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TRANSPOSABLE DOMAIN PAIRS AND DOMAIN DISTINCTION

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We divide pairs of domain states into three classes: completely, partially and non-transposable domain pairs. We show that two groups can be associated with a domain pair: the twinning group and the symmetry group of the pair. The twinning group determines which secondary order parameters are the same and which are different in two domain states of a domain pair. The symmetry group of a transposable domain pair allows one to express the order parameters and irreducible constituents of material property tensors in such a way that their components in two domain states are either the same or differ only in the sign. The analysis of domain distinction is illustrated on a simple example.

Keywords: Domain structures, ferroelastic domains, tensor distinction of domains, domain pairs, twinning group of a domain pair.

1. INTRODUCTION

Domain bulks (domain states) of two domains simultaneously observed by a single apparatus can exhibit different properties although their crystal structures are the same and differ only in their spatial orientation. Which properties are the same and which are different depends on the symmetry of one domain state and on the spatial relation between both domain states. We have shown that for all non-ferroelastic domain pairs[1] and for a class of ferroelastic domain pairs called completely transposable[2] this information can conveniently be expressed by a dichromatic (black and white) crystallographic point group and that the tensor components distinct in two domain states differ only in the sign.

In this paper we indicate how the analysis can be extended to other domain pairs. First, we divide domain pairs according to their internal symmetry expressed by possible transpositions of domain states. Then we discuss the distinction of domain states in terms of the primary and secondary order parameters of the phase transition and demonstrate the significance of so called twinning group in distinguishing domain states according to the secondary order parameters. Finally, we show how the symmetry of a domain pair can be utilized in domain distinction. The exposition is accompanied by a simple example which illustrates the main steps of the analysis. Similarly as in the preceding papers[1,2] we confine ourselves to the continuum description and point groups symmetry.

2. SYMMETRY GROUP AND TWINNING GROUP OF A DOMAIN PAIR

We consider crystalline domains which arise in a phase transition from a parent (ordered) phase of symmetry G to a distorted (disordered) phase of symmetry F, where F is a subgroup of G, $F < G$. We shall refer to the bulk structures of these domains in polydomain samples as *domain states*. Several disconnected domains of possibly different shape can have the same domain state. Consequently, domain states of a polydomain sample represent structures that appear in the sample, irrespective of in which domain and irrespective of the domain's shape. We shall confine ourselves to single domain states which we shall denote $S_1, S_2, \ldots, S_n$. Their number n equals the index of F in G, $n = |G| : |F|$, where $|G|$ and $|F|$ is the number of symmetry operations of G and F, respectively. Single domain states are symmetrically equivalent in the group G, i.e. each two domain states can be related by an operation g from G.

Most often, domains are distinguished by their bulk properties, i.e. according to their domain states. Then the problem of domain distinction is reduced to the distinction of domain states. To solve this task we have to describe in a convenient way the distinction of any two of all possible n domain states. For this purpose we use the concept of a *domain pair* denoted by $\{S_i, S_j\}$ and defined as an unordered set consisting of two domain states S_i and S_j.

Let us first examine the symmetry of a domain pair $\{S_i, S_j\}$. If F_i and F_j are the symmetry groups of S_i and S_j, respectively, then any operation f that belongs both to F_i and to F_j, $f \in F_i \cap F_j \equiv F_{ij}$, is a symmetry operation of $\{S_i, S_j\}$. If, moreover, there exists such $g^*_{ij} \in G$ which transposes (interexchanges) S_i and S_j, $g^*_{ij} S_i = S_j$, $g^*_{ij} S_j = S_i$, then all operations from the left coset $g^*_{ij} F_{ij}$ do so as well (the asterisk denotes operations interexchanging two domain states). These operations are also symmetry operations of $\{S_i, S_j\}$ since for unordered sets (domain pairs) $\{S_i, S_j\} = \{S_j, S_i\}$. Thus the *symmetry group J_{ij} of the domain pair* $\{S_i, S_j\}$ can be, in a general case, expressed in the following way[1]:

$$J_{ij} = F_{ij} + g^*_{ij} F_{ij}. \tag{1}$$

Domain pairs can be classified according to their symmetry J_{ij}. Pairs for which g^*_{ij} exists we call *transposable* (or *ambivalent*[1]) *domain pairs*, pairs for which an interexchanging operation g^*_{ij} cannot be found are called *non-transposable* (or *polar*[1]) *domain pairs*. The symmetry of a non-transposable pair is reduced to F_{ij}.

