Geodesic Gaussian Processes for the Reconstruction of a Free-Form Surface

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Abstract

Reconstructing a free-form surface from 3-dimensional noisy measurements is a central problem in inspection, statistical quality control, and reverse engineering. We present a new method for the statistical reconstruction of a free-form surface based on 3-dimensional cloud point data. The surface is represented parameterically, with each of the three cartesian coordinates (x, y, z) a function of surface coordinates (u, v). This avoids having to choose one euclidean coordinate (say, z) as a "response" function of the other 2 coordinate "locations" (say, x and y), as commonly used in previous euclidean Kriging models of manufacturing data. In the proposed method, parameterization algorithms from the manifold learning and computer graphics literature are applied to find the (u, v) surface coordinates. These are then used as locations in a spatial Gaussian Process model that considers correlations between two points on the surface a function of their *geodesic* distance on the surface, rather than a function of their euclidean distances over the xy plane. It is shown how the proposed Geodesic Gaussian Process (GGP) approach better reconstructs the true surface, filtering the measurement noise, than when using a standard euclidean Kriging model of the 'heights', i.e., $z(x, y)$. The methodology is applied to simulated surface data and to a real dataset obtained with a non-contact laser scanner. The parametric surface representation is compatible with computer-aided-design (CAD) models and allows differential-geometry manipulations such as the computation of surface areas.

Keywords: Manifold data analysis, Anisotropic Covariance, Non-contact sensed data.

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