CONREG User's Manual and Installation Guide

1 Introduction

CONREG (CONstrained Confidence REGion) is a MATLAB program that computes and displays the confidence regions for <u>constrained</u> optima of a response surface optimization. The underlying methodology for computing the confidence regions is discussed by Peterson, Cahya, and Del Castillo (2002). See also a detailed discussion on the program implementation by Cahya (2002) (available from ~~http://etda.libraries.psu.edu/theses/available/etd-1213101-155040~~). PSU's library For <u>unconstrained</u> optimization, another program codenamed BH can be used for computing the confidence regions for the optima (or stationary points). Both CONREG and BH programs can be downloaded from ~~http://lb350e.ie.psu.edu/software.htm~~. http://sites.psu.edu/engineeringstatistics/computer-codes/

The current version of CONREG handles response surface problems with the following characteristics:

- the model has to be linear in the parameters. The model can be nondifferentiable;

- the model includes up to five controllable factors;

- the constraining region can be spherical or simplex (mixture experiments).

Unlike the BH program where the input is the data (experimental runs and response observations), the input for CONREG is the fitted model (model functional form, parameter estimates, and covariance matrix of the parameter estimates). This allows flexibility for users to provide the parameter estimates using their own tools or methodologies.

CONREG is able to generate two types of confidence regions for problems with more than three factors: *unconditional* and *conditional* confidence regions. CONREG has a several easy-to-use graphical user interface (GUI) features not found in the BH program. Refer to Section 4 for details.

The CONREG program runs under the MATLAB software, version 5.3 release 11. The program consists of 6 M-files (.m) and 2 GUI files (.fig). These files are packaged into a compressed file CONREG.zip with additional 4 example files (.txt).

2   Program Installation

The installation of CONREG is straightforward: uncompress the CONREG.zip and copy the uncompressed files into a directory of preference. The next section discusses the steps for running the program.

3   Running the Program

1. Open the MATLAB window and set the working directory by clicking the *file* menu option, and then click *Set Path*. A Path-Browser window will pop up. Click on *browse* and then select the directory where the CONREG program files are located and then hit *OK* button when the directory is found.

2. Type `open('MainFig.fig')` on the MATLAB command prompt and hit the *enter* key. The CONREG main window will pop up (shown below).
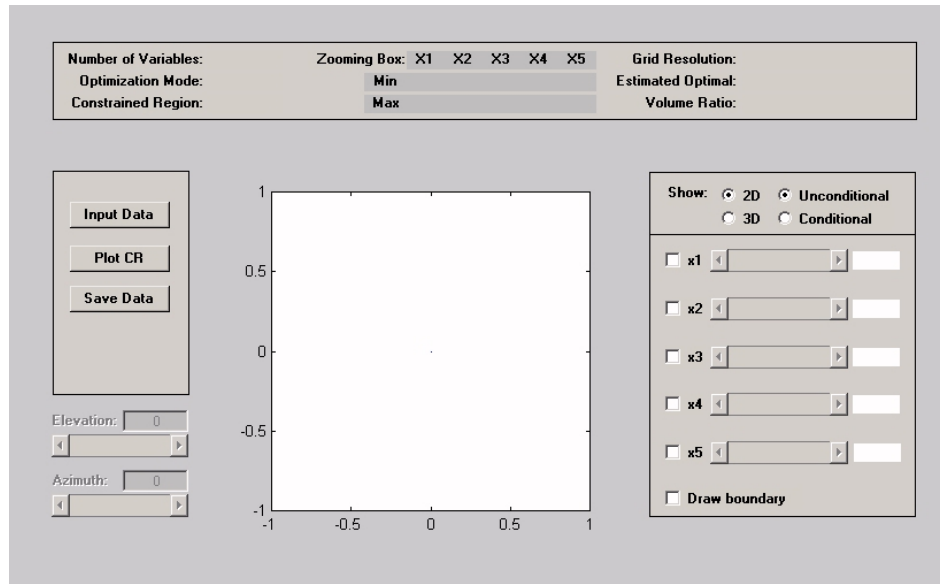
3. Click on the *Input Data* button and then find where the input file is located. When the input file is found, select the file and click the *open* button. Four input file examples are included in the package. A new input file can be created following a specific format guideline discussed in Section 5.

