text{H}_3\text{O}^+ \) and \( e^- \)) are short-lived byproducts of combustion so dissimilar in their properties that the larger positively charged molecule is effectively “frozen” in the bulk flow, while tiny electron diffuses quickly and responds to electric fields so easily as to effectively ignore the bulk flow [20]. This represented a severe deviation from the theory that applied in sparse plasmas where much of the prior literature had focused [16, 25, 26].

Recent studies show that simplified models can replicate atmospheric tests with highly simple geometries [22, 24], but the oxyfuel cutting flame is a more complicated problem. Measurements conducted by the author [27] demonstrated clear repeatable changes in the current-voltage characteristic with changes in standoff distance and fuel/oxygen ratio, but effects due to total flow rate and the work surface temperature were erratic and unrepeateable. The present paper presents measurements demonstrating electrical signatures for chemical action at the work surface superimposed on those attributed to the combustion reaction.

1.3 Prior Measurements

Previous experiments in the range ±10V demonstrated that the oxyfuel flame’s current-voltage characteristic exhibited semiconductor behaviors forming three distinct regimes; which the author numbered from left to right [27]. The slope (i.e. resistance) in regime 2 was reliably linearly related to the standoff distance and the transition current (saturation current) between regimes 1 and 2 was related to F/O ratio with a Gaussian shape [28, 29]. The regime 3 behaviors showed dependencies on the work temperature and total flow rate, but these measurements were not repeatable, indicating that there were additional influences not controlled in these experiments.

Recent models and measurements by Speelman et al. [23,24] shed light on the behavior at negative voltages (when the torch is asked to source electrons). Their system is a burner-stabilized flat flame, which they show to be a thin source of ions some distance from the burner surface. As is also true for the oxyfuel flame, their experiment’s surface temperatures are far too low for thermionic emission to come into play, so electrons are only...
emitted when surfaces are impacted by a positive ion. The total flow of electrons from a surface is the difference between the rate of impact of the fluid’s positive ions and electrons, so an electric field is strong enough to evacuate all of the electrons from the vicinity of a metal surface, there is an abrupt discontinuity in the flow of current like that of a semi-conductor. In this condition, the flow of current is controlled entirely by the rate of bombardment of the large sluggish positive ions.

So much was already understood a half-century ago, but these authors demonstrated an unusual abrupt transition between linear characteristics identical to the one at the left-hand-side of Figure 1. The slopes in these regimes are due to the distance that charged particles must migrate through a resistive plasma, which explains why the slopes are so linearly related to standoff.

In the oxyfuel torch, the space inside the inner cone has the happy advantage of being insensitive to the happenings at the work surface, so the behaviors are quite regular even when the condition of the work piece is not. The oxyfuel torch differs from most flame studies in several important ways: the oxyfuel flame is hot enough that it has an “outer cone” where species remain dissociated in a state of incomplete combustion, the flame geometry and the surrounding flow field are much more complicated, and the work piece is not a chemically inert conductor. Is one of these responsible for the unrepeatable regime 3 behaviors seen in the oxyfuel flame [27]? If so, can it be used as the basis of a novel sensing technique?

2 Experiment

The torch under test is an Oxweld C-67 with a 1/2-inch tip burning methane. Two experimental configurations were used to isolate effects of ions that originate in the flame and ions that are formed at the work surface in a preheat operation. In the first configuration, the work is replaced by a cooled copper disc. Unlike the oxides of iron, copper oxides are quite resilient so oxide layers that form on a copper work piece will stabilize, and the chemical action at the surface should halt (or at least equilibrate). On the other hand, oxides of iron and steels form and shed from the work surface continuously, making any experiment extremely difficult to control in a repeatable way.

In all tests, thermal mass flow meters were used to monitor the gas flow rates to the torch. Gas metering was performed by regulators and needle valves from a control panel in a separate room to prevent thermal drift. The torch was electrically isolated, and a programmable power supply was used to interrogate the current-voltage characteristic in the same way described previously [27].

2.1 Copper disc tests

A series of copper coupons were constructed by pressing step features on copper discs onto short lengths of brass pipe. A concentric tube-and-pipe arrangement shown in Figure 2 delivered a mixture of water and air in the center tube and vented it from an annulus formed by the outer pipe. A type-K thermocouple was cemented into a small indentation in the copper above the pipe.

This arrangement permitted discs of different diameters to be used to test for the impact of the area of engagement between the disc and the flame. Rectangular coupons would have been much simpler to manufacture, but could have caused irregular effects if the corners were accidentally aligned with features of the flame’s starburst pattern. The mass fraction of water supplied to the coolant permitted extreme variations in the coupon temperature independently of other experimental conditions.

