

A SOCIAL NETWORK ANALYSIS OF THE ORIE FACULTY HIRING NETWORK

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ABSTRACT. We study the Operations Research/Industrial-Systems Engineering (ORIE) faculty hiring network, consisting of 1,179 faculty origin and destination data together with attribute data from 83 ORIE departments. A social network analysis of the faculty hires can reveal patterns that have an impact on the dissemination of educational innovations within a profession. We first statistically test for the presence of a linear hierarchy in the network and for its steepness. We proceed to find a near linear hierarchical order of the departments, which we contrast with other indicators of hierarchy, including published rankings. A single index is not able to capture the structure of a complex network, so we next fit a latent variable exponential random graph model for the faculty hires, which is able to reproduce the main observed network characteristics: high incidence of self-hiring, skewed out-degree edge distribution, low density (except at the top of the hierarchy) and clustering. Finally, we simplify the network to one where faculty hires take place among three groups of departments. We discuss the implications of these findings for the flow of education and teaching ideas within the ORIE discipline and compare our findings with those reported for other related disciplines, Computer Science and Business.

Keywords: academic market analysis, hierarchical networks, exponential random graph models, latent location graph model.

1. INTRODUCTION

A faculty hiring network is represented by a graph $G = (V, E)$ composed of vertices $v_i \in V, i = 1, \dots, n$ denoting university departments in a given academic discipline, and directed arcs $e_{ij} \in E, i, j = 1, 2, \dots, n$ whose integer value attribute denotes the number of faculty hired by department v_j who received their Ph.D.'s in department v_i . A department hiring a Ph.D. from another department generates a directed edge in the network going from the sender department to the receiver department. In the pre-internet era, the study of faculty hiring networks was confined to departments of Sociology (Schichor, 1970; Burris,

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2004) where directories with the necessary faculty information existed in print. Today, with most departments and faculty posting their information in personal web pages, studying a faculty hiring network has become easier to do even if no directories are available, provided the faculty information is gathered. There now exist analyses of faculty hiring networks in a broader range of disciplines in the US, such as Political Science (Fowler *et al.* , 2007; Schmidt & Chingos, 2007), Mathematics (Myers *et al.* , 2011), Communication (Mai *et al.* , 2015), Business (Clauset *et al.* , 2015), Computer Science (Clauset *et al.* , 2015; Huang *et al.* , 2015), Law (Katz *et al.* , 2011), and History (Clauset *et al.* , 2015).

Researchers have studied the hiring and placement patterns of their academic fields for a variety of reasons. One concept these studies attempt to clarify is the *prestige* of a department, a rather elusive and inadequately theorized concept (Burris, 2004) but popular among university administrators. Departmental prestige can be studied as an effect due to the position of the department in a hierarchy existing in the faculty hiring network, as these positions provide a ranking within a discipline. Some authors have considered hiring networks for the purpose of determining the inequality in the production of Ph.Ds (Clauset *et al.* , 2015), an apparently general phenomena in which a small number of departments produce a large fraction of the Ph.Ds hired as professors. Others have studied these networks to determine inequalities in the hiring process (e.g., of women, see Way *et al.* (2016)), or to study the sociological aspects of a discipline (Fowler *et al.* , 2007; Katz *et al.* , 2011), e.g., whether communities or a dominance hierarchy exists among departments. Hiring and placing of Ph.D. students is also studied in some areas to understand how new ideas are disseminated through a profession. In particular, these studies can provide insights about the flow of teaching and educational innovations in a field. This is the object of the present paper with respect to the Operations Research/Industrial Engineering/Systems engineering (herein, ORIE) field.

In this paper we make available and study the ORIE hiring faculty network, an effort which, to our knowledge, has not been undertaken before. The network is composed of departments/programs/schools (from now on, "departments") within these 3 inter-related fields. Appendix 1 gives a list of all the 83 ORIE departments considered and how they were selected for this study. The ORIE dataset was compiled in summer 2016. We first perform statistical tests on the existence and steepness of a linear dominance hierarchy among the 83 ORIE departments. Having found strong evidence of such hierarchy, we then introduce the notion of a minimum violation and strength (MVS) measure of departmental prestige, and use it to compute a near linear hierarchy for the ORIE network, contrasting it with other more common measures of individual importance in a social network, including published ORIE rankings (US News and NRC). Single measures of department prestige give an inherently incomplete description of a complex network, we further model the positions of each department in the network as latent variables in an exponential random graph model that allows us to find groups of related departments. With these positions, we reduce the ORIE network to a simpler network of faculty hires between and within 3 groups of departments. Throughout this study, we compare the ORIE faculty hiring network with similar networks in related academic disciplines. We conclude with a discussion of the

implications of our findings for the dissemination of teaching and educational ideas within the ORIE profession.

2. DESCRIPTIVE STATISTICS OF THE ORIE NETWORK COMPARED TO THOSE FROM RELATED DISCIPLINES.

ORIE faculty data were collected during May-June of 2016 from faculty web pages working in institutions in the USA. A list of all Ph.D. granting institutions considered and the criteria used for their inclusion are given in Appendix 1. In total, 1179 faculty from 83 ORIE departments were considered in the analysis. Table 1 lists general descriptive statistics of the ORIE network compared to those of two closely related fields, Computer Science and Business schools, networks analyzed by Clauset *et al.* (2015), excluding their “Earth” (outside US) department and links from and to this vertex). The ORIE network is considerably smaller, with an average dept. size of 14.2 faculty compared to 21.4 faculty for CS and the much bigger schools of Business with an average number of 70.1 faculty. Two

TABLE 1. Descriptive statistics of the ORIE network compared to that of Computer Science departments and Schools of Business.

Network	Vertices	Edges(% Female)	Self-edges/edges	Density	Assortativity		
					MVR	Degree	Reciprocity
ORIE	83	1179(19.5)	0.1399	0.1121	0.4614	0.1452	0.1538
CS	205	4388(25.6)	0.0711	0.0688	0.5327	0.2964	0.1264
Business	112	7856(16.8)	0.0556	0.2738	0.4330	0.2661	0.2197

distinctive characteristics of the ORIE network relative to CS and Business (see Table 1) are *a*) its much larger proportion of “incestuous” Ph.D. hiring (14% of all faculty, almost three times that in Business and close to twice that of CS) and *b*) its lower degree assortativity. Figure 1 (top) depicts the number of self-hires among the 83 ORIE departments, sorted in a hierarchy that will be explained below. Note the heavy self-hiring at the top of the hierarchy, and how prevalent self-hiring is: 74% of the ORIE departments currently have (as of summer 2016) at least one former Ph.D. student hired as faculty. This is similar than in Business schools (74%) but higher than in Computer Science departments (58%). Almost 14% of all ORIE faculty are hired by their originating department. In some departments, self-hires account for more than 40% of the faculty (see Figure 1, bottom). The high level of self hiring relative to other disciplines should be a concern, as it reduces the transfer of information and education-knowledge between departments. Our dataset, however, lacks information about cases when a faculty returns to its alma mater after working in other departments, that is, cases where the self-hiring was not direct from graduate school to faculty.

The *assortativity* index is a correlation measure of how much vertices with similar values of a given scalar tend to be connected together (for this and other descriptive network statistic definitions, see Newman (2010)). Shown in Table 1 is the assortativity with respect to the total degree of each vertex (department). This indicates a lower tendency for ORIE departments, compared to those in CS and Business, to establish hiring-placement connections with departments with a similar total number of hires and placements. The

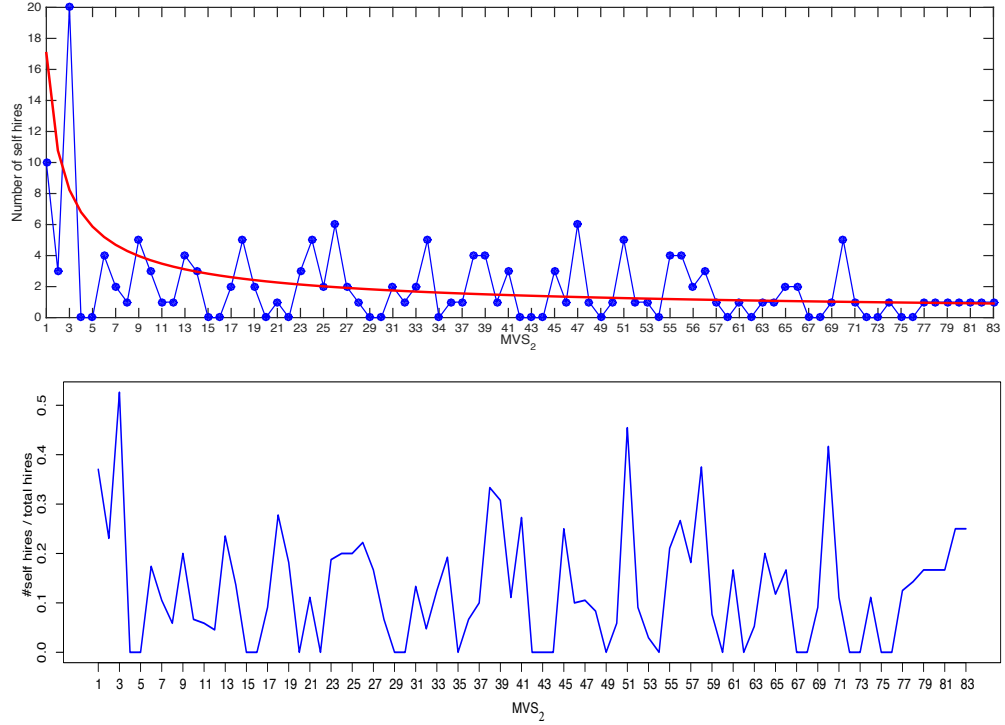


FIGURE 1. Top: in blue, observed number of self-hires among the 83 ORIE departments sorted by MVS_2 index described later in this paper. In red, number of self-hires (μ_{ii}) as predicted by the latent location ERGM model described in section 5. Bottom: self hires as a percentage of all faculty in each department, sorted by MVS_2 index. While self-hires are more numerous in the top departments, self-hires as a proportion of total faculty are significant across all the ORIE departments.

assortativity with respect to the MVR indices is an indication of how likely departments establish hiring-placement relations with departments equally ranked in a hierarchy that will be discussed in section 4.