All operations that transform S_i into S_j (irrespectively how they transform S_j) are contained in the left coset $g_{ij}F_i$ since $g_{ij}F_iS_i = g_{ij}S_i = S_j$. If $\{S_i, S_j\}$ is a transposable pair and, moreover, $F_i = F_j = F_{ij}$ then *all* operations of the left coset $g_{ij}F_i$ transform simultaneously S_j into S_i. We call such pairs *completely transposable domain pairs*. The symmetry group J_{ij} of a completely transposable pair $\{S_i, S_j\}$ is

$$J_{ij} = F_i + g^*_{ij} F_i. \tag{2}$$

If $F_i \neq F_j$ then $F_{ij} < F_i$ and the number of transposing operations of a transposable domain pair is smaller than the number of operations transforming S_i into S_j. We call, therefore, such pairs *partially transposable domain pairs*.

The transformation properties of a domain pair $\{S_i, S_j\}$ are described by the symmetry group F_i of S_i and by all operations $g_{ij}F_i$ that transform S_i into S_j. The smallest subgroup K_{ij} of G that contains both F_i and $g_{ij}F_i$ will be called the *twinning group of the domain pair* $\{S_i, S_j\}$.[4] For the completely transposable pairs the union of F_i and $g_{ij}F_i$ forms already a group, $K_{ij} = F_i + g^*_{ij}F_i$, which is identical with the symmetry group J_{ij} of the domain pair (see Equation (2)), $K_{ij} = J_{ij}$. For the partially transposable and non-transposable domain pairs the group K_{ij} comprises besides F_i and $g_{ij}F_i$ further left cosets and can be written in the following form[3,4]

$$K_{ij} = F_i + g_{ij}F_i + \cdots + g_{ik}F_i. \tag{3}$$

The twinning group K_{ij} plays a basic role in the symmetry analysis of domain pairs.

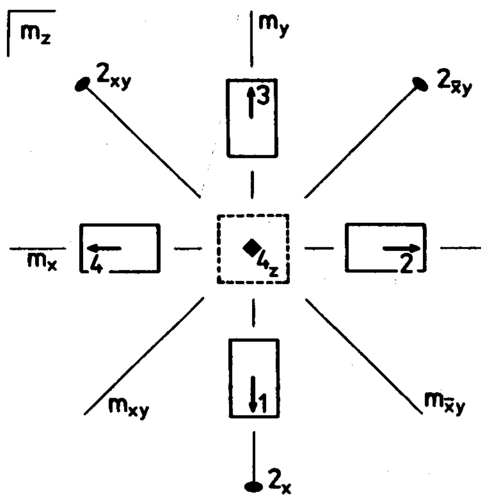

FIGURE 1 Exploded view of the single domain states of the distorted phase resulting from the phase transition with the symmetry reduction *4/mmm* ↘ *2mm*. Four single domain states 1, 2, 3, and 4 are represented by the solid rectangles, the arrows correspond to the spontaneous polarization (primary order parameter of the transition). The dashed square in the center represents the parent phase. Single domain states are related by depicted symmetry operations of the parent phase.

