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Tensor distinction of domains in ferroic crystals*

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Abstract. Ferroic crystals contain two or more domains and may be distinguished by the values of components of tensorial physical properties of the domains. We have extended Aizu's global tensor distinction by magnetization, polarization, and strain of all domains which arise in a ferroic phase transition to include distinction by toroidal moment, and from phases invariant under time reversal to domains which arise in transitions from all magnetic and non-magnetic phases. For determining possible switching of domains, a domain pair tensor distinction is also considered for all pairs of domains which arise in each ferroic phase transition.

PACS. 75.60.Ch Domain walls and domain structure – 77.80.Dj Domain structure; hysteresis – 61.50.Ks Crystallographic aspects of phase transformations; pressure effects

1 Introduction

A ferroic crystal contains two or more equally stable *domains*, volumes of the same homogeneous crystalline structure in different spatial orientations. The homogeneous bulk structure of these domains, in a polydomain sample, are referred to as *domain states*. Domain states may be distinguished by the values of components of macroscopic tensorial physical properties. Several disconnected domains may have the same domain state, consequently the distinction of domain states determines the distinction of all domains. Crystals in which the domain states may be distinguished by spontaneous polarization, magnetization, or strain are called *primary* ferroic crystals [1,2]. Crystals in which domain states are distinguished by the piezoelectric tensor is an example of a secondary ferroic crystal [3–5].

A ferroic crystal arises in a ferroic phase transition from a phase of higher symmetry to a phase of lower symmetry where there is a change in the point group symmetry. In Section 2 we use this change in point group symmetry to classify ferroic phase transitions extending Aizu's classification [6] to phase transitions from all non-magnetic and magnetic phases.

In Section 3, we consider the tensor distinction of domain states in a ferroic crystal. Let $\mathbf{T}_{\mathrm{MPP}}$ denote a macroscopic tensorial physical property. As the mathematical tensorial representation (vector, psudovector, rank two tensor etc.) of more than a single macroscopic tensorial physical property can be the same, we denote by $\mathbf{T}$ the type of mathematical tensorial representation corresponding to a macroscopic tensorial physical property. For example, magnetic, electric, and toroidal susceptibility are, using Jahn notation [7], tensors $\mathbf{T} = [V^2]$. We shall refer to $\mathbf{T}$ as a *tensor type*. We are interested in the distinction of the domain states which arise in a ferroic phase transition by the components of tensor types $\mathbf{T}$.

There are two types of tensor distinction which we shall consider:

(a) *Global tensor distinction*: we consider whether a tensor of type $\mathbf{T}$ can distinguish among all, some, or none of the domain states.
(b) *Domain pair tensor distinction*: for each pair of domain states, we consider whether or not a tensor of type $\mathbf{T}$ can distinguish between the two domain states of the domain pair.

Both of these distinction problems consider whether or not a tensor type as a whole can distinguish domain states without consideration of which tensor components can or cannot distinguish the domain states. If a tensor of a specific type can distinguish between domain states, then it would follow that one would want to know which components are the same and which are different in the domain states. We shall refer to this as *tensor component distinction* (Litvin [7]). While we do not intend to focus on this problem here, we comment on this in Section 4.

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Table 1. Groups of the reduced superfamily of $\mathbf{G} = \mathbf{2}_z/\mathbf{m}_z$. The right-hand-side column gives the magnetic point group in *primed* notation.

1. The group $\mathbf{G}$:	$\mathbf{2}_z/\mathbf{m}_z = \{1, 2_z, \bar{1}, m_z\}$	$= \mathbf{2}_z/\mathbf{m}_z$
2. The group $\mathbf{G1'}$:	$\mathbf{2}_z/\mathbf{m}_z\mathbf{1'} = \{1, 2_z, \bar{1}, m_z, 1', 2'_z, \bar{1}', m'_z\}$	$= \mathbf{2}_z/\mathbf{m}_z\mathbf{1'}$
3. Groups $\mathbf{G(H)}$:	$\mathbf{2}_z/\mathbf{m}_z(\mathbf{2}_z) = \{1, 2_z, \bar{1}', m'_z\}$	$= \mathbf{2}_z/\mathbf{m}_z{}'$
	$\mathbf{2}_z/\mathbf{m}_z(\bar{1}) = \{1, 2'_z, \bar{1}, m'_z\}$	$= \mathbf{2}_z{}'/\mathbf{m}_z{}'$
	$\mathbf{2}_z/\mathbf{m}_z(\mathbf{m}_z) = \{1, 2'_z, \bar{1}', m_z\}$	$= \mathbf{2}_z{}'/\mathbf{m}_z$

