TRANSPOSABLE DOMAIN PAIRS AND
DOMAIN DISTINCTION

V. JANOVEC,† D. B. LITVIN‡ and J. FUKSA†
†Institute of Physics, Academy of Sciences of the Czech Republic, Na Slovance 2,
180 40 Prague 8, Czech Republic;
‡Department of Physics, The Pennsylvania State University, The Berks Campus,
P.O. Box 7009, Reading, PA 19610-6009, U.S.A.

(Received September 6, 1994)

We divide pairs of domain states into three classes: completely, partially and non-transposable domain
pairs. We show that two groups can be associated with a domain pair: the twinning group and the
symmetry group of the pair. The twinning group determines which secondary order parameters are the
same and which are different in two domain states of a domain pair. The symmetry group of a trans-
posable domain pair allows one to express the order parameters and irreducible constituents of material
property tensors in such a way that their components in two domain states are either the same or differ
only in the sign. The analysis of domain distinction is illustrated on a simple example.

Keywords: Domain structures, ferroelastic domains, tensor distinction of domains, domain
pairs, twinning group of a domain pair.

1. INTRODUCTION

Domain bulks (domain states) of two domains simultaneously observed by a single
apparatus can exhibit different properties although their crystal structures are the
same and differ only in their spatial orientation. Which properties are the same
and which are different depends on the symmetry of one domain state and on the
spatial relation between both domain states. We have shown that for all non-
deroelastic domain pairs¹ and for a class of ferroelastic domain pairs called com-
pletely transposable² this information can conveniently be expressed by a dichro-
matic (black and white) crystallographic point group and that the tensor components
distinct in two domain states differ only in the sign.

In this paper we indicate how the analysis can be extended to other domain
pairs. First, we divide domain pairs according to their internal symmetry expressed
by possible transpositions of domain states. Then we discuss the distinction of
domain states in terms of the primary and secondary order parameters of the phase
transition and demonstrate the significance of so called twinning group in distin-
guishing domain states according to the secondary order parameters. Finally, we
show how the symmetry of a domain pair can be utilized in domain distinction.
The exposition is accompanied by a simple example which illustrates the main steps
of the analysis. Similarly as in the preceding papers¹,² we confine ourselves to the
continuum description and point groups symmetry.
2. SYMMETRY GROUP AND TWinning GROUP OF A DOMAIN PAIR

We consider crystalline domains which arise in a phase transition from a parent (ordered) phase of symmetry $G$ to a distorted (disordered) phase of symmetry $F$, where $F$ is a subgroup of $G$, $F < G$. We shall refer to the bulk structures of these domains in polydomain samples as domain states. Several disconnected domains of possibly different shape can have the same domain state. Consequently, domain states of a polydomain sample represent structures that appear in the sample, irrespective of in which domain and irrespective of the domain's shape. We shall confine ourselves to single domain states which we shall denote $S_1, S_2, \ldots, S_n$. Their number $n$ equals the index of $F$ in $G$, $n = |G| : |F|$, where $|G|$ and $|F|$ is the number of symmetry operations of $G$ and $F$, respectively. Single domain states are symmetrically equivalent in the group $G$, i.e. each two domain states can be related by an operation $g$ from $G$.

Most often, domains are distinguished by their bulk properties, i.e. according to their domain states. Then the problem of domain distinction is reduced to the distinction of domain states. To solve this task we have to describe in a convenient way the distinction of any two of all possible $n$ domain states. For this purpose we use the concept of a domain pair denoted by $\{S_i, S_j\}$ and defined as an unordered set consisting of two domain states $S_i$ and $S_j$.