Other descriptive statistics in the ORIE network tend to be somewhere between those of the CS and the Business networks. The connectance or density of the ORIE network, defined as the number of non-zero entries in the adjacency matrix \mathbf{Y} divided by the maximum number of possible connections between vertices is somewhere between CS (which is very sparse) and Business (which is densest). As it will be shown below, the ORIE faculty hiring network is actually quite sparse except for a group of departments, those at the top of a hierarchy, which connect more often among them. The proportion of female faculty in ORIE is 19.5%, higher than in CS (16.8%) but lower than in Business schools (25.6%), an inequality that calls for further analysis that is beyond the present study. The *reciprocity*, defined as the number of edges that are reciprocated between two vertices, is for ORIE

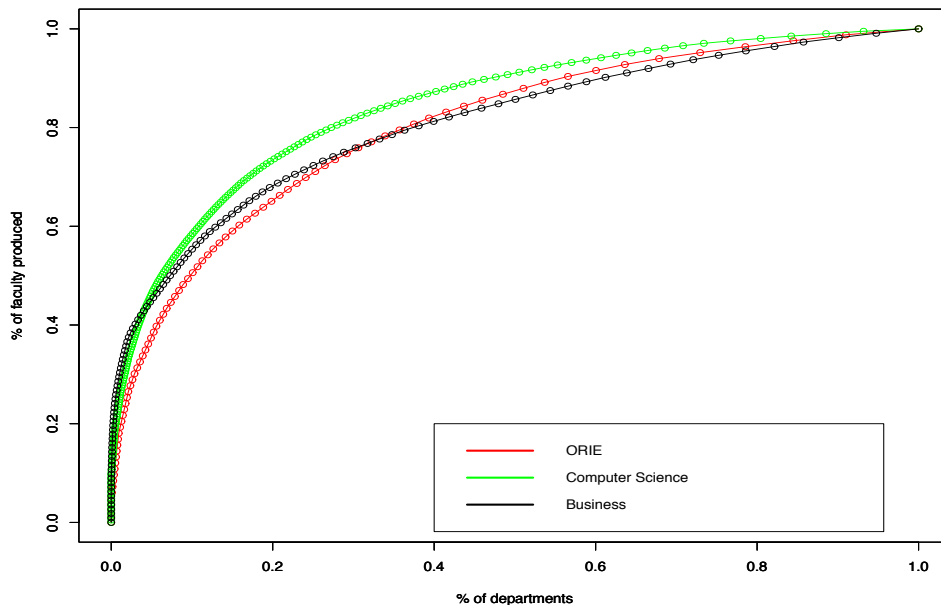


FIGURE 2. Lorenz curves indicating the inequality in the percentage of faculty production in ORIE, Computer Science, and Business as a percentage of the number of departments. About 10% of ORIE departments generate close to half of all ORIE faculty.

somewhere between CS and Business. This is a measure of how likely departments tend to establish exchanges of mutual hires over the years.

Finally, the inequality in the production of Ph.D.'s among ORIE departments, although quite large, is also between that of CS departments and Business schools. Figure 2 shows the Lorenz curves for the proportion of Ph.D.'s produced. Approximately, 10% of the ORIE departments generate about 50% of the faculty, although this is not as impressive as in Business schools, which have a very steep Lorenz curve near zero, with around 3% schools generating 40% of the faculty.

3. EXISTENCE OF A LINEAR DOMINANCE HIERARCHY

Given a faculty hiring network, a question of interest is if it is possible to order the departments in such a way to form a *linear dominance hierarchy*. The concept of dominance, popular in social networks in ecology and competitive sports, refers to an attribute of the pattern of repeated interactions between two individuals, characterized by a consistent outcome in favor of the same individual in a dyad. The consistent winner is dominant and the loser subordinate (de Vries, 1995). Individuals form a *linear dominance hierarchy* if and only if a) for every dyad (i, j) either i dominates j or j dominates i and b) every triad is transitive, i.e., for any individuals i , j and k it is true that if i dominates j and

j dominates k , then i dominates k . Transitivity is equivalent to a dominance hierarchy that is acyclic (Ali *et al.*, 1986). The notion of a dominance hierarchy has been studied in faculty networks by Clauset *et al.* (2015), Huang *et al.* (2015) and Mai *et al.* (2015). Applied to faculty hiring networks, if a department v_j hires more Ph.Ds from department v_i than the number v_i hires from v_j , the relationship between these two vertices implies a “dominance” relation of department i with respect to department j , given that j wishes more strongly to have access to the students produced in department i and not viceversa (ties are therefore allowed). In his study of animal societies, Landau (1951) defined a score structure $W = (w_1, w_2, \dots, w_n)$ where each w_j equals the number of individuals dominated by element j . A *hierarchy* is then a score structure

$$W = (w_1 = n - 1, w_2 = n - 2, \dots, w_n = 0)$$

so that the members of the society can be ordered as $1 \succ 2 \succ 3 \succ \dots \succ n$, with each dominating all the members below it, and being dominated by all those above. At the opposite extreme is an “egalitarian” society where assuming n odd, $W = (w_1 = \frac{n-1}{n}, w_2 = \frac{n-1}{n}, \dots, w_n = \frac{n-1}{n})$. Landau (1951) then introduced his “hierarchy index” h , a measure of the variability of the w_j ’s normalized so that $h = 0$ implies equality (egalitarian dominance) and $h = 1$ implies a perfect linear hierarchy, with h equal to

$$h = \frac{12}{n(n^2 - 1)} \sum_{j=1}^n \left(w_j - \frac{n-1}{2} \right)^2.$$

Of course, a perfect linear hierarchy may not exist in a society, and some violations to a perfect linear hierarchy may exist, a topic we discuss in section 4.

A practical problem when determining if a linear hierarchy exists in a society is that we may have few data points about individuals interacting, with many not interacting during the observation period. De Vries (1995) introduces a randomization test for the hypothesis of no linear hierarchy based on Landau’s h statistic that takes into account unknown tied relationships (in our case, these are departments that have never hired each other’s Ph.D. students). To perform the test, each dyad (i, j) is randomized m times and the h statistic is computed for each random sociomatrix giving a set of h_s numbers. h is also computed with the observed sociomatrix. If the empirical p-value of the test, defined as $1 - \#(h_s > h)/m$ is small, this is evidence the observed linear hierarchy is statistically different than that of a random matrix, which has no hierarchy. Ordering individuals in a linear or near-linear hierarchy is justified only if there is statistical evidence in favor of such hierarchy comparing the existing hierarchy to what could be obtained from a random matrix. A perfectly linear hierarchy (one with $h = 1$) may not be possible; finding a near-linear hierarchy ($h < 1$) is a hard combinatorial problem about which we comment below. Figure 3 shows results of the deVries tests applied to the ORIE, CS and Business networks. The empirical p-values are zero for all 3 disciplines, implying there is a statistically significant linear hierarchy in each of the three networks.

If a linear or near-linear hierarchy is significant, a related question is how steep is the hierarchy. De Vries *et al.* (2006) introduces the concept of the *steepness* of a linear hierarchy, namely, the size of absolute differences between adjacently ranked individuals in

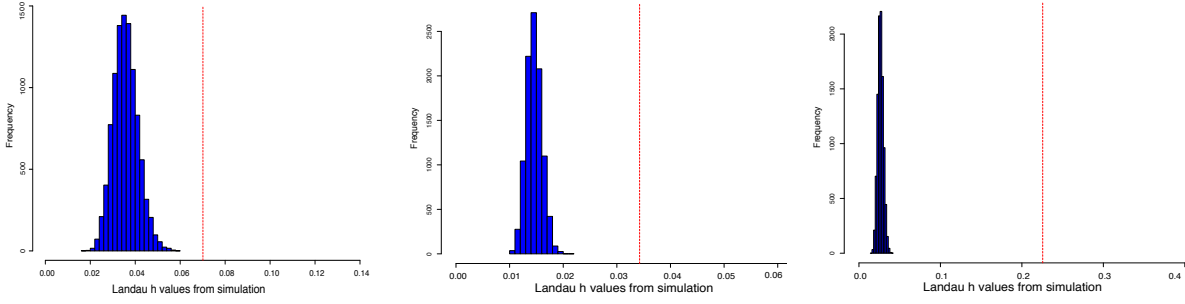


FIGURE 3. DeVries’ test for the presence of a linear dominance hierarchy in the complete ($n = 83$) ORIE network (Left), the Computer Science network (Middle, $n = 205$) and the Business network (Right, $n = 112$). Randomization distribution (10K simulations) under the null hypothesis of random graph of Landau’s h values. Red vertical lines are the observed h values. In all cases, the empirical p-values are 0.0, indicating strong evidence of the presence of a hierarchy in each of three faculty networks.

their overall success in winning dominance encounters. When these differences are large, the hierarchy is said to be steep and is called shallow otherwise. DeVries’ steepness measure is based on the D_i score by David (1987), a measure of the dominance success of an individual i given by the unweighted and weighted sum of the individual’s dyadic proportions of “wins” (Ph.D. placed in other departments) combined with an unweighted and weighted sum of its dyadic proportion of “losses” (Ph.D’s hired from other departments). The proportions of “wins” P_{ij} are defined as $P_{ij} = y_{ij}/n_{ij}$ where y_{ij} is the (i, j) entry in the sociomatrix \mathbf{Y} and n_{ij} is the total number of interactions between i and j (i.e., $n_{ij} = y_{ij} + y_{ji}$). David’s (normalized) scores are defined as:

$$D_i = \left(w_i + w_{2i} - l_i - l_{2i} + \frac{n(n-1)}{2} \right) / n$$

where $w_i = \sum_{j \neq i} P_{ij}$, $w_{2i} = \sum_{j \neq i} w_j P_{ij}$, $l_i = \sum_{j \neq i} P_{ji}$ and $l_{2i} = \sum_{j \neq i} l_j P_{ji}$. The factors containing the total number of individuals scale this term to make it vary between 0 and $n - 1$. To measure the steepness of a hierarchy, order the individuals according to D_i . Call R the rank of the individuals. Then a simple linear regression model $D = \beta_1 R + \beta_0$ is fit to the data. The estimated coefficient $\hat{\beta}_1$ is an estimate of the steepness of the hierarchy.

To test for the significance of the steepness of a hierarchy using a permutation test, assume as null hypothesis that the steepness is that given by a randomly formed network. Fitting linear regression models $D = \beta_1 R + \beta_0$ from the randomly generated networks each with ranks R will result in an empirical distribution of the $\hat{\beta}_1$ coefficient, which can then be compared to the $\hat{\beta}_1$ estimate from the actual network. Small empirical p-values imply the hierarchy is significantly steeper than that given by a simple random graph.

Figure 5 shows the fitted regression models to the David statistics for the ORIE, CS and Business networks. The ORIE and CS networks appear to have a steep hierarchy only for the first 10 departments or so. This is in contrast to Business schools which have a

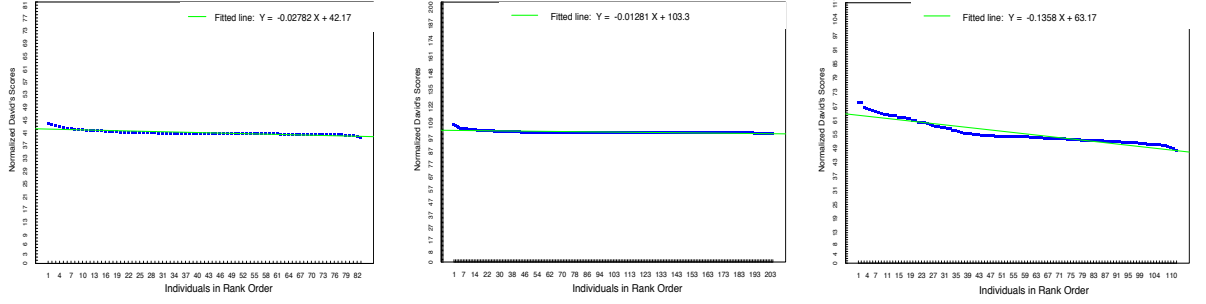


FIGURE 4. Fitted line and observed David's statistics for a linear hierarchy. Left: ORIE network ($n = 83$, slope = -0.02782). Middle: Computer Science ($n = 205$, slope = -0.01281). Right: Business ($n = 112$, slope = -0.1358). Note how for ORIE and CS the observed absolute slope increases for the first 10 departments, indicating a steeper dominance at the top of the hierarchy relative to other departments, but otherwise the hierarchy is quite flat.