2 Point group classification of ferroic phase transitions

Consider a ferroic phase transition from a phase of higher symmetry to a phase of lower symmetry where there is a change in the point group symmetry. The phase transition can then be characterized by the pair of point groups in the two phases. As our interest includes magnetically ordered crystals, we consider the magnetic point group symmetry of the phases.

Opechowski [8] has classified the 122 classes of magnetic point groups into *reduced superfamilies*: let $\mathbf{G}$ denote a group of one of the 32 classes of non-magnetic crystallographic point groups. The reduced superfamily of the group $\mathbf{G}$ consists of:

1. The group $\mathbf{G}$.
2. The group $\mathbf{G1'}$ where the time inversion group $\mathbf{1'}$ consists of the identity 1 and time inversion $1'$.
3. One group of each class of groups $\mathbf{G(H)} = \mathbf{H} + (\mathbf{G} - \mathbf{H})1'$, where $\mathbf{H}$ is a subgroup of index two of $\mathbf{G}$.

As an example, the reduced superfamily of the group $\mathbf{G} = \mathbf{2}_z/\mathbf{m}_z$ is given in Table 1. A complete listing of the 122 types of magnetic point groups in groupings of reduced superfamilies can be found in [8,9], and other listings, not in groupings of reduced superfamilies in [10,11]. A computerized tabulation of group theoretical properties of these magnetic point groups, including products of operations, subgroups, and coset and double-coset decompositions has been given by Schlessman and Litvin [12].

We shall use the Aizu symbol $\mathbf{M}F\mathbf{N}$ to characterize a ferroic phase transition consisting of the point group $\mathbf{M}$ of the high symmetry phase, the letter F to denote a ferroic phase transition, and the point group $\mathbf{N}$ of the low symmetry phase. While $\mathbf{M}F\mathbf{N}$ will be used as a general symbol for such a phase transition, we shall categorize these symbols into three kinds $\mathbf{G}F\mathbf{Sub}$, $\mathbf{G1'}F\mathbf{Sub}$, and $\mathbf{G(H)}F\mathbf{Sub}$ to correspond to the three kinds of magnetic point groups of the high symmetry phase. The symbol $\mathbf{Sub}$ represents a subgroup of the point group on the left-hand-side of the symbol F.

Phase transitions $\mathbf{M}F\mathbf{N}$ can be classified into classes as follows: let $\mathbf{M}$ denote a group of one of the 122 magnetic point group types, and let $\mathbf{M}F\mathbf{N}_1$ denote a ferroic phase transition. The classification into types depends on the criterion chosen to decide if a second ferroic phase transition $\mathbf{M}F\mathbf{N}_2$ belongs to the same class as $\mathbf{M}F\mathbf{N}_1$. Aizu [13] has introduced four criteria. Two ferroic phase transitions $\mathbf{M}F\mathbf{N}_1$ and $\mathbf{M}F\mathbf{N}_2$ belong to the same class of phase transitions if

(c1) $\mathbf{N}_1 = \mathbf{N}_2$.
(c2) $\mathbf{N}_1$ and $\mathbf{N}_2$ are conjugate subgroups of $\mathbf{M}$, i.e. there is an operation m of $\mathbf{M}$ such that $m\mathbf{N}_1 m^{-1} = \mathbf{N}_2$.
(c3) There is an operation r of the three-dimensional rotation group $\mathbf{R}$, not necessarily contained in $\mathbf{M}$, such that $\mathrm{r}\mathbf{M}\mathrm{r}^{-1} = \mathbf{M}$ and $\mathrm{r}\mathbf{N}_1\mathrm{r}^{-1} = \mathbf{N}_2$.
(c4) $\mathbf{N}_1$ and $\mathbf{N}_2$ belong to the same type of point groups, there is an operation r of $\mathbf{R}$ such that $\mathrm{r}\mathbf{N}_1\mathrm{r}^{-1} = \mathbf{N}_2$.