Let us first examine the symmetry of a domain pair $\{S_i, S_j\}$. If $F_i$ and $F_j$ are the symmetry groups of $S_i$ and $S_j$, respectively, then any operation $f$ that belongs both to $F_i$ and to $F_j$, $f \in F_i \cap F_j = F_{ij}$, is a symmetry operation of $\{S_i, S_j\}$. If, moreover, there exists such $g_{ij}^* \in G$ which transposes (interchanges) $S_i$ and $S_j$, $g_{ij}^* S_i = S_j$, $g_{ij}^* S_j = S_i$, then all operations from the left coset $g_{ij}^* F_{ij}$ do so as well (the asterisk denotes operations interchanging two domain states). These operations are also symmetry operations of $\{S_i, S_j\}$ since for unordered sets (domain pairs) $\{S_i, S_j\} = \{S_j, S_i\}$. Thus the symmetry group $J_{ij}$ of the domain pair $\{S_i, S_j\}$ can be, in a general case, expressed in the following way:

$$J_{ij} = F_{ij} + g_{ij}^* F_{ij}. \quad (1)$$

Domain pairs can be classified according to their symmetry $J_{ij}$. Pairs for which $g_{ij}^* F_{ij}$ exists we call transposable (or ambivalent') domain pairs, pairs for which an interchanging operation $g_{ij}^*$ cannot be found are called non-transposable (or polar') domain pairs. The symmetry of a non-transposable pair is reduced to $F_{ij}$.

All operations that transform $S_i$ into $S_j$ (irrespective how they transform $S_j$) are contained in the left coset $g_{ij} F_{ij}$ since $g_{ij} F_i S_i = g_{ij} S_i = S_j$. If $\{S_i, S_j\}$ is a transposable pair and, moreover, $F_i = F_j = F_{ij}$ then all operations of the left coset $g_{ij} F_{ij}$ transform simultaneously $S_i$ into $S_j$. We call such pairs completely transposable domain pairs. The symmetry group $J_{ij}$ of a completely transposable pair $\{S_i, S_j\}$ is

$$J_{ij} = F_i + g_{ij}^* F_i. \quad (2)$$

If $F_i \neq F_j$ then $F_{ij} < F_i$ and the number of transposing operations of a transposable domain pair is smaller than the number of operations transforming $S_i$ into $S_j$. We call, therefore, such pairs partially transposable domain pairs.

The transformation properties of a domain pair $\{S_i, S_j\}$ are described by the symmetry group $F_i$ of $S_i$ and by all operations $g_{ij} F_i$ that transform $S_i$ into $S_j$. The
smallest subgroup $K_{ij}$ of $G$ that contains both $F_i$ and $g_y F_i$ will be called the twinning group of the domain pair $(S_i, S_j)$. For the completely transposable pairs the union of $F_i$ and $g_y F_i$ forms already a group, $K_{ij} = F_i + g_y F_i$, which is identical with the symmetry group $J_{ij}$ of the domain pair (see Equation (2)), $K_{ij} = J_{ij}$. For the partially transposable and non-transposable domain pairs the group $K_{ij}$ comprises besides $F_i$ and $g_y F_i$ further left cosets and can be written in the following form:

$$K_{ij} = F_i + g_y F_i + \cdots + g_y^k F_i.$$  \[(3)\]

The twinning group $K_{ij}$ plays a basic role in the symmetry analysis of domain pairs.

---

**FIGURE 1** Exploded view of the single domain states of the distorted phase resulting from the phase transition with the symmetry reduction 4/mmm \( \not\sim \) 2mm. Four single domain states 1, 2, 3, and 4 are represented by the solid rectangles, the arrows correspond to the spontaneous polarization (primary order parameter of the transition). The dashed square in the center represents the parent phase. Single domain states are related by depicted symmetry operations of the parent phase.