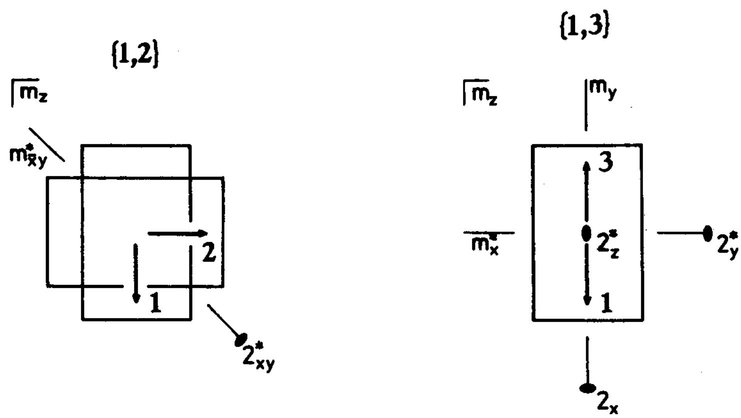

FIGURE 2 Graphical representation of domain pairs {1, 2} and {1, 3} depicted as two superimposed domain states. Symmetry operations marked by an asterisk interexchange the two domain states of the domain pair.

To illustrate our exposition we consider a phase transition in which the symmetry group of the parent phase is $G = 4/mmm$ and the domain state S_1 in the distorted phase has the symmetry $F_1 = 2_x m_y m_z$. This is a proper (full) ferroelectric and improper (partial) ferroelastic transition with spontaneous polarization as the primary order parameter and spontaneous deformation as the secondary order parameter. The distorted phase can appear in $n = |4/mmm| : |2mm| = 16 : 4 = 4$ single domain states $S_1 \equiv 1$, $S_2 \equiv 2$, $S_3 \equiv 3$, $S_4 \equiv 4$ which are presented in a graphical form in Figure 1. From these domain states one can form six non-trivial domain pairs which can be divided into two classes of crystallographically equivalent pairs. The representative domain pairs of these classes are {1, 2} with perpendicular polarization and {1, 3} with antiparallel polarization. Both these domain pairs are represented graphically in Figure 2 as superpositions of corresponding domain states.

The domain pair {1, 2} is partially transposable since there exist operations g^*_{ij} interexchanging domain states 1 and 2, e.g. $m^*_{\bar{x}y}$, and $F_1 \cap F_2 = 2_x m_y m_z \cap m_x 2_y m_z = \{m_z\}$ (see Figure 2). The symmetry group of this domain pair is

$$J_{12} = \{m_z\} + 2^*_{xy} \cdot \{m_z\} = 2_{xy} m_{\bar{x}y} m_z. \tag{4}$$

It is easy to show that the twinning group of this domain pair equals

$$K_{12} = \{2_x m_y m_z\} + 4 \cdot \{2_x m_y m_z\} + 4^2 \cdot \{2_x m_y m_z\} + 4^3 \cdot \{2_x m_y m_z\} = 4/mmm. \tag{5}$$

On the other hand, the domain pair {1, 3} is a completely transposable domain pair since there exist interexchanging operations g^*_{13}, e.g. m^*_x, and $F_1 = F_3 = 2_x m_y m_z$ (see Figure 2). The symmetry group and the twinning group of this pair are, therefore, identical and equal

$$J_{13} = K_{13} = \{2_x m_y m_z\} + m^*_x \cdot \{2_x m_y m_z\} = m_x m_y m_z. \tag{6}$$

3. ORDER PARAMETERS AND DOMAIN DISTINCTION

Generalizing slightly Aizu's concept of full and partial ferroelectric (ferromagnetic, ferroelastic) phases[5] one can divide quantities (material properties) of domain states into three categories:

1. Quantities that are the same in all domain states. If we denote by $\gamma^{(i)}$ and $\gamma^{(j)}$ the value of such a quantity γ in the domain state S_i and S_j, respectively, then $\gamma^{(i)} = \gamma^{(j)}$ for all domain pairs $\{S_i, S_j\}$.
2. Quantities that are different in all domain states. A quantity φ will belong to this category if $\varphi^{(i)} \neq \varphi^{(j)}$ for all non-trivial domain pairs $\{S_i, S_j\}$, $i \neq j$.
3. Quantities that are different in some but not in all domain pairs. The quantity λ will belong to this category if there exist at least two different domain pairs $\{S_i, S_j\}$ and $\{S_l, S_m\}$ such that $\lambda^{(i)} \neq \lambda^{(j)}$ and $\lambda^{(l)} = \lambda^{(m)}$, $i \neq j$, $m \neq l$.