4. To plot the confidence region, click on the *Plot CR* button. Whenever a confidence region plot has been saved in the input file, a message box will pop up asking whether the user wants to view the existing plot or recompute the confidence region. If some of the parameters in the input file has been modified, choose to recompute the confidence region.

5. Whenever recomputing the confidence region option is chosen, a progress window will appear (shown below) to show the current progress of the computation. There are times when the chosen grid resolution is too high for the problem at hand and thus the computation progresses very slowly, the computation can be cancelled by clicking the *Cancel Computation* button. The computation will stop after a certain waiting period, and the resolution can be decreased to expedite the computation.
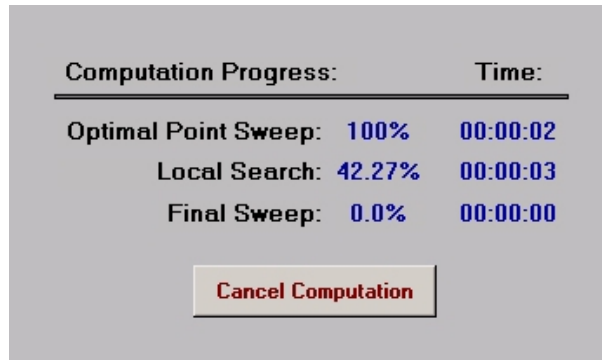
6. Once the computation is finished, the progress window will disappear and the resulting confidence regions will be shown on the CONREG main window (shown below).
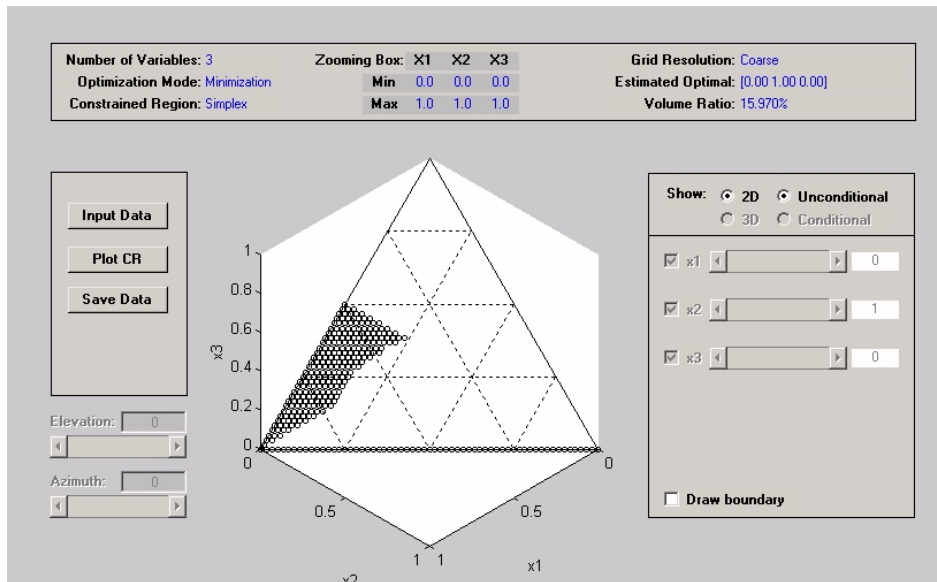

7. The confidence region plot can be saved for latter viewing by clicking the *Save Data* button.

4   Special Features

### Three-dimensional Rotation

For problems with three or more factors, a three-dimensional confidence regions (or projections from higher dimensional confidence regions) can be plotted. The confidence region object can be rotated to see it from different angles. Users can rotate the object by varying the value of the Elevation angle