According to which of the four criteria is used, for non-magnetic phase transitions $\mathbf{G}F\mathbf{Sub}$ there are, respectively, 433, 247, 212, and 190 classes of ferroic phase transitions. (The 247 classes, using criterion c2, of non-magnetic phase transitions $\mathbf{G}F\mathbf{Sub}$ have been tabulated by Litvin [7].) Following Aizu [6], we shall use here criterion c3. Using this criterion, there are 212 classes, or species, of phase transitions $\mathbf{G}F\mathbf{Sub}$ [6,14], 773 classes $\mathbf{G1'}F\mathbf{sub}$ [6,14], and 616 classes $\mathbf{G(H)}F\mathbf{sub}$ [14]. In Table 2 we list all classes of all three kinds of ferroic phase transitions with the point group $\mathbf{G} = \mathbf{m}_x\mathbf{m}_y\mathbf{2}_z$.

3 Tensor distinction

In a ferroic phase transition $\mathbf{M}F\mathbf{N}$, if a domain appears in the low symmetry phase, it can appear as any one of $n = |\mathbf{M}|/|\mathbf{N}|$ *single domain states* $S_1, S_2, \ldots, S_n$ where $|\mathbf{M}|$ and $|\mathbf{N}|$ denote the number of operations in $\mathbf{M}$ and $\mathbf{N}$, respectively. Single domain states have the same crystalline structure and differ only in their orientation in space. The orientations of all single domain states are related by operations of the high symmetry point group $\mathbf{M}$. *Domain states* will refer to the bulk structures, with their specific orientations in space of domains in a polydomain sample. In nonferroelastic polydomain phases, the orientation of each domain state coincides with the orientation of a single domain state. The number of domain states is therefore the same as the number of single domain states.

In ferroelastic polydomain phases disorientations, rotations of domains, arise as a result of the requirement that neighboring domains in a polydomain sample must meet along a coherent boundary. Consequently, domain states in general differ in their orientation from single domain states, their number is in general greater than the number of single domain states, and they are not all related by operations of $\mathbf{M}$. We shall consider here

Table 2. Classes of ferroic phase transitions $\mathbf{G}F\mathbf{Sub}$, $\mathbf{G1'}F\mathbf{sub}$, and $\mathbf{G(H)}F\mathbf{sub}$ with $\mathbf{G} = \mathbf{m}_x\mathbf{m}_y\mathbf{2}_z$. "T", "M", "P", and "$\varepsilon_{ij}$" denote toroidal moment, magnetization, polarization and strain tensor types, respectively. At the intersection of a column headed by a tensor type and a row with a ferroic phase transition class symbol $\mathbf{M}F\mathbf{N}$ is the global tensor distinction by that tensor type of that class of ferroic phase transitions. "F", "P", "N", and "Z" denotes, respectively, full, partial, null, and zero tensor distinction.