---

**FIGURE 2** Graphical representation of domain pairs \{1, 2\} and \{1, 3\} depicted as two superimposed domain states. Symmetry operations marked by an asterisk interchange the two domain states of the domain pair.
To illustrate our exposition we consider a phase transition in which the symmetry group of the parent phase is \( G = 4/mmm \) and the domain state \( S_1 \) in the distorted phase has the symmetry \( F_1 = 2_\infty m \). This is a proper (full) ferroelectric and improper (partial) ferroelastic transition with spontaneous polarization as the primary order parameter and spontaneous deformation as the secondary order parameter. The distorted phase can appear in \( n = \{4/mmm\} : \{2mm\} = 16 : 4 = 4 \) single domain states \( S_1 = 1, S_2 = 2, S_3 = 3, S_4 = 4 \) which are presented in a graphical form in Figure 1. From these domain states one can form six non-trivial domain pairs which can be divided into two classes of crystallographically equivalent pairs. The representative domain pairs of these classes are \( \{1, 2\} \) with perpendicular polarization and \( \{1, 3\} \) with antiparallel polarization. Both these domain pairs are represented graphically in Figure 2 as superpositions of corresponding domain states.

The domain pair \( \{1, 2\} \) is partially transposable since there exist operations \( g_x^* \) interchanging domain states 1 and 2, e.g. \( m_x^* m_y \), and \( F_1 \cap F_2 = 2_\infty m \). The symmetry group of this domain pair is

\[
J_{12} = \{m_z\} + 2_{2y} \cdot \{m_z\} = 2_{2y} m_y m_z.
\]

(4)

It is easy to show that the twinning group of this domain pair equals

\[
K_{12} = \{2_\infty m, m_z\} + 4 \cdot \{2_\infty m, m_z\} + 4^2 \cdot \{2_\infty m, m_z\} = 4/mmm.
\]

(5)

On the other hand, the domain pair \( \{1, 3\} \) is a completely transposable domain pair since there exist interchanging operations \( g_1^* \), e.g. \( m_1^* \), and \( F_1 = F_3 = 2_\infty m, m_z \) (see Figure 2). The symmetry group and the twinning group of this pair are, therefore, identical and equal

\[
J_{13} = K_{13} = \{2_\infty m, m_z\} + m_x^* \cdot \{2_\infty m, m_z\} = m_x m_y m_z.
\]

(6)

3. ORDER PARAMETERS AND DOMAIN DISTINCTION

Generalizing slightly Aziz's concept of full and partial ferroelectric (ferromagnetic, ferroelastic) phases one can divide quantities (material properties) of domain states into three categories:

1. Quantities that are the same in all domain states. If we denote by \( \gamma^{(i)} \) and \( \gamma^{(j)} \) the value of such a quantity \( \gamma \) in the domain state \( S_i \) and \( S_j \), respectively, then \( \gamma^{(i)} = \gamma^{(j)} \) for all domain pairs \( \{S_i, S_j\} \).

2. Quantities that are different in all domain states. A quantity \( \varphi \) will belong to this category if \( \varphi^{(i)} \neq \varphi^{(j)} \) for all non-trivial domain pairs \( \{S_i, S_j\}, i \neq j \).

3. Quantities that are different in some but not in all domain pairs. The quantity \( \lambda \) will belong to this category if there exist at least two different domain pairs \( \{S_i, S_j\} \) and \( \{S_i, S_m\} \) such that \( \lambda^{(i)} \neq \lambda^{(j)} \) and \( \lambda^{(i)} = \lambda^{(m)} \), \( i \neq j, m \neq i \).

To which category certain property \( \lambda \) belongs is determined by a group \( L_i \), called the stabilizer of \( \lambda^{(i)} \). This is a group which consists of all operations \( g \in G \) that leave \( \lambda^{(i)} \) invariant,

\[
L_i = \{g \in G | g \lambda^{(i)} = \lambda^{(i)}\}.
\]

(7)

If \( L_i = G \) then \( \lambda = \gamma \) is an invariant of \( G \) and belongs to the first category. If \( L_i = F_i \), where \( F_i \) is the symmetry group of the domain state \( S_i \), then \( \lambda \) belongs to
the second category and is identical with the primary order parameter $\varphi$ of the transition $G \rightharpoonup F_i$. Finally, if the stabilizer $L_i$ is an intermediate group,

$$ F_i < L_i < G, $$

(8)

then $\lambda$ belongs to the third category and can be identified with the secondary order parameter $\lambda$ of the transition $G \rightharpoonup F_i$ (and the primary parameter of the transition $G \rightharpoonup L_i$).