and Azimuth angle (see the figure below). The angles can be varied by adjusting the slider or fill in the angle values on the appropriate text boxes.
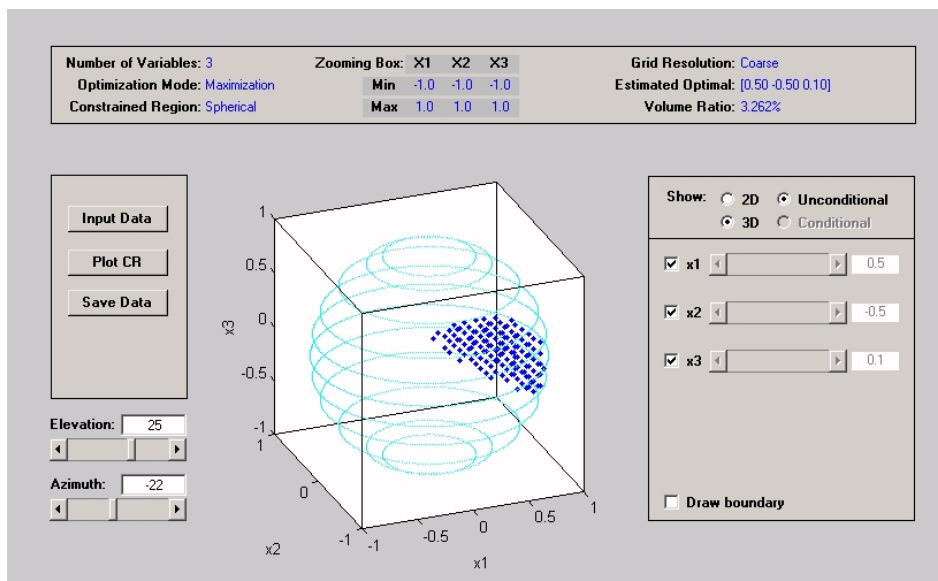

Interactive Projection

For problems with more than three factors, one way to see the confidence region is to see it through its projections into lower dimensional subspaces. There are a number of subspaces that can be chosen. CONREG allows the confidence region to be projected into different subspaces interactively. An example is shown in the figure below. The confidence region has a dimension of three. The plot below shows its projection into $x_2 - x_3$ subspace (x_2 and x_3 checkboxes are checked). Projection into other subspaces can be chosen by simply clicking the checkbox of the appropriate factor.
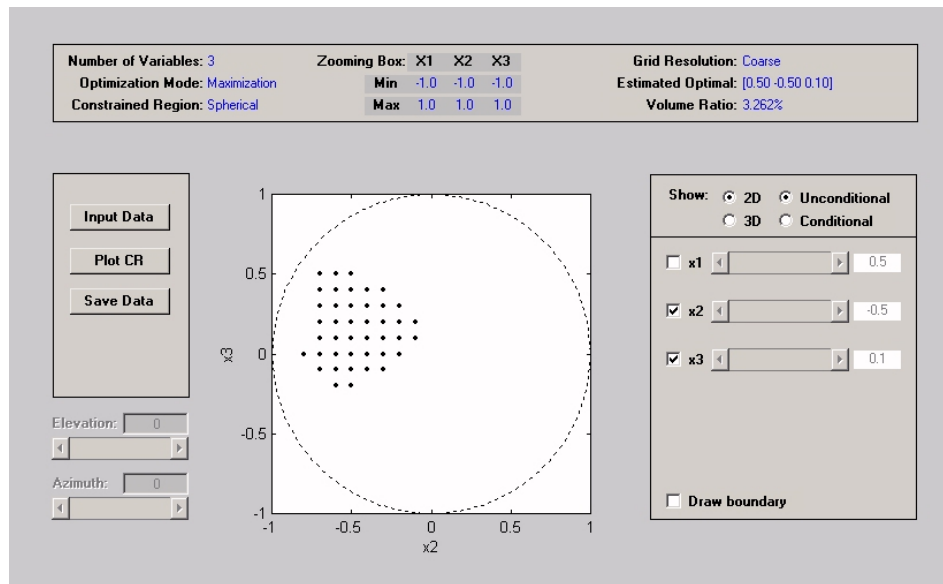
Interactive Slicing