		T	M	P	ε_{ij}
$\mathbf{G}F\mathbf{Sub}$:	$\mathbf{m}_x\mathbf{m}_y\mathbf{2}_zF\mathbf{1}$	F	F	F	F
	$\mathbf{m}_x\mathbf{m}_y\mathbf{2}_zF\mathbf{2}_z$	N	F	N	F
	$\mathbf{m}_x\mathbf{m}_y\mathbf{2}_zF\mathbf{m}_x$	F	F	F	F
$\mathbf{G1'}F\mathbf{sub}$:	$\mathbf{m}_x\mathbf{m}_y\mathbf{2}_z\mathbf{1'}F\mathbf{1}$	F	F	F	F
	$\mathbf{m}_x\mathbf{m}_y\mathbf{2}_z\mathbf{1'}F\mathbf{1'}$	Z	Z	F	F
	$\mathbf{m}_x\mathbf{m}_y\mathbf{2}_z\mathbf{1'}F\mathbf{2}_z$	P	P	N	F
	$\mathbf{m}_x\mathbf{m}_y\mathbf{2}_z\mathbf{1'}F\mathbf{2}_z'$	F	F	N	F
	$\mathbf{m}_x\mathbf{m}_y\mathbf{2}_z\mathbf{1'}F\mathbf{2}_z\mathbf{1'}$	Z	Z	N	F
	$\mathbf{m}_x\mathbf{m}_y\mathbf{2}_z\mathbf{1'}F\mathbf{m}_x$	F	P	F	F
	$\mathbf{m}_x\mathbf{m}_y\mathbf{2}_z\mathbf{1'}F\mathbf{m}_x'$	P	F	F	F
	$\mathbf{m}_x\mathbf{m}_y\mathbf{2}_z\mathbf{1'}F\mathbf{m}_x\mathbf{1'}$	Z	Z	F	F
	$\mathbf{m}_x\mathbf{m}_y\mathbf{2}_z\mathbf{1'}F\mathbf{m}_x\mathbf{m}_y\mathbf{2}_z$	F	Z	N	N
	$\mathbf{m}_x\mathbf{m}_y\mathbf{2}_z\mathbf{1'}F\mathbf{m}_x'\mathbf{m}_y\mathbf{2}_z'$	F	F	N	N
	$\mathbf{m}_x\mathbf{m}_y\mathbf{2}_z\mathbf{1'}F\mathbf{m}_x'\mathbf{m}_y'\mathbf{2}_z$	Z	F	N	N
$\mathbf{G(H)}F\mathbf{sub}$:	$\mathbf{m}_x'\mathbf{m}_y\mathbf{2}_z'F\mathbf{1}$	F	F	F	F
	$\mathbf{m}_x'\mathbf{m}_y\mathbf{2}_z'F\mathbf{2}_z'$	F	F	N	F
	$\mathbf{m}_x'\mathbf{m}_y\mathbf{2}_z'F\mathbf{m}_x'$	F	F	F	F
	$\mathbf{m}_x'\mathbf{m}_y\mathbf{2}_z'F\mathbf{m}_y$	F	N	F	F
	$\mathbf{m}_x'\mathbf{m}_y'\mathbf{2}_zF\mathbf{1}$	F	F	F	F
	$\mathbf{m}_x'\mathbf{m}_y'\mathbf{2}_zF\mathbf{2}_z$	F	N	N	F
	$\mathbf{m}_x'\mathbf{m}_y'\mathbf{2}_zF\mathbf{m}_x'$	F	F	F	F

ferroelastic polydomain phases in the so called high symmetry approximation [15], also called the parent-clamping approximation [16], which disregards the disorientations. As we disregard the disorientations, the number and orientations of domain states in ferroelastic polydomain samples also coincides with the number and orientations of single domain states, as in nonferroelastic polydomain samples, and are related by operations of $\mathbf{M}$.

As the set of domain states represents the structure of all domains in a polydomain phase, we consider the tensor distinction of the domain states. We denote by T_i, $i = 1, 2, \ldots, n$, the form of the tensors of a tensor type $\mathbf{T}$ in the set of domains states of a polydomain sample. The tensors T_i, $i = 1, 2, \ldots, n$, are all given in a single coordinate system, e.g. the coordinate system of the high symmetry phase or of one of the domain states. A tensor type $\mathbf{T}$ is said to be able to distinguish between two domain states, with corresponding tensors T_i and T_j of type $\mathbf{T}$, if $T_i \neq T_j$.

3.1 Global tensor distinction

We consider here whether or not tensors T_i, $i = 1, 2, \ldots, n$, of the type $\mathbf{T}$ can distinguish among all domain states S_i, $i = 1, 2, \ldots, n$, which arise in a ferroic phase transition. Following the terminology of Aizu [6], if the set of tensors T_i, $i = 1, 2, \ldots, n$, are all distinct, then we say that the tensor type $\mathbf{T}$ provides a *full* distinction of the domain states S_i, $i = 1, 2, \ldots, n$. If the set of tensors T_i, $i = 1, 2, \ldots, n$, are not all distinct but not all identical, then we say that the tensor type $\mathbf{T}$ provides a *partial* distinction. If the tensors are all identical, then the terminology is that a type $\mathbf{T}$ tensor provides a *null* distinction. Litvin [17] subdivided the "null" category into two: if the tensors are all identical and non-zero the terminology remains "null distinction". If the tensors are all identical and zero then the terminology becomes "zero distinction". In tabulations full, partial, null, and zero distinction are denoted, respectively, by the letters "F", "P", "N", and "Z".