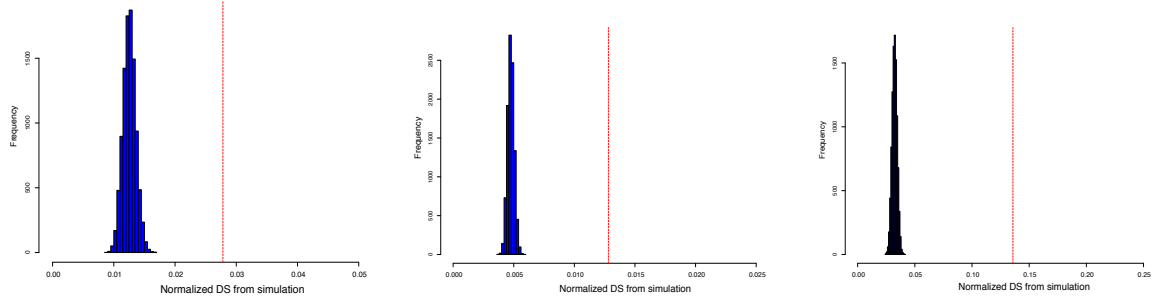


FIGURE 5. Randomized distribution (10K simulations) of the De Vries' test statistic for the significance of the steepness in the dominance hierarchy of (Left) the complete ORIE network, (Middle) the Computer Science network, and (Right) the Business network. Red vertical lines are the observed David's D_i statistics. In all cases, the empirical p-values are 0.0, indicating significance steepness in the hierarchy of each faculty network.

steeper slope for about half of the departments. Figure 5 indicates, however, that the slope is significantly different to that of a random graph in all 3 cases.

Both significance tests, for the existence of a linear hierarchy itself and for its steepness, are based on the idea that in a very sparse adjacency matrix, some observed non-zero edges may be simply ‘noise’, and should be treated as such. We will consider this potential stochasticity of the network again when both trying to find a dominance hierarchy and doing further network modeling. Similarly to the tests in this section, we will treat the observed network as a noisy realization of possible networks, and following Clauset *et al.* (2015) we will use a “bootstrapping” approach to network creation to help perform an analysis that considers other possible (sparse) networks that could have been observed.

The test for the significance of a linear hierarchy and a significant slope was repeated for the top ORIE departments in the hierarchy¹ in order to look at this group of departments. Table 2 shows the results of these tests applied to an increasing number of (sorted) departments. The slope is quite steep and significant only for the first 10 departments. Overall, for the 83 departments the slope is statistically different from zero, but its actual value is numerically small.

TABLE 2. Dominance hierarchy linearity and steepness test results for different ORIE subnetworks. The hierarchy is steeper closer to the top, where it is also much denser.

No. of departments	Density(connectance)	Observed h	p-value	Slope	p-value
Top 10	0.6000	0.5590	0.0307	-0.4281	0.0000
Top 20	0.4025	0.3036	0.0098	-0.2061	0.0000
Top 50	0.1912	0.1328	0.0004	-0.0688	0.0000
All 83	0.1121	0.0701	0.0000	-0.0278	0.0000

3.1. A caveat: “observational zeroes” in a sparse sociomatrix. The preceding analysis indicates that the 83 ORIE departments can be ranked according to a statistically significant near linear hierarchy using hire-placement data, even though the hierarchy appears to be quite flat for most of the n departments except at the very top of the hierarchy. The de Vries’ steepness test depends on the number of interactions between the actors or individuals. In animal behavior, it is well known that the ideal way to find a hierarchy is to observe pairwise competitions in a balanced (designed) “tournament” (David, 1987), but in the field of social network data an abundance of “observational zeroes” makes it more difficult to determine a hierarchy. Although no formal power analysis is available, de Vries (1995) gives some numerical evidence to suggest that the power of the linearity test to detect an existing hierarchy goes down when the frequency of observational zeroes increases, and it is quite possible the same occurs in the steepness test. Table 2 shows how the connectance (density) of the departments among the top of the hierarchy is much higher than among the rest, and this implies the David’s D_i statistics have more information about “wins” and “losses” (placements and hires) within this group than among dyads that include at least one individual ranked lower in the hierarchy.

The steepness results shown are related to the connectedness (density) of the 3 networks, since the CS network is the most sparse, followed by ORIE and then Business. The predominance of zeroes in a sociomatrix may not necessarily indicate there are no dominance relations among the dyads. As warned by de Vries *et al.* (2006) one should be careful when interpreting linearity and steepness tests for societies in which there are very few pairwise interactions recorded. This warning is by extension worth keeping in mind when trying to find a dominance relation or ranking in a hiring network. We can distinguish between three types of “observational zeroes” in the adjacency matrix (or sociomatrix): a) in animal behavior, if two individuals are not observed to interact, it may be because

¹This is the MVS₂ hierarchy discussed further below.

there is a dominance relation present, with the subordinate individual avoiding the dominant individual. We will call the similar case of departments avoiding hiring from higher ranked departments *deference* or *avoidance* zeroes. Alternatively, b) there may simply be no dominance-subordinate relation between two individuals, a case analogous to departments that do not have hiring-placement relations but “respect” each other, perhaps a less common case that we will call *equality* zeroes. Finally, c) it could be that we either have not observed long enough the network and the question of whether there is a dominance between two individuals is unresolved. This would be the case when relatively young departments have not established hiring-placement relations with many departments simply because their young age, or there is not enough edge data available publicly for some departments. These two possible cases will simply be called *unresolved* zeroes. It is not possible in practice, however, to determine what kind of “observational zeroes” one is dealing with in a particular hiring network.

With these precautionary notes we now proceed to find a near linear hierarchy in the ORIE network. For the CS and Business networks, see Clauset *et al.* (2015). The different connectance among strata of the hierarchy and its corresponding changes in steepness also justifies the search for groups of departments, a task we address in Section 5.

4. THE MINIMUM VIOLATIONS AND MINIMUM VIOLATIONS AND STRENGTH INDICES

Let \mathbf{Y} be the adjacency matrix of a faculty hiring network. Clauset *et al.* (2015) define a hierarchical index given by a permutation π that induces the minimum number of edges that “point up” the hierarchy. This is found by

$$(1) \quad \max_{\pi(\mathbf{Y})} S(\pi(\mathbf{Y})) = \max_{\pi(\mathbf{Y})} \sum_{i < j} Y_{ij} \text{sign}(\pi(j) - \pi(i))$$

Thus, if there is an edge from i to j ($Y_{ij} > 0$) and $\text{sign}(\pi(j) - \pi(i)) = -1$ (i.e., $\pi(j) < \pi(i)$) this means a lower ranked department in the hierarchy² has placed a faculty at a higher ranked department, in other words, we have a “violation” of the hierarchy implied by π and this will decrease $S(\pi(\mathbf{Y}))$. Likewise, if there is an edge from i to j and $\text{sign}(\pi(j) - \pi(i)) = +1$ (i.e., $\pi(j) > \pi(i)$), this is not a violation, and will make $S(\pi(\mathbf{Y}))$ increase. A permutation $\pi(\mathbf{Y})$ that maximizes (1) is called a *minimum violation ranking* (MVR) which has been proposed as the optimal way of rankings players in a round-robin tournament (Ali *et al.* , 1986). For networks where a perfect linear hierarchy does not exist ($h < 1$, thus violations are unavoidable) solving (1) is a hard combinatorial problem. Problem (1) is in particular equivalent to reordering the columns and rows of the adjacency matrix such that we get an upper triangular matrix, a problem which has been proved to be NP-complete (Gerbner *et al.* , 2016). Appendix table A1 lists the MVR rankings of the 83 ORIE departments considered in this study using the algorithm provided in (Clauset *et al.* , 2015).

In the field of animal behavior, de Vries (1998) proposes instead to use the *strength* of the violations in a sociomatrix as a criterion to minimize, in order to obtain a dominance

²Recall rank one is highest.

relation in a society of animals. This is defined as:

$$(2) \quad \min_{\pi(\mathbf{Y})} S(\pi(\mathbf{Y})) = \min_{\pi(\mathbf{Y})} \sum_{i>j} (i-j)Y_{ij}$$

where the difference $(i-j)$ (with $i > j$) measures the strength of a violation in the ranking (which exists if $Y_{ij} > 0$). Entries under the diagonal are *unexpected faculty hires*, where department j hires a number of faculty from a lower ranked department i . The resulting rankings from solving (2) will be referred in this paper as the *Minimum violations and strength* rankings and will be denoted by MVS_1 . Table A1 in the Appendix lists the MVS_1 rankings of the ORIE departments, obtained by modification of the stochastic search algorithm in Clauset *et al.* (2015).

A problem with both the MVR and MVS_1 rankings is that a department that places few of its own Ph.D. students, despite hiring from the top departments, may be ranked very low. A department that consistently hires from top ranked departments should be ranked higher than one that not only does not place its Ph.Ds but also does not have any hiring interactions with the top ranked group. To illustrate, a bar graph of the adjacency matrix \mathbf{Y} sorted according to the MVS_1 rankings indicates the nature of the problem (Figure 6, left): while both (1) and (2) tend to minimize the number of entries below the diagonal, the entries above the diagonal are not considered. Note the large Y_{ij} values in the upper right corner of the matrix plot; these correspond to low ranked departments under the MVS_1 criterion that hired repeatedly from the very top departments in the ranking, yet they ended up with a very low MVS_1 ranking. To correct this anomaly, we propose to also account for this type of secondary violation, which we will call *unexpected placements* i.e., when a top department places too many Ph.Ds in other lower ranked departments that should not be that attractive for their graduating students. Penalizing this kind of violation will result in an improved ranking for a department that “hires high”. A compounded criteria, including unexpected hires and unexpected placements is:

$$(3) \quad \min_{\pi(\mathbf{Y})} S(\pi(\mathbf{Y})) = \min_{\pi(\mathbf{Y})} \sum_{i>j} (i-j)Y_{ij} + (i-j)Y_{ji}$$

where Y_{ji} in the second term considers abnormal entries above the diagonal (since $i > j$) and $(i-j)$ measures the strength of this type of violation, similarly to (2). It would be wrong to use this compound criteria as it gives equal weight to the two types of violations, whereas unexpected hires (below the diagonal entries) should be accounted more severely than unexpected placements (above the diagonal). Assigning weights to each objective is *ad-hoc* and therefore not a solution. Instead, the pairwise stochastic switching algorithm that we utilize (from Clauset *et al.* (2015)) in solving (2) was modified to accept a new ranking (i.e., exchanging the rankings of two departments in question) if either:

- (1) the number of violations is lower after the switch, or
- (2) the number of violations is the same as before the switch, but the sum of the strengths of the two types of violations, $\sum \sum_{i>j} (i-j)Y_{ij} + (i-j)Y_{ji}$ is lower after the switch.

We refer to the resulting rankings as MVS₂ rankings. They are given in Appendix Table A1.

