SUBPERIODIC SUBGROUPS OF SPACE GROUPS
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A subperiodic group (SPG) $G^m_n$ ($m < n$) describes the symmetry of an object with $m$-dimensional periodicity in $n$-dimensional Euclidean space $E^n$. Crystallographic SPG's appear as subgroups of space groups $G^n_n$. For $n=3$ systematic attention has been paid only to site symmetries of points in $E^3$ with respect to $G^3_3$ described by site point groups $G^3_0$. Layer ($G^3_2$) and rod ($G^3_1$) subgroups of $G^3_3$ can be analogously treated as symmetries of planes and lines with respect to $G^3_3$.

We show how to generalize the concepts of orbits, Wyckoff positions and Wyckoff sets for planes and lines and how to determine and systematize corresponding layer and rod symmetries. We introduce classes (orbits) of crystallographically equivalent planes and lines analogously to crystallographic orbits of points. While the points are characterized by their location, the planes and lines are specified by location and orientation. Their orbits further split into suborbits with the same orientation. For a fixed orientation of planes and lines, their layer and rod symmetries with respect to a certain space group $G^3_3$ of point class $G$ are subgroups of an equitranslational subgroup $H$ of $G^3_3$ of the point class $H$ which preserves the orientation. The group $H$ is then reducible and the layer and rod symmetries are determined by scanning procedure /1/. Reducible space groups /2/ can be factorized by one or two dimensional translation subgroups which determine the scanning direction. The resulting factor groups are isomorphic to layer and rod groups which are in the same relationship with symmetries of planes and lines as the point group of a space group with symmetries of points. The whole scheme completes and refines the knowledge of groups of three-dimensional crystallography.

SPG's $G^3_2$ describe symmetry of interfaces, domain walls, twin planes, plane defects, crystal surfaces etc, $G^3_1$ specify symmetry of plane intersections, line defects, crystal edges etc.