A method of determining the global tensor distinction was given by Litvin [17] and the global tensor distinction of classes of ferroic phase transitions $\mathbf{G}F\mathbf{Sub}$ for all tensor types $\mathbf{T}$ invariant under time inversion of rank $n \leq 4$ is given in [18]. Aizu [6] has given the global tensor distinction of classes of ferroic phase transitions $\mathbf{G1'}F\mathbf{Sub}$ for magnetization, polarization, and strain. This was extended by Litvin [19] to include global distinction of classes $\mathbf{G1'}F\mathbf{Sub}$ by toroidal moment. In addition, in [19] there was given inverse tables to look up which ferroic phase transitions $\mathbf{G1'}F\mathbf{Sub}$ have particular kinds of distinctions by these four tensor types, and also the relationship to Schmid's concept [20] of ensembles of species.

Here we have extend the global distinction via toroidal moment, magnetization, polarization, and strain tensor types of ferroic phase transitions from those transitions $\mathbf{G1'}F\mathbf{Sub}$ to all classes $\mathbf{M}F\mathbf{N}$, i.e. to classes $\mathbf{G}F\mathbf{Sub}$ and $\mathbf{G(H)}F\mathbf{Sub}$ [14]. Examples of this global tensor distinction are given in Table 2.

3.2 Domain pair tensor distinction

Consider a ferroic phase transition $\mathbf{M}F\mathbf{N}$. Let $S_1, S_2, \ldots, S_n$ denote the domain states of the lower symmetry phase, tensors $\mathbf{T}$ a tensor type, and tensors T_i, $i = 1, 2, \ldots, n$, the form of the tensor type $\mathbf{T}$ in the domain states $S_1, S_2, \ldots, S_n$. The domain states are related by operations of $\mathbf{M}$ not in $\mathbf{N}$. We write the group $\mathbf{M}$ as a coset decomposition [27] with respect to the subgroup $\mathbf{N}$:

$$\mathbf{M} = \mathbf{N} + m_2\mathbf{N} + m_3\mathbf{N} + \ldots + m_n\mathbf{N}. \qquad (1)$$

The operations m_i, $i = 1, 2, \ldots, n$, of $\mathbf{M}$ are called the *coset representatives* of the coset decomposition. We denote the symmetry group of domain state S_i by $\mathbf{N}_i$. Assuming that the domain state S_1 is invariant under $\mathbf{N}_1 = \mathbf{N}$, the other domain states are related to S_1 by $S_i = m_i S_1$, and their symmetry groups are given by $\mathbf{N}_i = m_i \mathbf{N} m_i^{-1}$.

We consider the tensor distinction of ordered domain pairs $\{S_i, S_j\}$ $i \neq j$ by the tensors T_i and T_j. The tensors T_i and T_j can be determined as follows: T_i is the form of the tensor type $\mathbf{T}$ invariant under $\mathbf{N}_i$ and T_j can be determined from T_i via $T_j = m_{ij}T_i$, where m_{ij} is an operation of M which transforms the domain state S_i into the domain state S_j, i.e. $S_j = m_{ij}S_i$. Consequently, in a ferroic phase transition $\mathbf{M}F\mathbf{N}$ the tensor distinction of a domain pair $\{S_i, S_j\}$ is determined by the magnetic point group $\mathbf{N}_i$ and the operation m_{ij} of $\mathbf{M}$.

Instead of determining the tensor distinction of every domain pair, we first classify all possible domain pairs into classes as follows [7]: two domain pairs $\{S_i, S_j\}$, whose tensor distinction is determined by the magnetic point group $\mathbf{N}_i$ and the operation m_{ij}, and $\{S_p, S_q\}$, whose tensor distinction is determined by the magnetic point group $\mathbf{N}_p$ and the operation m_{pq}, are said to be in the same class of domain pair if there exists an operation r of the three-dimensional rotation group $\mathbf{R}$ such that:

$$r\mathbf{N}_i r^{-1} = \mathbf{N}_p \text{ and } r m_{ij}\mathbf{N}_i r^{-1} = m_{pq}\mathbf{N}_p$$

$m_{ij}\mathbf{N}_i$ represents the set of *all* operations of M which transform the domain state S_i into domain state S_j. Two domain pair which belong to the same class of domain pairs have the same domain pair tensor distinction, i.e. are distinguished by the same set of physical property tensors. This classification into classes of domain pairs which arise in a ferroic phase transition $\mathbf{M}F\mathbf{N}$ is also a classification into classes of the pairs $\{\mathbf{N}_i, m_{ij}\mathbf{N}_i\}$ which arise in the same transition, and there is a one-to-one correspondence between the two sets of classes.