A concept related to the two types of violations, unexpected hires and unexpected placements, found in the area of sport teams rankings, is that of an adjacent matrix in “hillside” form. In sport team rankings, the adjacency matrix contains the point or goal differential between all pairs of teams in a tournament. “Hillside” form is then defined as follows:

Definition. (Pedings *et al.* (2012)). A matrix \mathbf{Y} is in *hillside* form if it is ascending along all rows and descending along all columns, i.e., if

$$\begin{aligned} Y_{ij} &\leq Y_{ik}, & \forall i, \forall j \leq k, & \text{ and} \\ Y_{ij} &\geq Y_{kj}, & \forall j, \forall i \leq k. \end{aligned}$$

A sociomatrix in a sports tournament contains goal differentials, and therefore the hillside form as defined above is the ideal form of a hierarchy. But in a faculty hiring network the definition of a hillside form adjacency matrix needs to be modified since we wish to find a hierarchy such that we have both descending columns *and descending rows* as well. In contrast to winning by more goals against decreasingly ranked opponents, higher ranked departments should hire fewer faculty in decreasingly ranked departments.

The MVS₁-ranked matrix (see Figure 6, left) is not in hillside form since there is a “peak” on the upper right cell of the matrix. A perfect hillside form may not exist, and finding a ranking closest to hillside form requires an LP formulation with $O(n^3)$ constraints (Pedings *et al.* , 2012). Although Pedings *et al.* proposed an approximation algorithm, there are no warranties as to how good the solutions may be. Furthermore, these methods do not consider the strength of the violations, only their number. For these reasons, the stochastic pairwise switching search method was utilized instead, as described above. Figure 6 (right) shows the adjacency matrix for the ORIE departments sorted according to the MVS₂ indices. Note how the matrix is closer to “hillside” form, with both rows and columns closer to being both descending.

Bootstrapping the observed network. We provide more robust MVS₂ indices by optimizing 1000 randomly sampled (with replacement) ORIE networks. This “bootstrapping” approach works by sampling edges, with replacement, from the observed network, such that the probability of sampling an edge is proportional to the number of faculty in that edge. Each bootstrapped network was then optimized with respect to their MVS₁ and MVS₂ hierarchies, and the resulting MVS values were recorded for each department. The results for MVS₂ are shown on Figure 7. The final indices MVS₁ and MVS₂ reported on Table A1 are those corresponding to the ranks computed based on the average MVS values from the 1000 bootstrapped networks. Note how there is less uncertainty in the MVS₂ values in the extreme rankings, and more uncertainty for those ranked in the middle, a phenomenon also reported for CS and Business networks (Clauset *et al.* , 2015).

Correlations between different ranks. Table 3 shows the Spearman correlation coefficients between various rankings computed for the ORIE departments, including two published rankings (NRC and US News & World Report, see Appendix) and the MVR

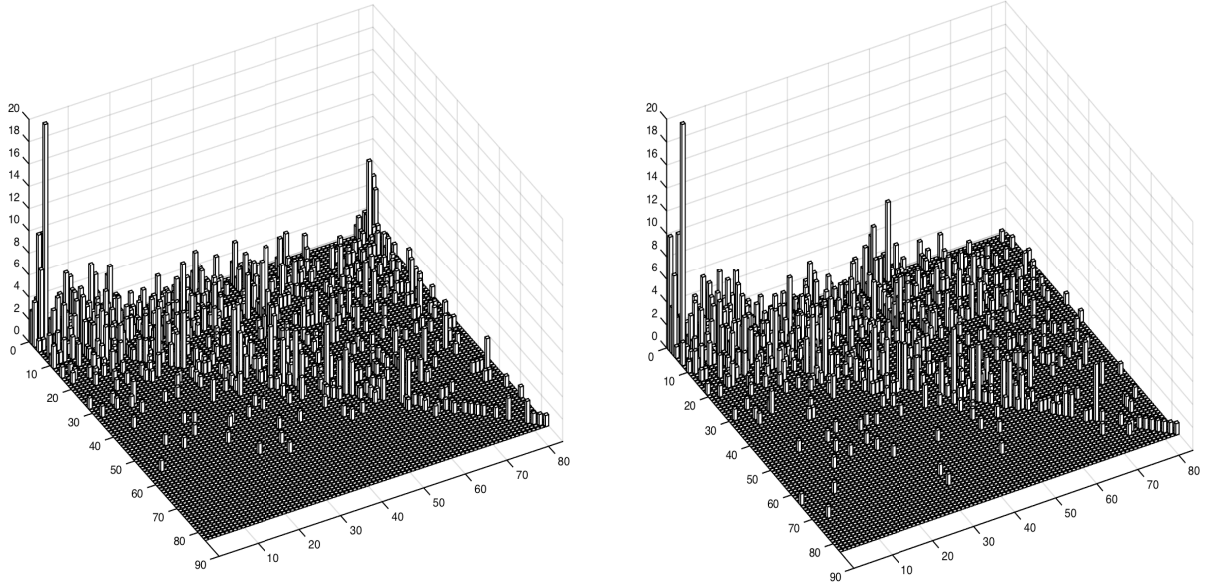


FIGURE 6. Left: matrix bar plot of the \mathbf{Y} adjacency matrix under the MVS_1 rankings, which considers the strength of a type 1 violation (unexpected placement of a faculty from a lower to a higher ranked department). Right: matrix bar plot of the \mathbf{Y} adjacency matrix under the MVS_2 rankings, which considers the strength of a type 1 violation and also of type 2 violations (unexpected number of hires from lower ranked departments). The plot on the right is closer to “hillside” than the one on the left, a consequence of the different objectives. Note the peak in the upper right corner in the plot on the left, these are departments that hire an unexpected number of faculty from the top departments, yet ended up, under the MVS_1 rankings, in a very low position. The MVS_2 rankings ameliorate this situation to a certain degree.

and MVS_2 rankings described earlier. All these rankings are shown in table A1 in the Appendix. Not surprisingly, the MVR and MVS_2 rankings are highly correlated, but a more unexpected high correlation is between MVS_2 rankings and “Hub” importance ranks. In a social network, *Hubs* and *Authorities* are types of vertices defined in an intertwined manner: a Hub is a vertex that points to many other vertices with high authority, and an authority is a vertex pointed to by many hubs (Newman, 2010). In a faculty hiring network, vertices with high hub ranking are departments that “feed” faculty to departments sought after by faculty candidates, thus Hub importance refers to *placement capacity* to departments that in turn are important (similarly the authority ranks refer to *hiring capacity*, the ability to attract and hire faculty from important departments). MVS_2 is also correlated with the out-degree rankings, but the correlation is not as high as with hub importance: a department must not only generate many Ph.Ds but must place them in the best possible departments. The PageRank, (left) Eigenvalue, and out degree rankings are all highly

correlated because the importance of “centrality” in a faculty hiring network is bestowed by out degree.

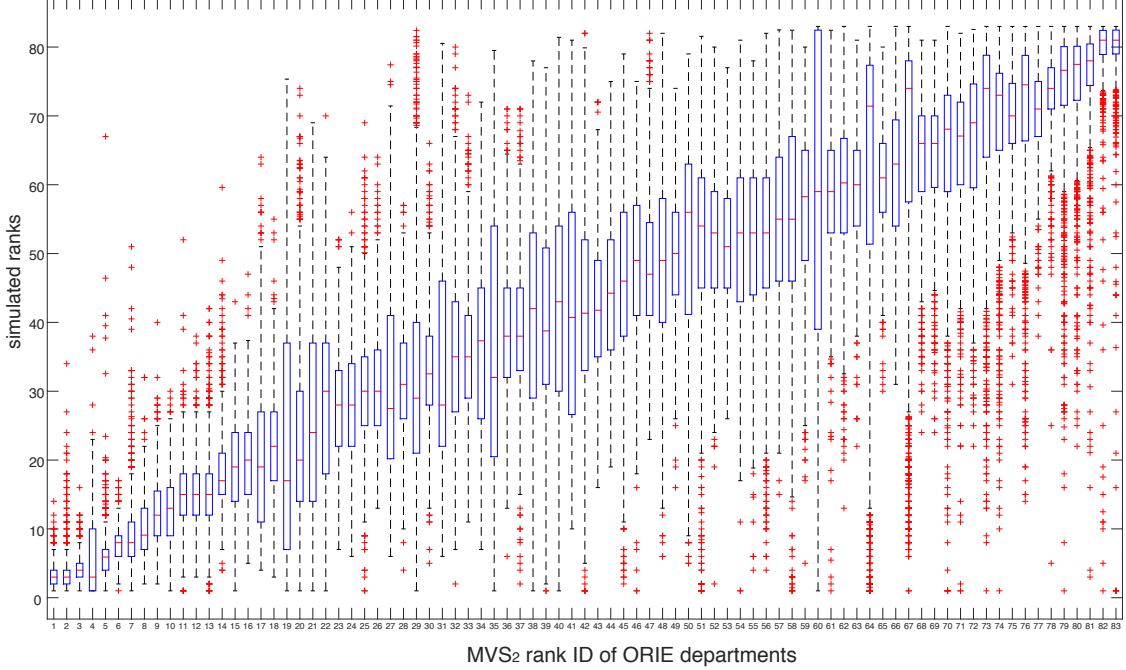


FIGURE 7. Boxplots of the bootstrapped Minimum Violation and Strength (MVS_2) ranks for all 83 departments in the ORIE network, sorted by mean MVS_2 value (1000 bootstrapped networks). The reported MVS_2 ranks correspond to the average values. Note how the extreme ranks are less uncertain, a characteristic also reported for other disciplines (Clauset *et al.*, 2015).

While there is significant correlation between their published rankings and the MVS_2 and MVR indices, there are significant differences especially among the top 20 departments (see Appendix table A1). For the top 20 MVS_2 departments, the Spearman correlation is 0.796 and 0.711 between the MVS_2 and US News and NRC rankings, respectively, with two departments in the top 20 MVS_2 indices not in the US News list.

5. LATENT LOCATION VARIABLES AND CLUSTERING OF ORIE DEPARTMENTS: A LATENT EXPONENTIAL RANDOM GRAPH MODEL

Descriptive statistics as those in section 2 can only provide partial information about the structure of a complex network. Single indices such as the MVR and MVS indices (or such as published rankings) that try to capture the “prestige” of an academic department are inherently incomplete. One feature that was evident from the descriptive analysis of the ORIE network (Sections 2 and 4) was that there is a core of departments at the top

TABLE 3. Spearman correlation coefficients between the ranks of common measures of vertex importance, including public rankings, in the observed ORIE network. Only pairs of entries with no NA values were considered. (USN = US News 2016 ranks, NRC = 2011 NRC ranks).