After high temperature tests, the cyclic thermal contraction and expansion of the copper weakened the press fit. Coupons that showed signs of weak joints were brazed to the pipe. This secured them in place, but rendered them unsuitable for further high-temperature tests.

The flow rate of water was recorded with a factory calibrated rotameter, the mass flow rate of air was metered by a choked orifice, and the inlet and outlet temperatures were measured by thermocouples. The heating rate was calculated by energy balance,

$$\dot{Q} = (\dot{m}_a c_{p,a} + \dot{m}_w c_{p,w})(T_{out} - T_{in}) + \dot{m}_{w,vap} h_{fg},$$  \hspace{1cm} (1)$$

where $\dot{m}_a$ and $\dot{m}_w$ are the mass flow rates of air and water, $c_{p,a}$ and $c_{p,w}$ are constant-pressure specific heats for air and water, $\dot{m}_{w,vap}$
mass of water in vapor form at the outlet, and \( h_{fg} \) is the enthalpy of vaporization of water at atmospheric pressure. The inlet and outlet temperatures, \( T_{in} \) and \( T_{out} \), and the total mass flow rates are measured directly, but the mass of water in vapor form must be calculated from the vapor pressure of water. By the definition of partial pressure,

\[
\frac{P_{w,\text{vap}}(T)}{P_{\text{atm}}} = \frac{n_{w,\text{vap}}}{n_{w,\text{vap}} + n_a}
\]

\[
\dot{m}_{w,\text{vap}} = \frac{m_w}{m_a} \dot{m}_a \left( \frac{P_{w,\text{vap}}(T)}{P_{\text{atm}} - P_{w,\text{vap}}(T)} \right)
\]

(2)

where \( m_{w,a} \) and \( m_{w,w} \) are the molecular weights of air and water.

Using this method, the heating rate was calculated to be approximately 1 to 1.2kW for the tests around 10 L/min and above total flow rate; about half of the power calculated from the lower heating value of the fuel. It should be emphasized that this analysis assumes that the cooling water is in equilibrium with dry air, and that some of the water is still liquid.

The thermal conductivity for the ETP copper used is tabulated at 391W/m/K [30], so heat added uniformly over a diameter of 12mm would result in a thermal gradient not greater than 22°C/mm at the plate surface. The thermocouple 6mm below the upper surface was, therefore, taken to be within 100°C of the surface temperature. This expectation for thermal uniformity of the copper was confirmed visually. Unlike steel samples, the entire coupon uniformly radiated visible light rather than showing a “hot-spot” under the flame.

### 2.2 Steel plate tests

The bottom side of half-inch (12.7mm) thick steel plates were drilled and a thermocouple was cemented 3mm deep; measured from the plate’s upper surface. No coolant was used, so tests were conducted by placing the torch directly over the thermocouple and monitoring the electrical characteristics of the flame as the plate heated.

To control for the quality of the plate surface, two types of samples were prepared. The first was a “clean” plate, which was not permitted to exhibit rust or particulates on the test surface. Prior to a test, the surface was wiped clean with a dry towel to remove any loose particulates or metal dust that might have settled.

The second was a “corroded” plate, which was left in the sunlight for a week with alternating doses of water and then mixtures of bleach and vinegar. Bleach and vinegar mixtures were not carefully controlled, but roughly equal parts bleach and vinegar should produce comparable results. Once evaporated, the bleach and vinegar mixture left behind salt deposits. These were removed by a warm water rinse, but no abrasive was used to preserve the rust formations. Figure 3 shows clean and corroded samples.
Because the thermal conductivity in steels are much lower than copper (about 51 W/m/K [31]), the embedded thermocouple is not a good estimate of the surface temperature. Presuming that the rate of heat addition was comparable to those measured in the copper disc tests, a model for the axisymmetric transient heating of a steel sample was used to calculate that starting 1.2 seconds into the process, the surface should be some 300 ± 80°C hotter than the thermocouple. A brief description of that calculation is in Appendix A. It should be noted that infrared thermometry was rejected as a means for measuring the steel surface temperature because of intense intermittent radiation produced by the iron oxides in the corroded plate tests.

3 Results and discussion

3.1 Ions originating in the flame

Figure 4 shows current-voltage characteristics of flames over a copper disc (coupon) while flow is held constant, but coupon temperature, diameter, and fuel/oxygen ratio are allowed to vary widely. The behaviors in regimes 1 and 2 agree with prior data, but regime 3 shows self-similarity never before observed in steel work piece tests.