As it is shown elsewhere, the twinning group $K_{ij}$, introduced in the preceding section, determines whether $\lambda$ is the same or different in $S_i$ and $S_j$ of a chosen domain pair $\{S_i, S_j\}$. If the stabilizer $L_i$ contains the twinning group $K_{ij}$ as its subgroup, i.e. if

$$ K_{ij} \subseteq L_i, $$

(9)

then $\lambda$ is the same in both domain states, $\lambda_i^{(\alpha)} = \lambda_j^{(\alpha)}$. In the opposite case, i.e. if

$$ L_i \cap K_{ij} \neq K_{ij}, $$

(10)

the domain states $S_i$ and $S_j$ differ in the secondary order parameter $\lambda$, $\lambda_i^{(\alpha)} \neq \lambda_j^{(\alpha)}$.

In our example there is just one intermediate group $L_1 = \mathbf{m}_x \mathbf{m}_y \mathbf{m}_z$ that fulfills condition (8). For the representative domain pair $\{1, 2\}$ with $K_{12} = 4/mmm$ the condition (10) is fulfilled, hence the domain states 1 and 2 differ in the secondary order parameter $\lambda$. On the contrary, for the pair $\{1, 3\}$ with $K_{13} = \mathbf{m}_x \mathbf{m}_y \mathbf{m}_z$, the condition (9) is obeyed and the secondary order parameter $\lambda$ has, therefore, the same value in both domain states 1 and 3.

The transformation properties of the quantities $\gamma$, $\varphi$, and $\lambda$, respectively, are specified by the representations $D^1$ (one-dimensional identity representation), $D^\varphi$ and $D^\lambda$ of the group $G$. The dimension $d_\varphi$ and $d_\lambda$ of $D^\varphi$ and $D^\lambda$ determines the number of components of the order parameter $\varphi$ and $\lambda$, $\varphi^{(\alpha)} = (\varphi_1^{(\alpha)}, \varphi_2^{(\alpha)}, \ldots, \varphi_{d_\varphi}^{(\alpha)})$, $\lambda^{(\alpha)} = (\lambda_1^{(\alpha)}, \lambda_2^{(\alpha)}, \ldots, \lambda_{d_\lambda}^{(\alpha)})$, respectively. The representations $D^\varphi$ and $D^\lambda$ of the order parameters can be either irreducible or can be decomposed into a sum of irreducible representations,

$$ D^\varphi = \sum_i m_i^\varphi D_i, $$

(11)

where $D_i$ are irreducible representations of $G$ and $m_i^\varphi$ are their multiplicities in $D^\varphi$. Similar decomposition holds for the reducible representation $D^\lambda$ of the secondary order parameter $\lambda$.

In the tables of Reference 7 we find that the primary order parameter $\varphi$ of the phase transition $4/mmm \rightharpoonup 2, \mathbf{m}_x \mathbf{m}_y \mathbf{m}_z$ transforms according to the 2-dimensional irreducible representation $D^\varphi = E_{\alpha}$ and that in the domain state 1 the second component $\varphi_2^{(\alpha)} = 0$. The representation $D^\lambda$ corresponding to the intermediate group $L_1 = \mathbf{m}_x \mathbf{m}_y \mathbf{m}_z$ is the one-dimensional representation $B_{1g}$ which defines the transformation properties of the only secondary order parameter $\lambda$.