	MVS ₂	MVR	USN	NRC	In-deg	Out-deg	Eigen	PgRank	Bet.	Hub	Auth.
MVS₂	1.000										
MVR	0.921	1.000									
USN	0.857	0.776	1.000								
NRC	0.849	0.819	0.876	1.000							
In-deg	0.521	0.275	0.690	0.499	1.000						
Out-deg	0.884	0.832	0.825	0.773	0.585	1.000					
Eigen	0.873	0.840	0.757	0.789	0.438	0.863	1.000				
PgRank	0.861	0.860	0.799	0.775	0.461	0.942	0.938	1.000			
Bet.	0.490	0.418	0.479	0.518	0.539	0.754	0.659	0.709	1.000		
Hub	0.923	0.855	0.857	0.845	0.569	0.933	0.917	0.906	0.637	1.000	
Auth.	0.724	0.557	0.801	0.693	0.776	0.613	0.607	0.574	0.341	0.712	1.000

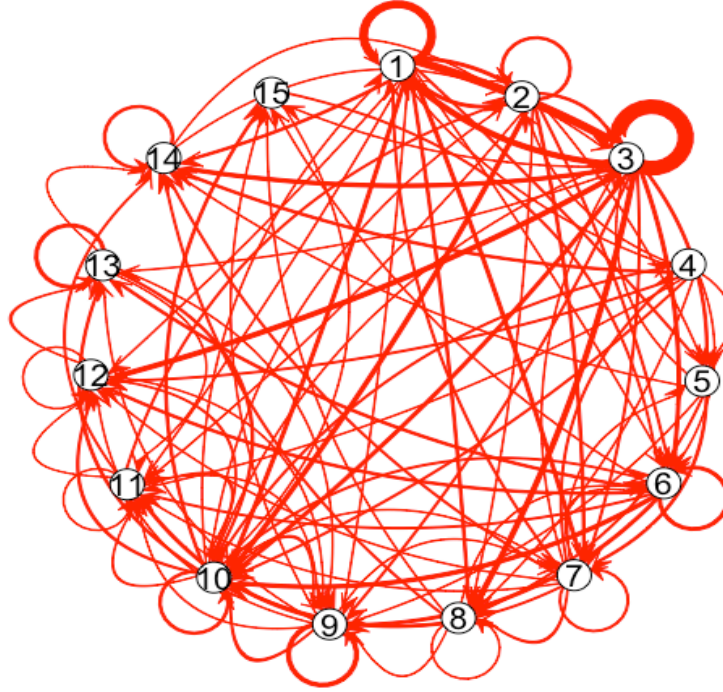


FIGURE 8. ORIE faculty hiring network for the top 15 departments sorted according to MVS₂ index. Edge width is proportional to number of hires. The edge density (or connectance) of the ORIE network is high only for the departments at the top of the hierarchy, but otherwise it is quite sparse, see Table 2.

of the hierarchy that form denser connections, while the periphery is much more sparsely connected. This indicates that in order to understand the structure of the network, rather than finding a linear hierarchy based on single indices, it is worth finding groups of similar departments.

In this section we consider the observed ORIE network as a noisy realization, or sample, from a stochastic network model. We consider a particular type of exponential random graph model (ERGM) in which the conditional probability of a tie between two actors, given covariates (if any), depends on the distance between actors in an unobserved or latent “social space” (Krivitsky & Handcock, 2008).

In a latent ERGM model, the sociomatrix (or adjacency matrix) \mathbf{Y} is viewed as a random variable that depends on parameters β , covariates \mathbf{x} and positions $\mathbf{Z} = \{\mathbf{Z}_i\}_{i=1}^n$ in the social space. The latent ERGM model we used is:

$$\begin{aligned} P(\mathbf{Y} = \mathbf{y} | \beta, \mathbf{x}, \mathbf{Z}) &= \prod_{i,j} P(Y_{i,j} = y_{ij} | \beta, \mathbf{x}, \mathbf{Z}) \\ P(Y_{ij} = y_{ij} | \beta, \mathbf{x}, \mathbf{Z}) &= f(y_{ij} | \mu_{ij}) = \frac{\exp(-\mu_{ij}) \mu_{ij}^{y_{ij}}}{y_{ij}!} \\ \log(\mu_{ij}) = E[Y_{ij} | \beta, \mathbf{x}, \mathbf{Z}] &= \eta_{ij}(\beta, \mathbf{x}, \mathbf{Z}) \\ \eta_{ij}(\beta, \mathbf{x}, \mathbf{Z}) &= \sum_{k=1}^p x_{k,i,j} \beta_k - |\mathbf{Z}_i - \mathbf{Z}_j| \end{aligned}$$

We thus adopted a Poisson density for the number of hires between two departments Y_{ij} and a link function $\eta = g(\mu) = \log(\mu_{ij})$ which is log-linear in the covariates and in the distances in the Euclidean latent space (defined by the \mathbf{Z}_i ’s). The latent positions are assumed to follow a mixture of spherical multivariate normals in the social space, which could have in principle any dimension d :

$$(4) \quad \mathbf{Z}_i \stackrel{\text{i.i.d.}}{\sim} \sum_{g=1}^G \lambda_g N_d(\boldsymbol{\mu}_g, \sigma_g^2 \mathbf{I}_d), \quad i = 1, \dots, n$$

where $0 \leq \lambda_g \leq 1$ is the prob. an individual belongs to group g ($\sum_g \lambda_g = 1$)

Fitting the model implies finding estimates for the parameters $\beta, (\lambda_g, \boldsymbol{\mu}_g, \sigma_g^2)$ for $g = 1, \dots, G$. A Bayesian formulation has been proposed by Krivitsky et al. (Krivitsky *et al.*, 2009) and an MCMC approach based on Gibbs sampling updates for $\beta, \boldsymbol{\mu}_g, \sigma_g, \lambda_g$ and a Metropolis-Hasting update for the latent positions \mathbf{Z}_i . This method has been implemented in the `latentnet` (Krivitsky & Handcock, 2008) R package³. We use the method described in Krivitsky & Handcock (2008) to setup the priors for all hyperparameters.

For the MCMC estimation, for each alternative model we used a warm-up period of 50,000 observations, and then ran the MCMC routine for 5,000,000 more iterations, collecting statistics only every 100 iterations (i.e., 50,000 more observations).

³The MCMC fitting routine is in function `ergmm`. This function allows one to use certain model terms that do consider self-loops, an important characteristic of the ORIE network as discussed in section 1.

There are not many techniques available for ERGM model selection. We selected a final model comparing the alternative models' Bayesian Information Criterion (BIC), considering whether the MCMC estimation procedure converged satisfactorily or not for a given model, and looking at model diagnostics involving the prediction performance of each model considered. Models with 2-dimensional ($d = 2$) location vectors \mathbf{Z}_i had difficulty converging; the chains did not show adequate mixing. For $d = 3, G = 2$ (2 groups or clusters in 3 dimensions), BIC=5171 but the β_j coefficients failed to converge; for $d = 3$ and $G = 3$, BIC=5221 (somewhat worse) but it had satisfactory MCMC convergence for all its parameters (see Figures 9 and 10).

The fitted final model, obtained from the posterior means of all parameters, is:

$$\begin{aligned}
 \log(\mu_{ij}) &= \log(E[Y_{ij}|\boldsymbol{\beta}, \mathbf{x}, \mathbf{Z}]) = \beta_0 + \beta_1 x_{ii} + \beta_2 x_i + \beta_3 x_j - |\mathbf{Z}_i - \mathbf{Z}_j| \\
 (5) \quad &= \underbrace{2.8363}_{(2.52, 3.10)} + \underbrace{0.1722}_{(0.09, 0.24)} x_{ii} \underbrace{-1.1190}_{(-1.21, -1.02)} x_i + \underbrace{0.2828}_{(0.19, 0.37)} x_j - |\mathbf{Z}_i - \mathbf{Z}_j|
 \end{aligned}$$

where $x_{ii} = \log(\text{MVS}_2(i))$ if $i = j$ and $x_{ii} = 0$ for $i \neq j$ is a self "loop" covariate, modeling self-hires, $x_i = \log(\text{MVS}_2(i))$ is a "sender" covariate and $x_j = \log(\text{MVS}_2(j))$ is a "receiver" covariate. Numbers in the under braces are the 95% credible posterior intervals for each coefficient. The sender and receiver covariates model the propensity of a department to either provide or receive faculty. The motivation for this model was that the MVS_2 indices can provide an indication of the number of faculty hired from a department by another. Taking the $\log(\text{MVS}_2)$ we are saying the effects are linear, given this is a log-linear model.

5.1. Model prediction performance. The fitted latent location ERGM model (5) adequately predicts the expected value of the number of hires within and between departments. Using $i = j$ in (5), the predicted mean self-hire values μ_{ii} are obtained, and these are contrasted with the observed self-hired values in the ORIE network in the top plot of Figure 1, showing excellent agreement. Figure 13 diagrammatically shows the fitted expected number of hires between all 83 ORIE departments, which can be compared to the plot on the right of Figure 6, again demonstrating very close behavior to the observations. More specific model diagnostics for ERGM models are based on Monte Carlo simulations of the fitted network and comparison of the observed statistics to those in the ensemble of simulations.

We simulated one thousand networks from the posterior of the fitted model using the `latentnet` R package and compared the simulated networks with the actual ORIE faculty network. Figure 11 contrasts the actual observed values for the in and out degree distribution (red) with the 95% intervals of the simulated values for these statistics (blue boxplots). Both distributions are approximately reproduced by the model, including the very skewed out-degree distribution. There is some over-generation of nodes with in-degrees equal to 6 and 7, and over-generation of nodes with out-degrees equal to one edge. Figure 12 (left) contrasts posterior simulations of the minimum geodesic distances between the vertices and the actual values, which is reproduced well by the model. The plot on the right contrasts the posterior simulations vs. actual values for the edge-wise shared partners, defined as

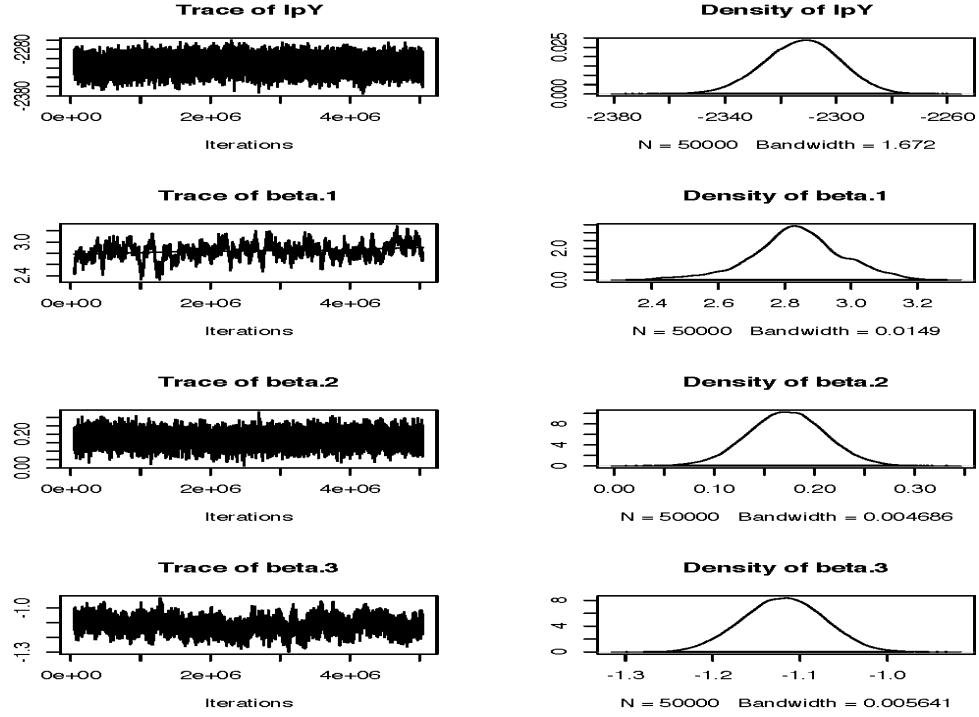


FIGURE 9. MCMC diagnostics for model (5): log-likelihood and values of β_0, β_1 , and β_2 . All convergence diagnostics are adequate.

the number of edges e_{ij} such that both i and j have k common neighbors. The peculiar shape of this distribution is very well reproduced by the latent ERGM model. Finally, in figure 14 (left) we first computed the posterior edge density distribution of the simulated networks and contrasted it with the actual value (red dotted line), which falls among the simulated values, indicating the model can reproduce networks with the correct density of edges. The number of departments that self-hire ranges from around 49 to 78 in the posterior simulated networks, and this includes the observed value (61) in the real ORIE network (red dotted line in figure 14, center). Finally, the proportion of Ph.Ds self hired by their original department ranges from about 12% to 20%, again including the observed 14% (red dotted line, Figure 14, right).