To which category certain property λ belongs is determined by a group L_i called the stabilizer of $\lambda^{(i)}$. This is a group which consists of all operations $g \in G$ that leave $\lambda^{(i)}$ invariant,

$$L_i = \{g \in G | g\lambda^{(i)} = \lambda^{(i)}\}. \tag{7}$$

If $L_i = G$ then $\lambda = \gamma$ is an invariant of G and belongs to the first category. If $L_i = F_i$, where F_i is the symmetry group of the domain state S_i, then λ belongs to the second category and is identical with the *primary order parameter*[6] φ of the transition $G \searrow F_i$. Finally, if the stabilizer L_i is an intermediate group,

$$F_i < L_i < G, \tag{8}$$

then λ belongs to the third category and can be identified with the *secondary order parameter*[6] of the transition $G \searrow F_i$ (and the primary parameter of the transition $G \searrow L_i$).

As it is shown elsewhere,[3,4] the twinning group K_{ij}, introduced in the preceding section, determines whether λ is the same or different in S_i and S_j of a chosen domain pair $\{S_i, S_j\}$. If the stabilizer L_i contains the twinning group K_{ij} as its subgroup, i.e. if

$$K_{ij} \leq L_i, \tag{9}$$

then λ is the same in both domain states, $\lambda^{(i)} = \lambda^{(j)}$. In the opposite case, i.e. if

$$L_i \cap K_{ij} \neq K_{ij}, \tag{10}$$

the domain states S_i and S_j differ in the secondary order parameter λ, $\lambda^{(i)} \neq \lambda^{(j)}$.

In our example there is just one intermediate group $L_1 = m_x m_y m_z$ that fulfills condition (8). For the representative domain pair {1, 2} with $K_{12} = 4/mmm$ the condition (10) is fulfilled, hence the domain states 1 and 2 differ in the secondary order parameter λ. On the contrary, for the pair {1, 3} with $K_{13} = m_x m_y m_z$ the condition (9) is obeyed and the secondary order parameter λ has, therefore, the same value in both domain states 1 and 3.

The transformation properties of the quantities γ, φ, and λ, respectively, are specified by the representations D^1 (one-dimensional identity representation), D^φ and D^λ of the group G. The dimension d_φ and d_λ of D^φ and D^λ determines the number of components of the order parameter φ and λ, $\varphi^{(i)} \equiv (\varphi_1^{(i)}, \varphi_2^{(i)}, \ldots, \varphi_{d_\varphi}^{(i)})$, $\lambda^{(i)} \equiv (\lambda_1^{(i)}, \lambda_2^{(i)}, \ldots, \lambda_{d_\lambda}^{(i)})$, respectively. The representations D^φ and D^λ of the order parameters can be either irreducible or can be decomposed into a sum of irreducible representations,

$$D^\varphi = \sum_i m_i^\varphi D^i, \tag{11}$$

where D^i are irreducible representations of G and m_i^φ are their multiplicities in D^φ. Similar decomposition holds for the reducible representation D^λ of the secondary order parameter λ.

In the tables of Reference 7 we find that the primary order parameter φ of the phase transition $4/mmm \searrow 2_x m_y m_z$ transforms according to the 2-dimensional irreducible representation $D^\varphi = E_u$ and that in the domain state 1 the second component $\varphi_2^{(1)} = 0$. The representation D^λ corresponding to the intermediate group $L_1 = m_x m_y m_z$ is the one-dimensional representation B_{1g} which defines the transformation properties of the only secondary order parameter λ.

The order parameters φ and λ can be associated with linear combinations of material property tensor components that transform according to D^φ and D^λ (e.g. in Reference 7 such low-rank tensorial basis functions that transform according to D^φ are given for all point group symmetry reductions). In another approach, which is described in detail elsewhere,[3] the material property tensors are decomposed into so called irreducible constituents which transform according to the irreducible representations of G and can be, therefore, associated with order parameters. An irreducible constituent of a tensor can be labelled by the symbol of the corresponding irreducible representation; in the decomposition of some tensors several constituents can appear that transform according to the same representation. The number of components of a D^i-irreducible constituent equals the dimension of the corresponding irreducible representation D^i. The components of an irreducible constituent can be expressed as linear combinations of the usual tensor components.