The order parameters $\varphi$ and $\lambda$ can be associated with linear combinations of material property tensor components that transform according to $D^\varphi$ and $D^\lambda$ (e.g. in Reference 7 such low-rank tensorial basis functions that transform according to $D^\varphi$ are given for all point group symmetry reductions). In another approach, which is described in detail elsewhere, the material property tensors are decomposed into so called irreducible constituents which transform according to the irreducible
representations of \( G \) and can be, therefore, associated with order parameters. An irreducible constituent of a tensor can be labelled by the symbol of the corresponding irreducible representation; in the decomposition of some tensors several constituents can appear that transform according to the same representation. The number of components of a \( D' \)-irreducible constituent equals the dimension of the corresponding irreducible representation \( D' \). The components of an irreducible constituent can be expressed as linear combinations of the usual tensor components.

This way of expressing tensors is appropriate for discussing tensor distinction of domain states. To illustrate this point we turn again to our example. In Table I three types of tensors, namely \( V \), \([V^2]\), and \([V^2]V\), are expressed in terms of irreducible constituents. It can be shown that two \( A_{1g} \)-irreducible constituents appear in the tensor \( T - [V^2] \) and that their values are \( A = \frac{1}{2}(T_{11} + T_{22}) \) and \( B = T_{33} \). These constituents have the same value in all four domain states. One \( E_u \)-irreducible constituent appears in the polar vector \( T - V \) (polarization) and its two components correspond to the first two components \( T_1 \) and \( T_2 \) of \( T - V \) with \( T_2 = 0 \) in the domain state 1 (in Table I the non-zero value of components is denoted by \( P \)). The tensor \( T - [V^2]V \) consists of five \( E_u \)-irreducible constituents that correspond to the following couples of tensor components: \( (T_{11}, T_{22}), (T_{21}, T_{12}), (T_{31}, T_{32}), (T_{33}, T_{43}), \) and \( (T_{62}, T_{66}) \). The values of the non-zero components of these constituents are in Table I denoted by \( Q, R, S, U, U \), and \( W \), respectively. Each of these couples can be associated with the primary order parameter \( \varphi \) and is, therefore, different in two domain states of any non-trivial domain pair. The \( B_{1g} \)-irreducible constituent is associated with the secondary order parameter \( \lambda \). It appears once in the tensor \( T - [V^2] \), where its value \( C = \frac{1}{2}(T_{11} - T_{22}) \).

**TABLE I**

Tensor distinction of representative domain pairs \{1, 2\} and \{1, 3\}. Tensor designation:

- \( V \) . . . polarization,
- \([V^2]\) . . . deformation, permittivity,
- \([V^2]V\) . . . piezoelectricity, electrooptics.

\( A, B \) . . . values of the \( A_{1g} \)-irreducible constituents,
\( C \) . . . value of the \( B_{1g} \)-irreducible constituent,
\( P, Q, R, S, U, W \) . . . values of the \( E_u \)-irreducible constituents.

<table>
<thead>
<tr>
<th>tensor</th>
<th>domain states</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>( V )</td>
<td>((P, 0, 0))</td>
</tr>
<tr>
<td>([V^2])</td>
<td>((0, 0, A + C))</td>
</tr>
<tr>
<td>([V^2]V)</td>
<td>((0, 0, 0))</td>
</tr>
<tr>
<td>(Q, R, S, U, W)</td>
<td>((0, 0, 0))</td>
</tr>
</tbody>
</table>

The following relations hold: \( P = P = \frac{1}{P}, Q = Q = \frac{1}{Q}, R = R = \frac{1}{R} \), \( S = S = \frac{1}{S}, U = U = \frac{1}{U}, W = W = \frac{1}{W} \).
Let us now consider a domain pair \( \{S_i, S_j\} \), where \( S_j = g_p S_i \). The value of the order parameter \( \varphi^{(j)} \) (and of the corresponding irreducible components) in the domain state \( S_j \) can be calculated from the value \( \varphi^{(i)} \) in the domain state \( S_i \) according to the following formula

\[
\varphi^{(j)}_i = \sum_{m=1}^{d_a} D^{(j)}_{m}(g_i)\varphi^{(i)}(g)_m, \quad l = 1, \ldots, d_a.
\]

(12)

The secondary order parameter \( \lambda \) and the corresponding irreducible constituents transform according to a similar formula.