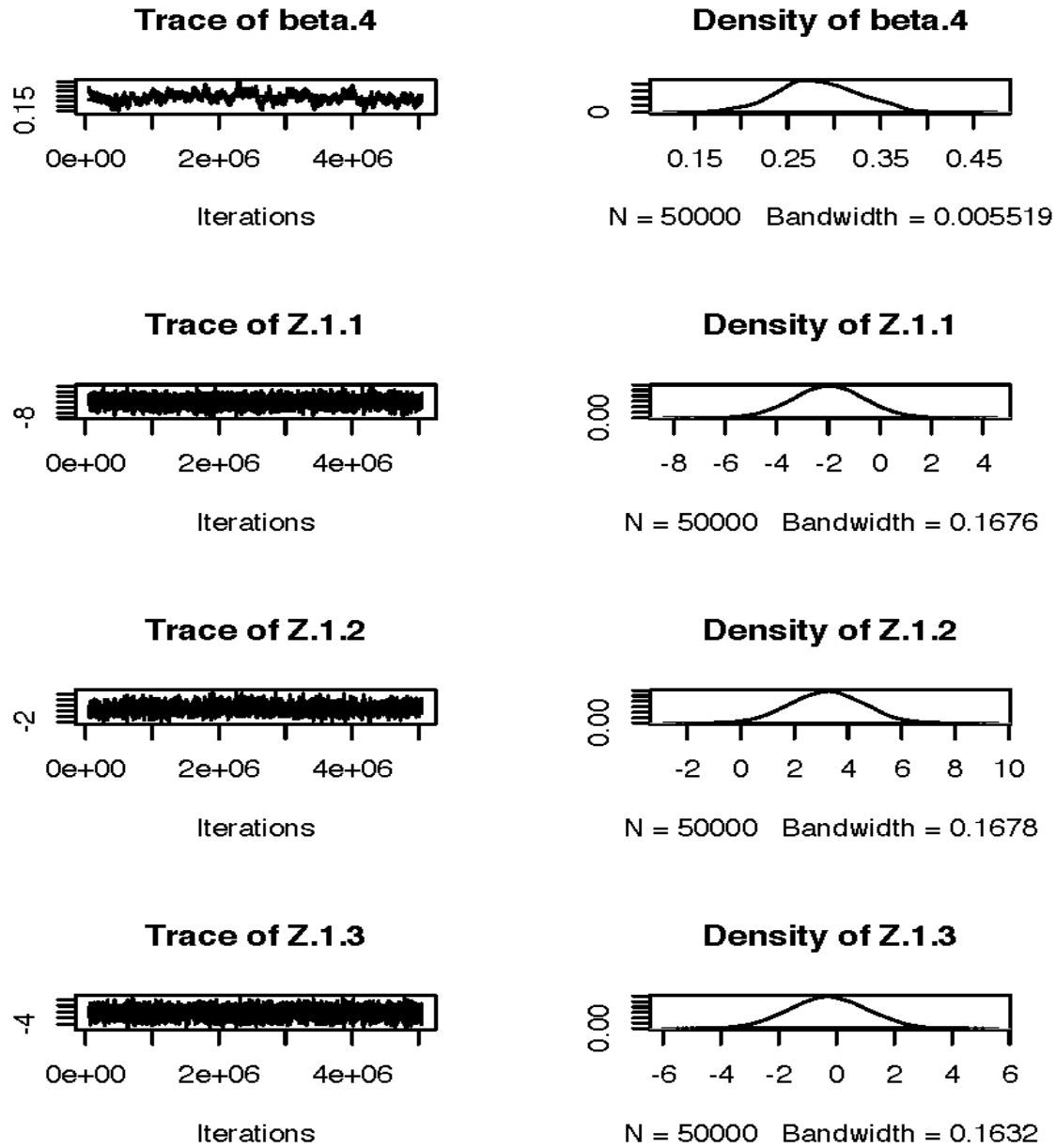


FIGURE 10. MCMC diagnostics for model (5), cont.:values of β_4 , Z_1 , Z_2 , and Z_3 . All convergence diagnostics are adequate.

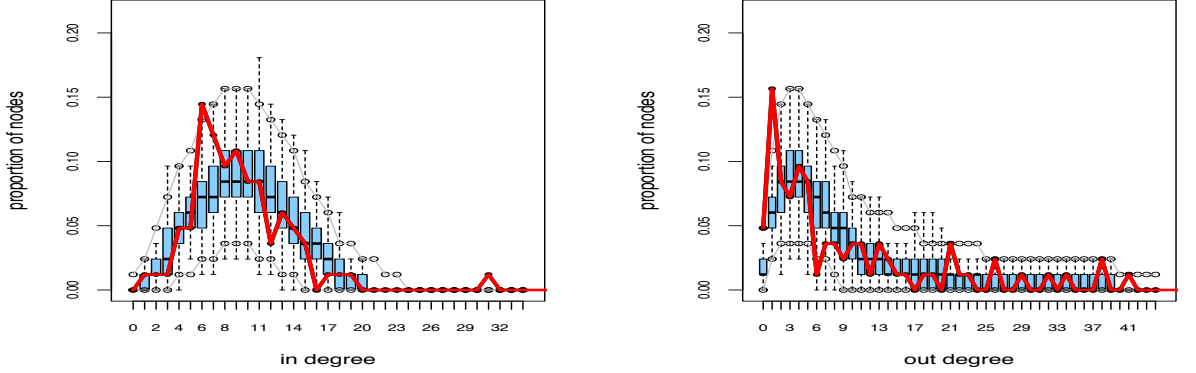


FIGURE 11. Diagnostic plots for the ERGM model (5) based on 1000 simulations. Left: in-degree distribution. Right: out degree distribution. Boxplots are simulated values from (5), red continuous lines are the observed values. With exception of a slight over-generation of vertices with in-degree 6 and 7 and out-degree equal to one, the fitted model reproduces the observed degree distributions.

We conclude from these “goodness of fit” posterior simulations that the general structure of the ORIE faculty hired network is captured by the fitted latent ERGM (5).

5.2. Determination of groups of departments. The latent ERGM model permits us to determine G groups in which the nodes are clustered in the location space \mathbf{Z} . We found the clustering provided by the ERGM model unreliable, the clusters did not seem to form in any particular meaningful way. Instead, to select groups of departments, we took the mean of the posterior of the latent locations \mathbf{Z}_i , $i = 1, \dots, 83$ and ran the PAM (Partitioned around medoids) algorithm (Kaufman & Rousseeuw, 2008) to find 3 groups (clusters) of departments. This provided a much better group separation than using the λ_g proportions in (4) as the basis of clustering. The number of clusters was found using PAM and the “Gap” statistics (Tibshirani *et al.*, 2001) that determines a goodness of clustering measure. The best number of clusters is either 1 (no clustering) or 3 from the gap statistics (see Figure 15). We therefore form 3 groups or clusters of departments, and the groups are displayed in different color in Figure 16 and listed in Appendix Table A1.

It is notable how well the latent groups detect departments who more consistently hire from top departments in the hierarchical MVS₂ order, and classify them in the same group. It is not possible to determine the groups if we only look at any of the single indices, including the MVS₂ indices.

Using the three groups of departments, we can reduce the ORIE network to a simple network where faculty flow within and between the three aggregate groups (Figure 17). Most of the hires (62 %) take place within groups, only 38% is between groups. Note how groups 3 and 1 are very thinly connected: only 3 faculty receiving their Ph.D. in the

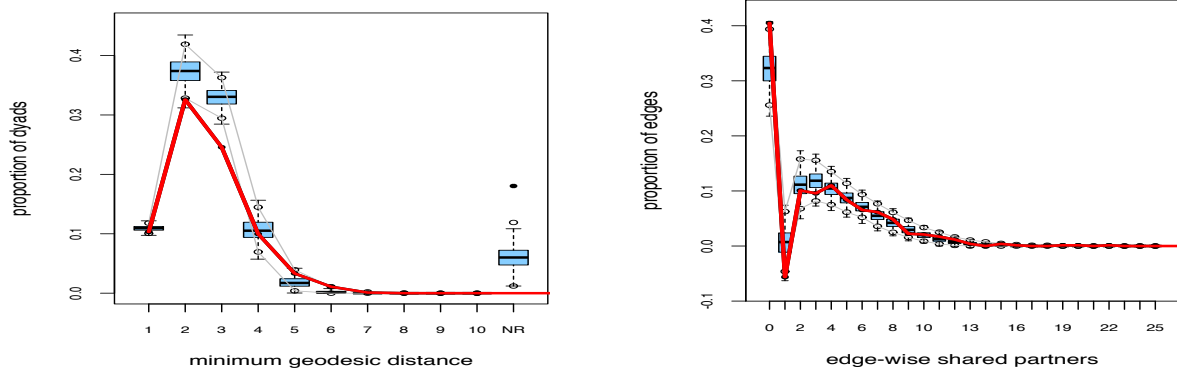


FIGURE 12. Diagnostic plots for the latent ERGM model (5) based on 1000 simulations. Left: geodesic distance distribution. Right: edge-wise shared partner distribution. The number of edge-wise shared partners is defined as the number of edges e_{ij} such that both i and j have k common neighbors (“shared partners”). Boxplots are simulated values from (5), red continuous lines are the observed values. The fitted model is able to reproduce the observed statistics. Note how almost 40% of the edges in the ORIE network have no common neighbors.

34 departments in group 3 have been hired by the 14 departments in group 1. Inversely, only 14 Ph.D.’s from group 1 have been hired in departments from group 3. Group 3 has also provided few (26) faculty to group 2. This indicates that the ORIE network is highly separated in clusters, with departments in the bottom of the hierarchy (group 3) producing almost no faculty for those on the top. Interestingly, the “intermediate” departments in group 2 (35 departments) include the top 7 departments as ranked by “betweenness” and overall have the lowest (i.e. most important) betweenness average ranks (38.3 for group 2, 42.2 for group 1, 44.7 for group 3). It could be argued that the intermediate departments in group 2 keep the ORIE as a single, connected discipline. It is an interesting question for further work to see if the degree of interaction between groups 1 and 3 is higher or lower than the interaction threshold between different (but close) disciplines, such as OR and Computer Science, for instance.