This way of expressing tensors is appropriate for discussing tensor distinction of domain states. To illustrate this point we turn again to our example. In Table I three types of tensors, namely V, $[V^2]$, and $[V^2]V$, are expressed in terms of irreducible constituents. It can be shown that two A_{1g}-irreducible constituents appear in the tensor $T \sim [V^2]$ and that their values are $A = \frac{1}{2}(T_{11} + T_{22})$ and $B = T_{33}$. These constituents have the same value in all four domain states. One E_u-irreducible constituent appears in the polar vector $T \sim V$ (polarization) and its two components correspond to the first two components T_1 and T_2 of $T \sim V$ with $T_2 = 0$ in the domain state 1 (in Table I the non-zero value of components is denoted by P). The tensor $T \sim [V^2]V$ consists of five E_u-irreducible constituents that correspond to the following couples of tensor components: (T_{11}, T_{22}), (T_{21}, T_{12}), (T_{31}, T_{32}), (T_{53}, T_{43}), and (T_{62}, T_{61}). The values of the non-zero components of these constituents are in Table I denoted by Q, R, S, U and W, respectively. Each of these couples can be associated with the primary order parameter φ and is, therefore, different in two domain states of any non-trivial domain pair. The B_{1g}-irreducible constituent is associated with the secondary order parameter λ. It appears once in the tensor $T \sim [V^2]$, where its value $C = \frac{1}{2}(T_{11} - T_{22})$.

TABLE I

Tensor distinction of representative domain pairs {1, 2} and {1, 3}. Tensor designation: V . . . polarization, $[V^2]$ . . . deformation, permittivity, $[V^2]V$ . . . piezoelectricity, electrooptics. A, B . . . values of the A_{1g}-irreducible constituents, C. . . value of the B_{1g}-irreducible constituent, P, Q, R, S, U, W . . . values of the E_u-irreducible constituents.

tensor	domain states		
	1	2	3
V	$(P,0,0)$ $(P_\gamma + P_\alpha, P_\gamma - P_\alpha, 0)$	$(0,P,0)$ $(P_\gamma - P_\alpha, P_\gamma + P_\alpha, 0)$	$(-P,0,0)$
$[V^2]$	$\begin{pmatrix} A+C & 0 & 0 \\ 0 & A-C & 0 \\ 0 & 0 & B \end{pmatrix}$	$\begin{pmatrix} A-C & 0 & 0 \\ 0 & A+C & 0 \\ 0 & 0 & B \end{pmatrix}$	$\begin{pmatrix} A+C & 0 & 0 \\ 0 & A-C & 0 \\ 0 & 0 & B \end{pmatrix}$
$[V^2]V$	$\begin{pmatrix} Q & 0 & 0 \\ R & 0 & 0 \\ S & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & U \\ 0 & W & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & R & 0 \\ 0 & Q & 0 \\ 0 & S & 0 \\ 0 & 0 & U \\ 0 & 0 & 0 \\ W & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} -Q & 0 & 0 \\ -R & 0 & 0 \\ -S & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -U \\ 0 & -W & 0 \end{pmatrix}$
	$\begin{pmatrix} Q_\gamma + Q_\alpha & R_\gamma - R_\alpha & 0 \\ R_\gamma + R_\alpha & Q_\gamma - Q_\alpha & 0 \\ S_\gamma + S_\alpha & S_\gamma - S_\alpha & 0 \\ 0 & 0 & U_\gamma - U_\alpha \\ 0 & 0 & U_\gamma + U_\alpha \\ W_\gamma - W_\alpha & W_\gamma + W_\alpha & 0 \end{pmatrix}$	$\begin{pmatrix} Q_\gamma - Q_\alpha & R_\gamma + R_\alpha & 0 \\ R_\gamma - R_\alpha & Q_\gamma + Q_\alpha & 0 \\ S_\gamma - S_\alpha & S_\gamma + S_\alpha & 0 \\ 0 & 0 & U_\gamma + U_\alpha \\ 0 & 0 & U_\gamma - U_\alpha \\ W_\gamma + W_\alpha & W_\gamma - W_\alpha & 0 \end{pmatrix}$	