In our example the \( 2 \times 2 \) matrices of \( D^s(g) = E_s(g) \) appearing in the transformation formula (12) can be easily constructed for the vector \( (P_x, P_y) \). All \( E_n \) irreducible constituents transform in the same way, e.g. when one switches from the domain state 1 to domain state 2 the non-zero components of the tensor \( [V^2]V \) are flipping in the same way as the non-zero component \( P \) of \( T \sim V \) (see Table I). The advantage of expressing tensor distinction in terms of irreducible constituents is obvious: to calculate the value of the tensor \( T^{(2)} \sim [V^2]V \) in the domain state 2 from the value \( T^{(1)} \sim [V^2]V \) in domain state 1 according to usual formulae for tensor transformations one has to construct a \( 18 \times 18 \) transformation matrix whereas only one \( 2 \times 2 \) matrix is needed when the tensor \( T \sim [V^2]V \) is expressed in terms of the irreducible constituents.

The representation \( D^s = B_{1g} \) is one-dimensional, therefore the parameter \( \lambda \) and corresponding irreducible component are either constant or have different sign in two domain states. Thus, e.g. for the tensor \( T \sim [V^2] \) in our example the transformation from the domain state 1 to domain state 2 is expressed just by the change of the sign of \( C \) (see Table I).

4. ROLE OF DOMAIN PAIR SYMMETRY

Distinction of domain states by order parameters or by irreducible constituents can be further simplified by making use of the symmetry group \( J_{ij} \) of the domain pair \( \{S_i, S_j\} \) defined in Section 2. For a transposable domain pair \( \{S_i, S_j\} \) we can associate with the group \( J_{ij} \) a virtual phase transition \( J_{ij} \setminus F_{ij} \). The order parameter \( \alpha \) of this transition, which we shall call the domain pair parameter, is different in \( S_i \) and \( S_j \). Since the index \( F_{ij} \) in \( J_{ij} \) equals 2, the domain pair parameter \( \alpha \) differs just in the sign, \( \alpha^{(i)} = -\alpha^{(j)} \). The representation \( D^{\alpha} \) of \( J_{ij} \), according to which \( \alpha \) transforms, is a one-dimensional alternating representation of \( J_{ij} \) which subduces in \( F_{ij} \), an identity representation \( A_1 \), \( (D^{\alpha} \downarrow F_{ij}) = A_1 \). If we restrict the representation \( D^{\alpha} \) to the group \( J_{ij} \), then this subduced representation \( (D^{\alpha} \downarrow J_{ij}) \) can be decomposed in the following way,

\[
(D^{\alpha} \downarrow J_{ij}) = m^{\alpha}_1 D^1 + m^{\alpha}_n D^n + m^{\alpha}_B D^B + \ldots,
\]

\[
(D^{\alpha} \downarrow F_{ij}) = A_1, \quad (D^B \downarrow F_{ij}) \neq A_1,
\]

(13)

where \( D^1 \) is the identity representation \( A_1 \) of \( J_{ij} \) and \( D^B \) is a representation of \( J_{ij} \) that does not become the identity representation \( A_1 \) in \( F_{ij} \). The multiplicities \( m^{\alpha}_1, m^{\alpha}_n \) and \( m^{\alpha}_B \) give the number of the components of \( \varphi \) that are the same, that
have opposite sign and that equal zero in domain states \( S_i \) and \( S_j \), respectively. Besides \( D^b \) other irreducible representations can appear in (13) which do not subdue \( A_i \) in \( F_{ij} \) and produce zero components similarly as \( D^b \). There is always \( m^\lambda \gamma > 0 \).