For each class of ferroic phase transitions $\mathbf{M}F\mathbf{N}$ one can list [21] one pair from each class of pairs $\{\mathbf{N}_i, m_{ij}\mathbf{N}_i\}$. This one pair can be chosen as a pair of the form $\{\mathbf{N}, m_{1j}\mathbf{N}\}$ which represents the information necessary to determine the tensor distinction between the pair of domains in the domain pair $\{S_1, S_j\}$ and all domain pair in the class to which this domain pair belongs. Table 3 gives examples of such listings. Tensor pair distinction provides information on the distinction of domain states which is not provided by global distinction. For example, for the phase transition $\mathbf{m3\text{-}m1'}F\mathbf{3m1'}$ of perovskite $BiFeO_3$ [28] the domain pairs fall into three classes of domain pairs. All domain pairs can not be distinguished by toroidal moment or magnetization, but can be by polarization. Global tensor distinction "P" for strain means some domain pairs can be distinguished by strain and some can not. The domain pair tensor distinction shows that only domain pairs not distinguished by strain are those with domains related by spatial inversion. A second example is the phase transition $\mathbf{M}F\mathbf{N} = \mathbf{m}_x\mathbf{'}\mathbf{m}_y\mathbf{'}\mathbf{m}_z F\mathbf{2}_z$, the global tensor distinction does not provide information to decide if there are pairs of domains which can be distinguished simultaneously by toroidal moment and polarization, since both physical properties have a partial global tensor distinction. Looking at the tensor pair distinction of the domain states which arise in this phase transition, see Table 3, one has that there are domain pairs which can be distinguished simultaneously by toroidal moment and polarization and other pairs which can be distinguished by one but not the other of these two physical properties. (For a complete listing of the tensor pair distinction for all ferroic phase transitions see [14].)

There is a special case where global tensor distinction is identical with tensor pair distinction. This is the case of *transposable* phase transitions $\mathbf{M}F\mathbf{N}$ where $\mathbf{N}$ is a subgroup of index two of the group $\mathbf{M}$. There are 380 classes transposable ferroic phase transitions $\mathbf{M}F\mathbf{N}$ [21]. Of these, 141 classes are non-ferroelastic magneto-electric transitions, i.e. the spontaneous strain is the same in the two domain states which arise in the transition and the magneto-electric tensor distinguishes between the two domain states [22]. A second subset is the 309 classes of transposable magnetic ferroelastic transitions [23].

4 Tensor component distinction

To determine the tensor component distinction, i.e. the specific components of a tensor type $\mathbf{T}$ which provide distinction in a ferroic phase transition $\mathbf{M}F\mathbf{N}$, one needs the set of tensor forms $\{T, m_2T, \ldots, m_nT\}$ where T is the form of the tensor type $\mathbf{T}$ invariant under $\mathbf{N}$ and m_2, $m_3, \ldots m_n$ are a set of coset representatives, see equation (1), of the coset decomposition of $\mathbf{M}$ with respect to $\mathbf{N}$. The form T of 36 magnetic and non-magnetic physical property tensors of rank 0, 1, 2, and 3 which are invariant under each of the 656 subgroups of $6_z/m_z m_x m_1 1'$ and $m_z\overline{3}_{xyz}m_{xy}1'$ has been tabulated [24]. The remaining

Table 3. For three ferroic phase transitions $\mathbf{M}F\mathbf{N}$ we give the global tensor distinction (GTD) with respect to the four physical properties toroidal moment (T), magnetization (M), polarization (P) and strain (ε_{ij}). For each we list the point group $\mathbf{N}$ and set of operations $m_{1j}\mathbf{N}$ of one domain pair from each class of domain pairs arising in the phase transition $\mathbf{M}F\mathbf{N}$ along with the domain pair tensor distinction (DPTD) of the domain pairs in that class.