The remarkable self-similarity of these data in regime 3 permits a number of important inferences. Independence on disc diameter demonstrates the flame starburst that spreads out over the plate surface does not participate; all transfer of charge occurs in the region where the core of the flame impinges on the plate surface. Combined with the insensitivity to fuel/oxygen ratio and plate temperature, this encourages the conclusion that the outer cone simply possesses a certain density of positive charge carriers, and when the electrons are driven away from the work, the rate of their delivery to the plate alone determines the current.

If that is so, then the positive saturation current should be strongly sensitive to the total gas flow rate. Figure 5 shows the current measured when 10V was applied between the torch and coupons of two diameters while the flow rate was varied. The line marked “flashback” represents the lowest flow rate that could be achieved without causing the flame to flash into the torch.

These data were collected both by applying 10V DC to the flame and by sampling only data at the top of a ±10V 5Hz sawtooth wave. In all cases, the measurements were collected at a 1kHz sample rate without filtration. The error bars are produced from the maximum and the minimum measurement found in the noise, and the markers are the mean values.

One could be tempted to interpret the data as linear were it not for the positive curvature evident in the extreme low-flow data. A linear trend passing through the origin would have indicated that there is a constant density of charged particles that is being delivered to the plate in proportion to the total flow of gas, so this paraboloid indicates that the density of charged particles must be declining with declining flow.

To determine how the charged particle count changes with flow, we divide the current (in fundamental charge units) by the total number of molecules that flow through the torch. This serves as an estimate of the fraction of molecules that carry charge. Of course, the mole count will not be preserved by chemical reaction, but (especially for methane) the total mole count is not likely to vary by more than an order of magnitude, and it should not have a significant dependence on flow rate. With a coefficient, \( A \), to account for the necessary conversion of units,
FIGURE 6. Ratio of charged particles to the number of reactant molecules versus total flow rate.

this provides

\[ \frac{N^+}{N_{cold}} = A \frac{I}{V}, \]  

where \( I \) is the saturation current, and \( V \) is the flow rate in standard volumetric units. It should be emphasized that standard volumetric units are actually a measure of molecular flow rate. When current is in \( \mu A \) and volume flow is in normal liters per minute, \( A \) is \( 13.9 \times 10^{-9} \).

Figure 6 shows a linear relationship between charge fraction and flow as calculated from Equation 3. If we take \( 30 \times 10^{-9} \) to be a typical scale for the charge fraction, we may estimate the charge density in an atmospheric flame (101.3kPa) at 3000K to be approximately \( 7 \times 10^{16} \) per cubic meter; a quantity consistent with Langmuir probe measurements taken inside of atmospheric propane flames [26], and quite typical for concentrations reported from a wide range of laboratory flames [14, pp406] including methane flames [23].

This reduction in the charge fraction at low flow rates is consistent with the longer time afforded the charged particles to recombine while they are in transit from the torch to the plate. Over a century ago, J. J. Thomson [32] observed that in a quasi-neutral flame, the concentrations of positive and negative charge carriers will be equal, \( n^+ = n^- = n \), so that if the formation of ions can be neglected, their number will decline according to

\[ \dot{n} = -\alpha n^2. \] (4)

Given some initial concentration formed in the inner cone, \( n_0 \), the local concentration of charged particles in a plug of gas traveling in the outer cone should decline with time according to

\[ n = (\alpha t + n_0^{-1})^{-1}. \] (5)

Without too great a leap in imagination, we may assert that the time, \( t \), over which the charges are allowed to recombine will be the torch standoff distance, \( s \), divided by the gas velocity, \( U \). If the term containing \( \alpha \) is large compared with the \( 1/n_0 \) term, we recover a linear relationship between fluid velocity and charge concentration above the work piece.

\[ n^- = n^+ \approx \frac{U}{\alpha s}. \] (6)

This is also consistent with prior observations that regime 3 currents rose when standoff was reduced [27]. These measurements are sufficient to indicate that lack of repeatability in prior measurements are attributed to a secondary source of ions at the work surface.

The reader may observe that we have argued that saturation occurs when there is a total evacuation of electrons at the work surface, which would seem to contradict the quasi-neutral plasma used to obtain the approximation in Equation 6. It must be emphasized that the evacuation of electrons does not occur globally, but in a thin sheath surrounding the so-called “absorbing” surface [14, 15, 25, 33].

3.2 Ions originating at the work surface

The evidence so far has served to demonstrate that ions are formed in the inner cone, partially recombine while traveling through the outer cone, and they impinge in low concentrations on the work piece. These have resulted in relatively low currents (about \( 20 \mu A \)) in the copper coupon tests, but tests over steel under comparable conditions have shown currents several times these levels [27], suggesting that there are other sources for charged particles in these cases.