Similar decomposition holds for the representation \( D^\lambda \) according to which transforms the secondary order parameter \( \lambda \),

\[
(D^\lambda \downarrow J_{ij}) = m_\gamma D^\gamma + m_a D^a + m_b D^b + \ldots ,
\]

with similar meaning of the multiplicities \( m_\gamma \), \( m_a \), and \( m_b \) as for the primary order parameter \( \varphi \). If \( J_{ij} \equiv L_i \), then the multiplicity \( m_a \) = 0, i.e. \( \lambda^{(i)} = \lambda^{(0)} \). If, on the other hand,

\[
L_i \cap J_{ij} \neq J_{ij},
\]

then the multiplicity \( m_a \) > 0, i.e. at least some of the components of \( \lambda \) and of \( \lambda \)-irreducible constituent(s) have opposite sign in domain states \( S_i \) and \( S_j \).

For the domain pair (1, 2) of our example the decomposition (13) reads:

\[
(E_u \downarrow 2_{sy} m_{sy}, m_z) = A_{1} + B_{1},
\]

where \( A' \) is the identity representation of the group \( m_s \). From these relations it follows that, e.g. the 2-dimensional \( E_u \)-irreducible constituent of the polarization \( \mathbb{P} \) can be decomposed into two components \( P_\gamma \) and \( P_a \), where the first one is constant and the second one has opposite sign in domain states 1 and 2. This decomposition has a simple geometrical meaning: \( P_\gamma \) is the projection of \( \mathbb{P} \) into the [110] direction and \( P_a \) into the [110] direction (see Figure 2). The polarization \( \mathbb{P} \) expressed in this way is given in the second row of Table I. We see that the flipping of the non-zero component \( P \) of \( \mathbb{P} \) is replaced by the sum \( P_\gamma + P_a \) and the difference \( P_\gamma - P_a \). The tensor \( [V^2]V \) can be decomposed in the similar manner (see the last row of matrices in Table I).

The decomposition (14) of the representation \( B_{1g} \) according to which transforms the secondary order parameter \( \lambda \) is

\[
(B_{1g} \downarrow 2_{sy} m_{sy}, m_z) = B_{1} + (B_{1} \downarrow m_z) = A',
\]

i.e. \( m_\gamma = 1 \). Consequently, the value \( C \) of the \( B_{1g} \)-irreducible constituent in the tensor \( [V^2] \) has opposite sign in domain states 1 and 2 (see Table I).

For the domain pair (1, 3) the decomposition (13) yields

\[
(E_u \downarrow m_z m_z, m_z) = B_{3u} + B_{2u},
\]

where \( A_{1} \) and \( B_{2} \) are irreducible representations of the group \( 2_{sy} m_s m_z \). From this decomposition we conclude that the value of the first component of the polarization has opposite sign in domain states 1 and 3 and the second component is zero in both of them. Similar conclusion holds for the values of the \( E_u \)-constituents in the tensor \( T \sim [V^2]V \) (see the last column in Table I).

From the decomposition (14), which reads \( (B_{1g} \downarrow 2_{sy} m_z m_z) = A_{1} \), it follows that the value \( C \) of the \( B_{1g} \)-irreducible constituent remains the same in domain states 1 and 3. This also follows directly from the equality \( L_1 = m_z m_z, m_z = J_{13} \).

Our example illustrates a general feature of tensor distinction in transposable
domain pairs. Material property tensors can be decomposed into two parts: one is the same and the other differs only in sign in the two domain states of a transposable domain pair.

ACKNOWLEDGEMENTS

We thank Dr. Z. Zikmund for useful discussions and help with figures. This work has been supported by the Grant Agency of the Czech Republic under the grant No. 202/93/0454, by the Grant Agency of the Academy of Sciences of the Czech Republic under the grant No. 19083 and by the National Science Foundation under grants DMR-9100418 and DMR-9205825.

REFERENCES

3. J. Fuksa and V. Janovec, to be published.
4. J. Fuksa and V. Janovec, this Symposium.