						T	M	P	ε_{ij}
$\mathbf{M}F\mathbf{N} = \mathbf{m3\text{-}m1'}F\mathbf{3m1'}$					GTD:	Z	Z	F	P
$\mathbf{N}$	$m_{1j}\mathbf{N}$						DPTD:		
$\mathbf{3(xyz)m(x\text{-}y)1'}$	2_x	$3_{xy\bar{z}}$	$3^2_{x\bar{y}z}$						
	m_{yz}	$\bar{4}^3_y$	$\bar{4}_z$						
	$2'_x$	$3'_{xy\bar{z}}$	${3^2_{x\bar{y}z}}'$						
	m'_{yz}	$\bar{4}^{3\prime}_y$	$\bar{4}'_z$			Z	Z	F	F
$\mathbf{3(xyz)m(x\text{-}y)1'}$	$\bar{1}$	$\bar{3}^5_{xyz}$	$\bar{3}_{xyz}$						
	$2_{\bar{x}y}$	$2_{\bar{x}z}$	$2_{\bar{y}z}$						
	$\bar{1}'$	${\bar{3}^5_{xyz}}'$	${\bar{3}_{xyz}}'$						
	$2'_{\bar{x}y}$	$2'_{\bar{x}z}$	$2'_{\bar{y}z}$			Z	Z	F	N
$\mathbf{3(xyz)m(x\text{-}y)1'}$	m_x	$\bar{3}_{xy\bar{z}}$	$\bar{3}^5_{x\bar{y}z}$						
	2_{yz}	4^3_y	4_z						
	m'_x	$\bar{3}'_{xy\bar{z}}$	${\bar{3}^5_{x\bar{y}z}}'$						
	${2_{yz}}'$	$4^{3\prime}_y$	${4_z}'$			Z	Z	F	F
$\mathbf{M}F\mathbf{N} = \mathbf{m}_x{}'\mathbf{m}_y{}'\mathbf{m}_z F \mathbf{2}_z$					GTD:	P	N	P	P
$\mathbf{N}$	$m_{1j}\mathbf{N}$						DPTD:		
$\mathbf{2}_z$	$\bar{1}$	m_z				F	N	F	N
$\mathbf{2}_z$	$2'_x$	$2'_y$				N	N	F	F
$\mathbf{2}_z$	m'_x	m'_y				F	N	N	F
$\mathbf{M}F\mathbf{N} = \mathbf{4}_z/\mathbf{m}_z\mathbf{1}'F\mathbf{2}_z/\mathbf{m}_z{}'$					GTD:	P	Z	Z	F
$\mathbf{N}$	$m_{1j}\mathbf{N}$						DPTD:		
$\mathbf{2}_z/\mathbf{m}_z{}'$	4_z	4^3_z	${\bar{4}_z}'$	$\bar{4}^{3\prime}_z$		N	Z	Z	F
$\mathbf{2}_z/\mathbf{m}_z{}'$	$\bar{1}$	m_z	$1'$	$2'_z$		F	Z	Z	N
$\mathbf{2}_z/\mathbf{m}_z{}'$	$\bar{4}_z$	$\bar{4}^3_z$	$4'_z$	$4^{3\prime}_z$		F	Z	Z	F

tensor forms in the set are m_iT, $i = 2, 3, \ldots, n$, where m_iT represents the transformation of the tensor form T under the operation m_i [25]. As for sets of tensor forms $\{T, m_2T, \ldots, m_nT\}$, only tabulations [26] for the physical properties magnetization and polarization and phase transitions $\mathbf{M}F\mathbf{N}$ where $\mathbf{M}$ is one of those 656 groups is known to this author.

To determine the specific components of a tensor type $\mathbf{T}$ which provide the domain pair distinction in the class $\{\mathbf{N}, m_{1j}\mathbf{N}\}$ requires the form T of the tensor type $\mathbf{T}$ invariant under $\mathbf{N}$ and $m_{1j}T$. For the 141classes of transposable non-ferroelastic magneto-electric transitions, such a tensor component distinction has been given for the twelve physical property tensors listed below with their Jahn notation [22]:

V	polarization
aeV	magnetization
$[V^2]$	strain
$[V^2]$	electric susceptibility
$[V^2]$	magnetic susceptibility
aeV^2	magnetoelectric susceptibility
$V[V^2]$	piezoelectric coefficient
$aeV[V^2]$	piezomagnetic coefficient
$[V^3]$	non-linear electric susceptibility
$ae[V^3]$	non-linear magnetic susceptibility
$aeV[V^2]$	first non-linear magnetoelectric susceptibility
$V[V^2]$	second non-linear magnetoelectric susceptibility

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