6. CONCLUSIONS.

We have demonstrated the existence of a near linear hierarchy in the ORIE faculty hiring network through the application of contemporary techniques in statistical analysis of social networks that exclude subjective assessments. A peculiarity of this network compared to other disciplines is the heavier occurrence of “incestuous” self-hires, more common in number at the top of the hierarchy but important as a percentage of the faculty hires throughout the 83 departments considered. While we provided an approximate linear

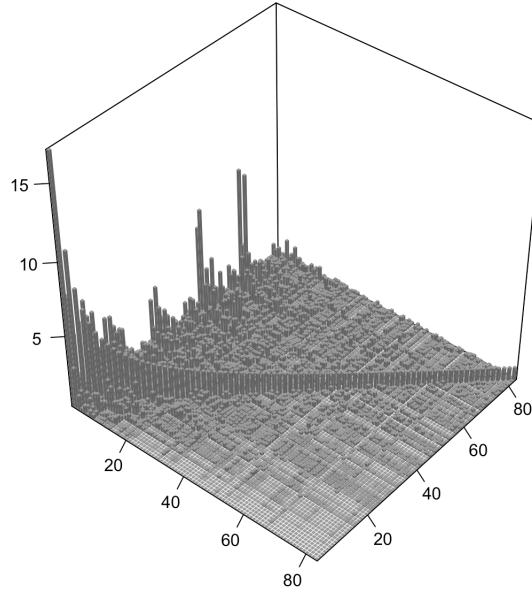


FIGURE 13. Predicted expected number of faculty hires in the ORIE network ($\hat{\mu}_{ij}$) given by the fitted latent location ERGM model (5). Height equals expected number of faculty, axes are the 83 departments sorted according to MVS_2 . Compare with the observed number of hires, the plot on the right of figure 6.

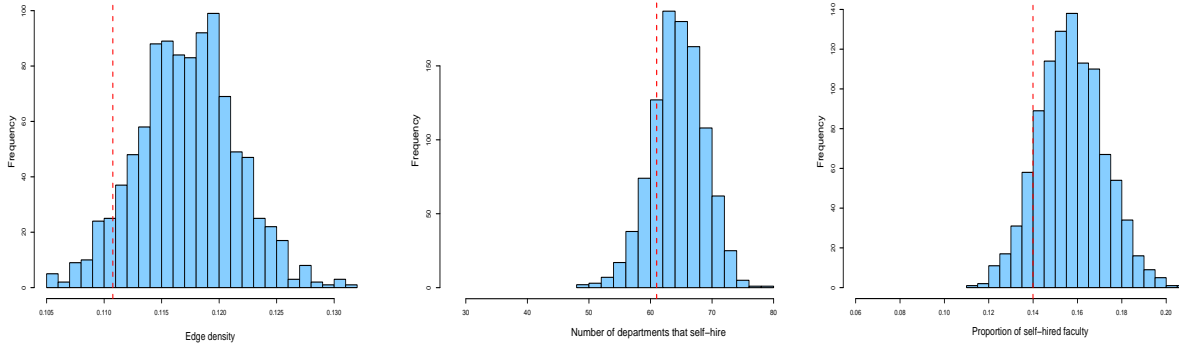


FIGURE 14. Diagnostic plots for the latent location ERGM model (5) based on 1000 simulations. Left: edge density distribution. center: number of departments that self-hire. Right: proportion of self-hired faculty. Observed values in ORIE network are given by red vertical lines. In all cases, the fitted model reproduces the observed values in the actual network.

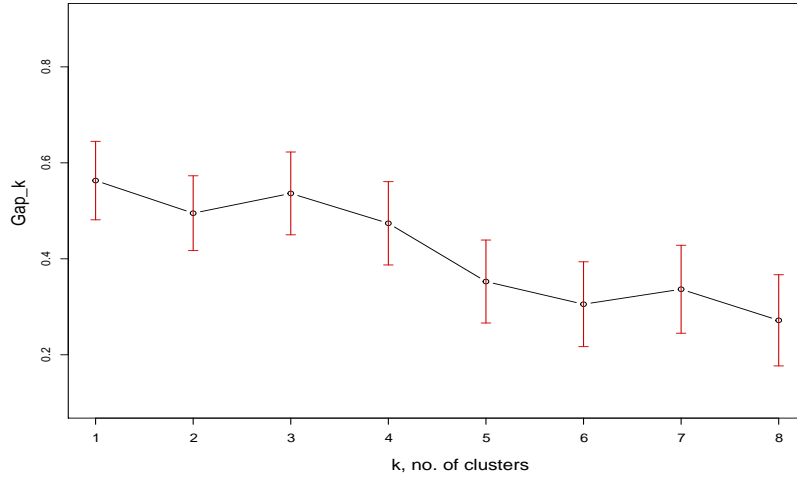


FIGURE 15. Simulated “gap” statistics (Tibshirani *et al.*, 2001) to determine the number of clusters in the latent variables \mathbf{Z}_i identified in the exponential random graph model (5). Either 1 or 3 clusters are identified, we chose 3 clusters due to better interpretability.

hierarchy index (Minimum Violation and Strength, MVS_2) obtained through bootstrapping the original network and therefore not sensitive to the unequal density of edges through the network (and that stand in contrast to published rankings), a single number fails to capture the structure of an academic hiring network. We therefore proposed and fitted a latent location Exponential Random Graph Model (ERGM) that uses MVS_2 only as a covariate, and successfully captures the main characteristics of the ORIE faculty network, including its node degree distributions, its incidence of self-hiring, and its edge density distribution among others. This model allowed us to simplify the ORIE network to one with only three groups of departments, with most hiring taking place within groups, also showing the little interaction between groups at the extremes of the hierarchy. Furthermore, departments in the intermediate group act as a “link” between the other two groups, keeping the network connected as a single discipline, with departments in this group having highest “betweenness” importance (Newman, 2010).

The hierarchical nature of the ORIE network has consequences for the dissemination of ideas among the department members. While publication and professional meetings are the usual ways faculty groups learn about research work done elsewhere, the same can usually not be said about the dissemination of teaching and educational novelties. The immediacy and impact that new hires can have in the educational life of a department cannot be understated. The transmission of educational ideas and techniques, despite journals that treat education topics, is much more limited than that of research topics moving through a hierarchical hiring network.

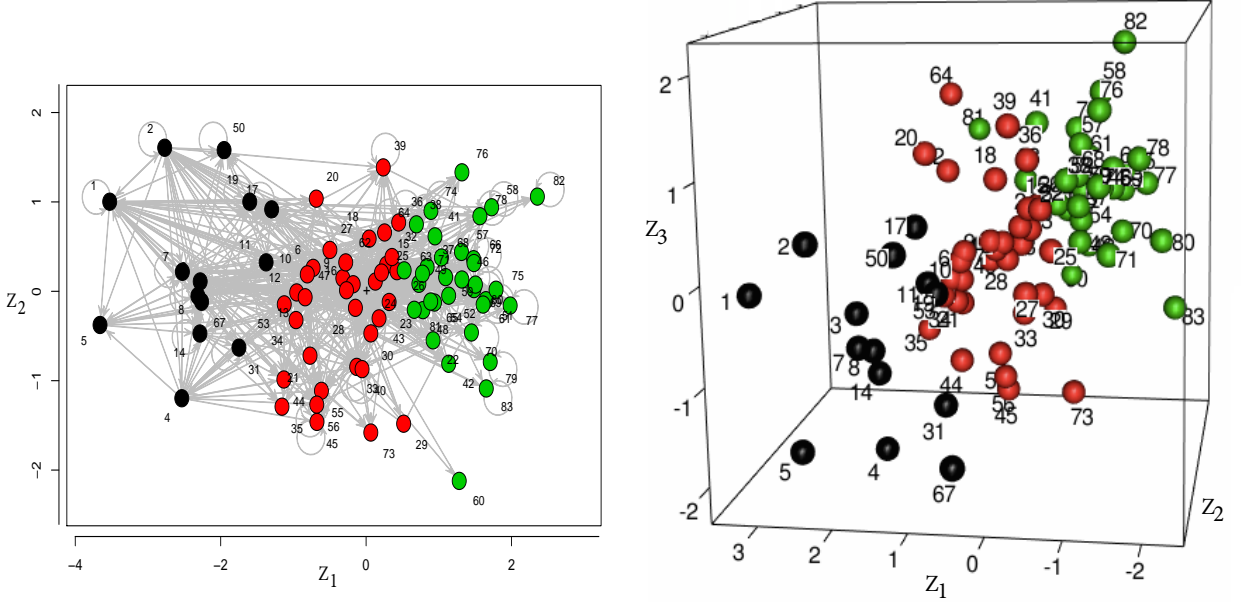


FIGURE 16. Latent Positions Z_1 , Z_2 , Z_3 after fitting (5). Vertices are colored according to the three identified groups (black=group 1, red=group 2, and green = group 3). Left: plot over first two latent variables, Right: plot over the 3-dimensional latent space (edges not shown). Numbers correspond to MVS₂ indices, see Table A1. It is evident the few connections between groups 1 and 3, and how group 2 contains departments with a high degree of “betweenness” (Newman, 2010).

Not only the existence of a hierarchy, but also a higher reliance on self-hires has consequences for the flow of new ideas within a discipline. ORIE is a field with a high self-hiring rate compared to Computer Science and Business Schools (close to two and three times, respectively). This characteristic does not permit as much flow of information as what could occur had the self-hiring rates be more moderate.

The sparseness of the ORIE network outside the top of the hierarchy also needs to be mentioned. While non-zero, low count edges may be just “noise”, which we handled in our analysis (via permutation tests or bootstrapping when finding evidence of a hierarchy or trying to determine an index or through a statistical model for the network) zero valued edges do not necessarily indicate the absence of a dominance relation. As discussed in section 3.1, it is not possible to determine from the available data whether an “observational zero” in a social network implies the lack of a dominance relation or rather its presence, with the dominated party trying to avoid interactions with the dominant one.

We are well aware that hiring decisions are not only made based on the “prestige” of the potential “sender” and “receiver” departments. They are also a consequence of complex politics, personalities and styles of the faculty in the “receiver” department and its college administration. Sometimes the prestige of the Ph.D. adviser, and not that of the sending

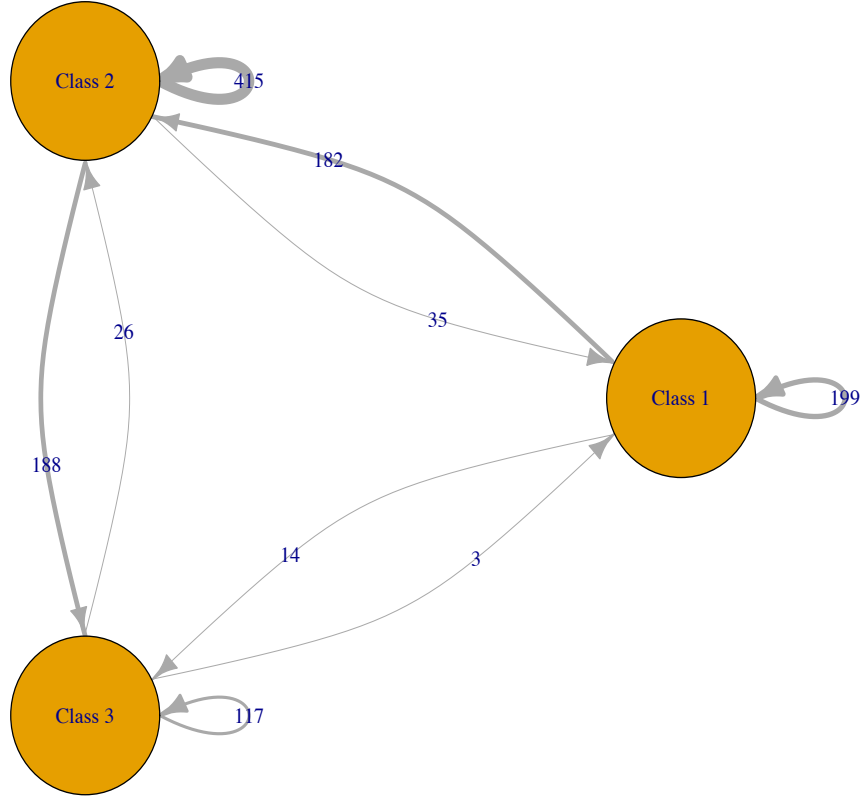


FIGURE 17. Simplified ORIE network, showing the faculty hires between the three identified groups of departments. Edge width is proportional to the number of faculty hires, which are the numbers shown on each edge. The bulk of the hire-placements occurs within group. Note also how there are very few connections between groups 1 and 3, with departments in group 3 contributing only to three hires in departments in group 1.

department, is what matters in a hiring decision. The minimum violation and strength rankings (MVS_2) we presented are based on finding dominance relations in the interactions between parties. But placing a Ph.D is not necessarily a “win”, particularly if to a lower department in the hierarchy, and hiring a Ph.D. is not necessarily a “loss” or “defeat”, especially if the originating department is higher in the hierarchy. This is the reason we believe our latent location ERGM model of section 5 and the corresponding cluster analysis is a more reasonable approach for understanding the characteristics of the ORIE faculty network, resulting in groups that do not necessarily follow the MVS_2 ranks, especially at the top of the hierarchy, and also resulting in simulated networks that closely resemble the actual ORIE network. To be in the first group of departments, a department needs to consistently hire from top departments, even if it does not place many Ph.D.’s in other top departments, a non-existent notion in competitive sports and ecology.