Conditional confidence regions ("Slices") are a powerful way to see the multivariate nature of a high dimensional confidence region. CONREG allows the user to see the confidence region slices at different levels by sliding the appropriate slider bar (shown below). To do this, the *Conditional* radio button needs to be checked first before the slider bars are activated. The current values of the factors are shown on the text box to the left of the slider bar.
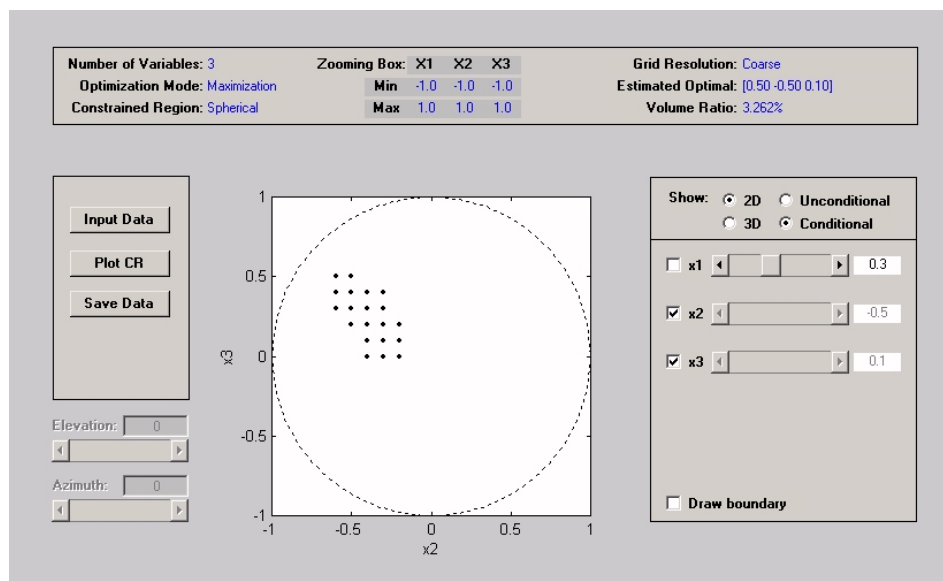
5   Input file format

The input file needs to be arranged in the following format:

MainParam

ConfidenceParam

BoundingBox

$z(x)$

$\widehat{\theta}$

$\widehat{V}$

0 0

where:

- *MainParam* is a 1×5 vector consisting: k, p, Grid-Resolution, Constrained-Region, and Optimization-Mode. Here, k is the number of factor and p is the number of parameters. Grid-Resolution has a value of 1, 2, or 3 for *coarse*, *medium*, and *fine* grid resolutions, respectively. Constrained-Region has a value of 1 or 2 for spherical or simplex regions, respectively. Optimization-Mode has a value of 1 or 2 for minimization or maximization, respectively.

- *ConfidenceParam* is a 1×2 vector consisting: confidence level $(1\text{-}\alpha)$ and the critical value c_α^2.

- *BoundingBox* is a $2 \times k$ vector that bounds the viewing region.

- $z(x)$ is a $1 \times p$ vector of model terms.

- $\widehat{\theta}$ is a $1 \times p$ vector of parameter estimates.

- $\widehat{V}$ is the estimate of covariance matrix of $\widehat{\theta}$.

As an example, consider the input data given in CG78ex.txt file (included in the package):

3.0	4.0	1.0	2.0	1.0
0.95	7.9646			
0	0	0		
1	1	1		

1, x(1), x(2), (x(1)*x(2))/(x(1)+x(2))

20.9843	28.1943	62.5543	-123.8237
0.4199	-0.5598	-0.5598	-0.1600
-0.5598	1.4929	0.7464	-1.2796
-0.5598	0.7464	1.4929	-1.2796
-0.1600	-1.2796	-1.2796	15.1727
0	0		

Here, the problem has 3 factors and 4 parameters. Coarse grid resolution, simplex constrained region, and minimization were requested. The confidence level is 95% with critical value equals to 7.9646. The region will be bounded between 0 and 1 for each of the factors. The model is

$$y = 20.9843 + 28.1943x_1 + 62.5543x_2 - 123.8237\frac{x_1 x_2}{x_1 + x_2} + \epsilon.$$

Note that to represent x_i in the input file, parentheses should be used to enclose the index i, i.e. $x(i)$. The last row of the input file should always be 0 0. These zeros are used by the program to denote the end of the input file. These zero numbers will be modified internally by the program once the resulting confidence region plot is saved for future use.

References

Peterson, J. J., Cahya, S., and Del Castillo, E. (2002), "A General Approach to Confidence Regions for Optimal Factor Levels of Response Surfaces," Accepted for publication in *Biometrics* (2002).

Cahya, S. (2002), *Sampling Properties of Optimal Operating Conditions of Single and Multiple Response Surface Systems*, Ph.D. thesis, Industrial Engineering Department, The Pennsylvania State University, University Park, PA.