Figure 7 shows the raw measurements of current with the torch at +10V DC while heating a clean plate. The gray line shows the data collected at a 1kHz sample rate, while the black line shows the same data after processing by a 41-element median value filter. The embedded zoomed plot shows that the noise (which was not present in the copper coupon tests) is comprised of brief individual peaks that persist for a sample interval or less. They are clearly asymmetrical (biased positive), and they are found approximately once every second or so. There is a lower level background noise that contains a pure tone varying between 170 to 190Hz, corresponding to a barely audible acoustic resonance in the torch hoses.

Prior to 200 seconds, except for the intermittent peaks, these data are entirely consistent with the currents measured in the copper coupon tests. However, at 200 seconds, there is an abrupt rise
of the median current and the frequency and magnitude of the intermittent peaks. In fact, starting between 250 and 300 seconds, the signal is dominated by these peaks, so that the median filter no longer rejects them.

The 20 μA at the start of the test is clearly due to the same transport of ions formed in the flame that were observed over copper. We attribute the intermittent peaks and the rise in median current to chemical action (specifically oxidation) at the work surface. The velocities in the outer cone should be roughly on the order of 100 m/s, and the size of the reaction zone is on the order of 10 mm, meaning the residence time of such particles near the work might only be 0.1 ms; quite consistent with the peaks observed in these data. The initial intermittent peaks are consistent with the formation and rapid oxidation of small metal particulates at the plate surface. As the work approaches its kindling temperature, more of the surface participates in oxidation, these events occur more frequently, and eventually they dominate the signal.

Figure 8 shows the median currents of both clean and corroded plates plotted against the plate surface temperature. Recall that the surface temperature is estimated by adding 300°C to the embedded thermocouple, and that a confidence no tighter than ±80°C should be attributed to these values. Both clean and corroded plates show an abrupt rise around 1050°C. While the clean plates demonstrate a flat initial current, corroded plates demonstrate a momentary current peak which decays away to the original 20 μA background. This difference between corroded and clean work pieces would seem to be due to chemical action with the protruding oxide formations.

The ignition temperatures of iron and steels in an oxygen atmosphere have been reported between 900 and 1000°C, with higher temperatures possible depending on the thickness of surviving oxide layers [34, 35]. The flame’s outer cone is not a pure oxygen atmosphere, so these temperatures cannot be directly compared to those of Figure 8. Just the same, the rise in current corresponds well with the temperatures at which oxygen can be expected to attack the metal surface.

If these currents really are due to reaction at the surface, then the presence of a substance that is so easily ionized as sodium chloride should produce a remarkable and similar result. Figure 9 shows two tests conducted on corroded samples which did not have their salt deposits removed. The response is a self-similar decay of current some ten times the magnitude of the currents observed over copper. The entire current-voltage characteristic mimics that of their unsalted counterparts, but exaggerated in positive magnitude.

Over the course of the tests shown in Figure 9 the superposition of current due to the salt deposits never decayed low enough to the characteristic rise seen in Figure 8. In addition to confirming that these signals are due to chemical action at the plate surface, this demonstrates that the high amounts of impurities on the work surface can dominate the currents due to reaction with the base metal.

4 Conclusions

4.1 Detecting ready-to-pierce

Voltages on the order of 10 V can be applied to an oxyfuel cutting torch, and the resulting current can be monitored to infer whether the work is ready for a pierce operation. Great care must
be taken in analyzing the signals, because these currents form from the superposition of three sources of ions:

1. Bombardment from ions produced in the inner cone;
2. Chemical action with surface particulates;
3. Chemical action with the base steel.

Only the last of these is any indication on the suitability for piercing.

Chemical action at the surface can arise from impurities attached to the metal surface (such as oils, metal dust, and especially salts). Of the samples considered here, only the deliberate deposition of extreme amounts of salt produced a signal in which the characteristic rise from the base material was not visible. Protruding oxide formations seem to have a preliminary signature that quickly subsides.

To develop a viable control system, it will be important to test whether these characteristic signals are extensible to fuels other than methane. Also, a focused study on the impact of protruding surface oxides would reveal whether there is a threshold for their severity that compromises the signal.

4.2 Detecting flame and flow

In cases where chemical action at the cathode can be ruled out, the remaining current is due entirely to bombardment of ions produced in the flame. These measurements demonstrate a regular parabolic curve with regard to total gas flow rate. If the torch were to be positioned over a chemically stable cathode (like copper) the cathode could serve as both an igniter and a sensing surface. A brief imposition of 10V on the torch could verify the presence of a stable flame and infer the total gas flow rate.