The analysis presented in this paper refers to data collected at a single point in time (summer 2016). This neglects the *dynamic* effect of hires over the years which could be modeled if more complete data about the years of subsequent employments of each single professor in the network were available (our datasets contain partial data, too incomplete to attempt such analysis). Dynamic exponential random graph models seem a good way to study such dynamic networks.

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APPENDIX 1: INSTITUTION DATA

The list of ORIE departments was formed by merging those Ph.D. granting departments in the 2016 US News & World report "Industrial/Manufacturing/Systems Engineering" graduate rankings (accessed 5/12/2016) with those in the 2011 National Research Council (NRC, 2011) rankings for "Operations Research, Systems Engineering and Industrial Engineering". The NRC lists both departments and programs, so an institution may appear more than once; we included each institution only once in the network (their NRC ranking shown below corresponds to the highest ranking of either program or department) and collected the faculty information as if it were a single department. Only departments with existing web pages as of May 2016 and that have a Ph.D. program were included (this excluded U. of Nebraska-Lincoln). The NRC rankings listed in Table A1 below are those given by the average of the 5% and 95% "R" rankings, listed in standard competition order. There are Industrial Engineering departments which are merged with other disciplines in a single department (e.g., Mechanical and Industrial Engineering, or Computer Systems and Industrial Engineering); an effort was made to include only those faculty in the ORIE field. It was assumed, in the absence of information, that a faculty working in an ORIE department obtained his/her Ph.D. in the ORIE department of the listed institution (a number of faculty only lists their alma mater institution but not their alma mater's department). Also, faculty that received their Ph.D. outside of the USA (or outside of our list of departments) were not considered. Only faculty in tenured or tenure-track positions

were included, although it was not always clear if some faculty positions were tenured or not.

It is important to point out that the edges (v_i, v_j) of the ORIE network are formed by faculty *currently* (as of summer 2016) in department j who received their Ph.D. in department i . This evidently neglects the movement of faculty through some of other intermediate departments between i and j and is a potential source of error. Our dataset contains partial information with respect to these intermediate departments where each faculty worked before their current job, information too incomplete to attempt an analysis.

The final ORIE dataset is contained in 2 files, one for the 83 institutions (vertices) and its attributes, and one for the 1179 faculty (edges) and its attributes. The data collection effort was undertaken in summer 2016 over a 4 week span. After data gathering, all faculty data was checked manually for errors by 3 persons. Table A1 contains the attributes in the institutions file, which includes ranks of various measures of vertex importance and the latent groups found in section 5.

TABLE A1. Latent group membership and importance measure ranks, ORIE departments, sorted by group and then by MVS₂ index.

		Ranks											
Institution	Group	MVS ₂	MVS ₁	MVR	USNews	NRC	In-deg	Out-deg	Eigen.	PageR.	Bet	Hub	Auth.
Stanford	1	1	2	2	4	1	5	2	2	2	17	2	2
UC Berkeley	1	2	1	1	2	4	36	6	3	3	26	3	18
MIT	1	3	3	3	6	3	3	1	1	1	18	1	1
Carnegie M.	1	4	4	4	NA	7	79	12	6	8	14	8	59
Princeton	1	5	5	5	NA	18	66	33	8	16	68	15	16
Cornell	1	7	7	7	7	8	16	10	4	6	25	7	12
Columbia	1	8	8	8	11	20	22	17	16	12	58	17	6
Northwest.	1	11	11	11	4	6	20	15	10	9	39	10	28
U. Penn	1	14	15	18	28	14	14	24	7	17	43	20	7
USC	1	17	19	19	12	29	12	28	22	22	35	25	10
UT (Aus)	1	19	18	16	19	23	46	29	12	15	33	22	21
UNC (Ch-H)	1	31	35	42	NA	34	32	58	23	45	48	38	36
Naval Post.	1	50	83	75	23	NA	23	69	69	78	70	60	14
Wash. U.	1	67	66	43	39	NA	69	71	71	68	67	71	34
U. Michigan	2	6	6	6	2	5	11	3	9	5	5	4	8
Purdue	2	9	9	9	9	9	9	5	13	7	6	6	13
GA Tech	2	10	10	4	1	2	5	4	1	5	1	5	3
U. Illi. (UC)	2	12	14	14	15	33	13	13	17	14	28	14	5
U. Wis (Ma)	2	13	12	12	7	9	21	14	15	11	10	9	22
U. Florida	2	15	13	13	19	23	56	9	19	13	36	16	38
Penn St.	2	16	16	15	12	12	7	8	18	10	4	13	15
Ohio St.	2	18	21	21	17	9	19	11	25	19	12	19	19
U. Minn.	2	20	20	20	32	46	61	39	37	38	61	29	31
U. Maryland	2	21	17	17	NA	16	60	22	11	21	37	11	32
U. Pitt	2	23	24	24	23	39	25	16	32	24	19	24	20
VPI	2	24	25	25	9	15	10	7	26	18	7	12	25
U. Arizona	2	25	27	27	28	35	53	26	41	37	55	27	51
NC State	2	26	33	32	12	12	6	19	40	30	15	30	29
Lehigh	2	27	23	26	18	28	39	34	30	29	47	43	40
SUNY Buf	2	28	29	31	28	37	28	21	27	25	20	26	33
U. Miss (Co)	2	29	26	23	58	31	78	54	34	51	69	45	63
Rutgers	2	30	28	30	21	38	47	35	21	23	34	44	49
Texas A& M	2	32	31	34	15	24	15	20	20	20	2	23	30
U. Virginia	2	33	37	35	28	23	27	37	48	42	38	34	23
U. Mass.	2	34	39	46	36	50	8	25	29	31	23	18	9
Boston U.	2	35	30	28	39	30	62	60	47	57	66	53	24
U. Arkansas	2	36	40	41	39	57	31	44	39	48	40	49	43
RPI	2	39	41	37	21	19	37	36	55	41	46	39	45
U. Wash.	2	40	42	40	26	35	57	48	49	52	59	42	35
Iowa St.	2	43	45	47	26	32	26	40	35	39	21	41	37
U. Conn.	2	44	59	54	NA	54	35	57	58	67	63	31	39
G. Wash. U.	2	45	44	51	53	53	43	42	33	40	50	37	27
Arizona St.	2	47	48	49	23	21	1	18	28	27	3	21	11
Northeastern	2	53	81	82	36	27	4	70	70	79	71	51	4
Stevens	2	55	61	74	39	NA	17	49	50	55	53	40	17
G. Mason	2	56	51	60	32	NA	30	38	36	43	22	35	42
NJIT	2	62	79	80	NA	57	34	83	82	83	82	80	26
Case West.	2	64	32	53	38	43	77	59	14	26	41	36	55
Worcester P	2	73	74	64	53	NA	73	80	75	80	74	82	46

TABLE A1. Latent group membership and importance measure ranks, ORIE departments, sorted by group and then by MVS₂ index (cont.).

		Ranks											
Institution	Group	MVS ₂	MVS ₁	MVR	USNews	NRC	In-deg	Out-deg	Eigen.	PageR.	Bet	Hub	Auth.
U. Iowa	3	22	22	22	39	22	80	31	24	35	54	28	71
U. S. Florida	3	37	38	36	46	54	54	27	42	34	11	33	67
Oklahoma St	3	38	34	38	39	41	40	30	38	36	45	32	44
Kansas St.	3	41	50	39	46	47	49	46	62	49	65	57	48
U. Illi. (Chi)	3	42	36	33	46	45	71	52	45	53	62	48	61
Texas Tech	3	46	47	48	53	40	55	23	51	33	9	47	68
U. Oklahoma	3	48	53	56	46	50	42	53	57	61	49	58	50
Clemson	3	49	58	58	32	47	29	51	63	58	44	61	52
Miss. U.	3	51	43	44	58	62	50	32	44	32	31	50	78
Auburn	3	52	49	52	32	42	48	41	54	47	29	52	66
Wayne St.	3	54	54	61	53	44	44	64	52	60	42	62	41
U. Louisville	3	57	57	50	66	62	51	55	65	54	56	65	58
U. Alabama	3	58	52	45	NA	61	67	47	64	44	57	63	57
UC Florida	3	59	55	57	39	54	38	45	56	50	24	59	62
UT (Dal)	3	60	46	29	58	NA	83	74	53	77	76	55	82
U. Houston	3	61	62	55	53	65	72	50	60	56	52	56	77
Air Force IT	3	63	71	77	46	NA	18	62	72	76	51	54	47
SUNY (Bin)	3	65	80	81	58	57	24	65	76	75	75	67	53
Oregon St.	3	66	77	72	46	49	41	63	73	69	72	69	69
Wichita St.	3	68	69	78	66	NA	33	76	67	73	60	78	56
W. Virginia	3	69	63	70	66	NA	52	56	59	59	13	66	64
UT (Arling)	3	70	60	66	58	NA	45	43	43	46	8	46	70
U. Tenn.	3	71	56	69	58	62	58	61	31	28	27	68	72
NCA&T St	3	72	78	71	66	NA	64	82	78	82	78	81	81
Florida St.	3	74	64	79	66	NA	59	66	46	62	16	64	65
U. NC (Ch.)	3	75	65	73	58	NA	63	73	66	74	32	73	76
U. Wis (Mil)	3	76	76	65	58	NA	74	81	77	81	77	83	60
Old Dom U	3	77	67	76	NA	66	65	68	68	66	64	74	83
Ohio U.	3	78	68	83	66	NA	68	67	61	63	30	72	80
New Mex St	3	79	73	63	NA	60	76	79	83	72	83	75	73
U. Miami	3	80	75	68	66	52	75	75	79	71	79	77	79
U. Ark. LR	3	81	82	67	46	NA	70	72	74	70	73	70	54
Montana St.	3	82	72	59	NA	NA	82	78	81	65	81	76	74
Florida IT	3	83	70	62	NA	NA	81	77	80	64	80	79	81