The following relations hold: $P_\gamma = P_\alpha = \frac{1}{2}P$, $Q_\gamma = Q_\alpha = \frac{1}{2}Q$, $R_\gamma = R_\alpha = \frac{1}{2}R$, $S_\gamma = S_\alpha = \frac{1}{2}S$, $U_\gamma = U_\alpha = \frac{1}{2}U$, $W_\gamma = W_\alpha = \frac{1}{2}W$.

Let us now consider a domain pair $\{S_i, S_j\}$, where $S_j = g_{ij}S_i$. The value of the order parameter $\varphi^{(j)}$ (and of the corresponding irreducible components) in the domain state S_j can be calculated from the value $\varphi^{(i)}$ in the domain state S_i according to the following formula

$$\varphi_l^{(j)} = \sum_{m=1}^{d_\varphi} D_{lm}^{\varphi}(g_{ij})\varphi_m^{(i)}, \quad l = 1, \ldots, d_\varphi. \tag{12}$$

The secondary order parameter λ and the corresponding irreducible constituents transform according to a similar formula.

In our example the 2×2 matrices of $D^\varphi(g) = E_u(g)$ appearing in the transformation formula (12) can be easily constructed for the vector (P_x, P_y). All E_u-irreducible constituents transform in the same way, e.g. when one switches from the domain state 1 to domain state 2 the non-zero components of the tensor $[V^2]V$ are flipping in the same way as the non-zero component P of $T \sim V$ (see Table I). The advantage of expressing tensor distinction in terms of irreducible constituents is obvious: to calculate the value of the tensor $T^{(2)} \sim [V^2]V$ in the domain state 2 from the value $T^{(1)} \sim [V^2]V$ in domain state 1 according to usual formulae for tensor transformations one has to construct a 18×18 transformation matrix whereas only one 2×2 matrix is needed when the tensor $T \sim [V^2]V$ is expressed in terms of the irreducible constituents.

The representation $D^\lambda = B_{1g}$ is one-dimensional, therefore the parameter λ and corresponding irreducible component are either constant or have different sign in two domain states. Thus, e.g. for the tensor $T \sim [V^2]$ in our example the transformation from the domain state 1 to domain state 2 is expressed just by the change of the sign of C (see Table I).

4. ROLE OF DOMAIN PAIR SYMMETRY

Distinction of domain states by order parameters or by irreducible constituents can be further simplified by making use of the symmetry group J_{ij} of the domain pair $\{S_i, S_j\}$ defined in Section 2. For a transposable domain pair $\{S_i, S_j\}$ we can associate with the group J_{ij} a virtual phase transition $J_{ij} \searrow F_{ij}$. The order parameter α of this transition, which we shall call the *domain pair parameter*, is different in S_i and S_j. Since the index F_{ij} in J_{ij} equals 2, the domain pair parameter α differs just in the sign, $\alpha^{(j)} = -\alpha^{(i)}$. The representation D^α of J_{ij}, according to which α transforms, is a one-dimensional alternating representation of J_{ij} which subduces in F_{ij} an identity representation A_1, $(D^\alpha \downarrow F_{ij}) = A_1$. If we restrict the representation D^φ to the group J_{ij} then this subduced representation $(D^\varphi \downarrow J_{ij})$ can be decomposed in the following way,

$$(D^\varphi \downarrow J_{ij}) = m_1^{\varphi J}D^1 + m_\alpha^{\varphi J}D^\alpha + m_\beta^{\varphi J}D^\beta + \ldots,$$
$$(D^\alpha \downarrow F_{ij}) = A_1, (D^\beta \downarrow F_{ij}) \neq A_1, \tag{13}$$

where D^1 is the identity representation A_1 of J_{ij} and D^β is a representation of J_{ij} that does not become the identity representation A_1 in F_{ij}. The multiplicities $m_1^{\varphi J}$, $m_\alpha^{\varphi J}$ and $m_\beta^{\varphi J}$ give the number of the components of φ that are the same, that have opposite sign and that equal zero in domain states S_i and S_j, respectively. Besides D^β other irreducible representations can appear in (13) which do not subduce A_1 in F_{ij} and produce zero components similarly as D^β. There is always $m_\alpha^{\varphi J} > 0$.