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A Thermal plate model

An estimate for the actual steel plate surface temperature is determined relative to the embedded thermocouple based on an analytical model for the transient heat conduction through a plate heated from above over a finite radius.

The cylindrical form of the conservation of energy in a solid provides a governing equation,

$$mc\frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) \tag{7}$$

where \(m\) is the material density, \(c\) is the specific heat, \(k\) is the thermal conductivity, \(T\) is the material temperature, \(r\) is the distance from the center of the axis of heating, and \(z\) is the depth in the plate (downward pointing z-axis).

If heat is added at a rate, \(\dot{Q}\), over an annulus with inner and outer radii, \(R_1\) and \(R_2\), the boundary condition on the upper surface is

$$\frac{\partial T}{\partial z}\bigg|_{z=0} = -\frac{\dot{Q}}{k\pi(R_2^2 - R_1^2)} \begin{cases} 1 : R_1 < r < R_2 \\ 0 : \text{otherwise} \end{cases} \tag{8}$$

At the centerline, and on the lower surface,

$$\frac{\partial T}{\partial r}\bigg|_{r=0} = \frac{\partial T}{\partial r}\bigg|_{z=L} = 0. \tag{9}$$

We also select some arbitrary choice of radius, \(R_\infty\), presumed to be “far” from the centerline, which we will hold equal to the initial temperature.

$$T|_{t=0} = T|_{r=R_\infty} = T_0 \tag{10}$$

The boundary condition at \(R_\infty\) is not a physical one; it is merely a convenient way to ensure a quasi-steady solution since convection is neglected in the upper and lower boundary conditions.

We adopt a nondimensionalization scheme based on the plate thickness, \(L\), a temperature scale, \(\Delta T\), the ambient temperature, \(T_0\), and a time scale, \(t_s\),

$$z = Lx \quad r = L\rho \quad t = t_s\tau \quad T = \theta\Delta T + T_0$$
The time and temperature scale can be related to the heating rate and the plate properties,

\[ t_s = \frac{mcL^2}{k} \quad \Delta T = \frac{\dot{Q}L}{k\pi(R_2^2 - R_1^2)}. \]

The method of separation of variables leads to a Bessel function expansion

\[ \theta = -\sum_{\gamma} a_{\gamma} J_0(\gamma \rho) \left[ \frac{\cosh \gamma(x - 1)}{\sinh \gamma} \right. \]

\[ - \frac{1}{\gamma} e^{-\gamma^2 x} \left. + \sum_{\beta} \frac{2\beta}{\alpha} e^{\alpha x} \cos \beta x \right] \]  

(11)

for parameters,

\[ \gamma = \frac{\lambda_1}{\rho_1}, \frac{\lambda_2}{\rho_2}, \frac{\lambda_3}{\rho_\infty}, \ldots \]  

(12)

\[ \beta = \pi, 2\pi, 3\pi, \ldots \]  

(13)

\[ \alpha = -\beta^2 - \gamma^2 \]  

(14)

\[ a_{\gamma} = \frac{2(\rho_2 J_1(\gamma \rho_2) - \rho_1 J_1(\gamma \rho_1))}{(\gamma \rho_\infty J_1(\gamma \rho_\infty))^2} \]  

(15)

where \( \lambda_k \) represents the series of Bessel function, \( J_0 \), roots. The parameters, \( \rho_1 \) and \( \rho_2 \) are the dimensionless inner and outer radii of heat addition, and \( \rho_\infty \) is the dimensionless outer radius.

Figure 10 shows the thermocouple location relative to the isotherms in steel at (top) steady-state, (bottom-left) 1.2 seconds into heating, and (bottom-right) 4.8 seconds into heating from a 1kW source over a half-inch diameter. While the isotherms process substantially as the plate heats, the shape near the heat source is quite stable.

In fact, analysis of the difference between the peak surface temperature and the thermocouple shows that after approximately 1 second into heating, there is a constant temperature difference that is independent of choice of boundary conditions, \( R_\infty \). It is, however, sensitive to the choice of heating intensity and shape.

Since these are approximated parameters, it is important to establish their impact on the estimate. Figure 11 shows the peak surface temperature difference from the sensor location versus heating radius. Each curve represents a different ratio between the inner radius and outer radius; 0 representing no inner radius, and 0.5 representing an annulus with an inner circle half the size of the outer.

From this, we conclude that the surface temperature of the steel plate may be approximated as 300°C above that of the thermocouple 1/8-inch below the plate surface. Because of the strong sensitivity to the shape of the heat addition, that estimate should not be trusted to better than ±80°C. Since the temperatures are on the order 1000°C, this amounts to 8% error.

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