Similar decomposition holds for the representation D^λ according to which transforms the secondary order parameter λ,

$$(D^\lambda \downarrow J_{ij}) = m_1^{\lambda J} D^1 + m_\alpha^{\lambda J} D^\alpha + m_\beta^{\lambda J} D^\beta + \ldots, \tag{14}$$

with similar meaning of the multiplicities $m_1^{\lambda J}$, $m_\alpha^{\lambda J}$ and $m_\beta^{\lambda J}$ as for the primary order parameter φ. If $J_{ij} \leq L_i$, then the multiplicity $m_\alpha^{\lambda J} = 0$, i.e. $\lambda^{(i)} = \lambda^{(j)}$. If, on the other hand,

$$L_i \cap J_{ij} \neq J_{ij}, \tag{15}$$

then the multiplicity $m_\alpha^{\lambda J} > 0$, i.e. at least some of the components of λ and of λ-irreducible constituent(s) have opposite sign in domain states S_i and S_j.

For the domain pair {1, 2} of our example the decomposition (13) reads:

$$(E_u \downarrow 2_{xy} m_{\bar{x}y} m_z) = A_1 + B_1, (B_1 \downarrow m_z) = A', \tag{16}$$

where A' is the identity representation of the group m_z. From these relations it follows that, e.g. the 2-dimensional E_u-irreducible constituent of the polarization **P** can be decomposed into two components P_γ and P_α, where the first one is constant and the second one has opposite sign in domain states 1 and 2. This decomposition has a simple geometrical meaning: P_γ is the projection of **P** into the [110] direction and P_α into the [1$\bar{1}$0] direction (see Figure 2). The polarization **P** expressed in this way is given in the second row of Table I. We see that the flipping of the non-zero component P of **P** is replaced by the sum $P_\gamma + P_\alpha$ and the difference $P_\gamma - P_\alpha$. The tensor $[V^2]V$ can be decomposed in the similar manner (see the last row of matrices in Table I).

The decomposition (14) of the representation B_{1g} according to which transforms the secondary order parameter λ is

$$(B_{1g} \downarrow 2_{xy} m_{\bar{x}y} m_z) = B_1, (B_1 \downarrow m_z) = A', \tag{17}$$

i.e. $m_\alpha^{\lambda J} = 1$. Consequently, the value C of the B_{1g}-irreducible constituent in the tensor $[V^2]$ has opposite sign in domain states 1 and 2 (see Table I).

For the domain pair {1, 3} the decomposition (13) yields

$$(E_u \downarrow m_x m_y m_z) = B_{3u} + B_{2u}, (B_{3u} \downarrow 2_x m_y m_z) = A_1,$$
$$(B_{2u} \downarrow 2_x m_y m_z) = B_2, \tag{18}$$

where A_1 and B_2 are irreducible representations of the group $2_x m_y m_z$. From this decomposition we conclude that the value of the first component of the polarization has opposite sign in domain states 1 and 3 and the second component is zero in both of them. Similar conclusion holds for the values of the E_u-constituents in the tensor $T \sim [V^2]V$ (see the last column in Table I).

From the decomposition (14), which reads $(B_{1g} \downarrow 2_x m_y m_z) = A_1$, it follows that the value C of the B_{1g}-irreducible constituent remains the same in domain states 1 and 3. This also follows directly from the equality $L_1 = m_x m_y m_z = J_{13}$.

Our example illustrates a general feature of tensor distinction in transposable domain pairs. Material property tensors can be decomposed into two parts: one is the same and the other differs only in sign in the two domain states of a transposable domain pair.

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