An Analysis and MIMO Extension of a Double EWMA Run-to-Run Controller for Non-squared systems

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Abstract

The double EWMA (exponentially weighted moving average) control method is a popular algorithm for adjusting a process from run to run in semiconductor manufacturing. Until recently, the dEWMA controller had been applied only for the single controllable factor (or input), single quality charcteristic (or output) case. Recently, Del Castillo and Rajagopal [4] propose a multivariate double EWMA controller for squared multiple-input, multiple-output (MIMO) processes, where there is an equal number of inputs and outputs. This paper extends the MIMO dEWMA controller for non-squared systems. Two different MIMO dEWMA controllers are presented and their performance studied with application to a Chemical-Mechanical Polishing (CMP) process, a critical semiconductor manufacture processing step that exhibits non-linear dynamics.

1 Introduction

The application of the exponentially-weighted-moving-average (EWMA) statistic for process adjustment purposes, particularly in semiconductor manufacturing, has gained popularity in recent times. The double EWMA (dEWMA) controller was first proposed by Butler and Stefani [1] for a polysilicon gate etch process used in semiconductor processing, where the process drifts as the reactor ages from run to run. Del Castillo [2], presents an analysis of the dEWMA controller that adjusts a single controllable factor (or input) applied to a single quality characteristic (or output; this is the SISO case). This controller uses two EWMA statistics to predict the coupled effect of the offset and the drift in the process response. Conditions for stability and robustness of dEWMA controllers for a process with deterministic trend were derived and later extended to random walk with drift and IMA (1,1) disturbances in [3].

Recently, Tseng *et al.* [9] proposed a multivariate extension to a single EWMA controller (i.e., a controller with multiple inputs and multiple outputs, or MIMO). Del Castillo and Rajagopal [4] present multivariate extensions of the dEWMA controller, and analyze the robustness and stability conditions. The controller proposed in that paper is restricted to squared processes (i.e., processes with equal number of inputs and outputs). The purpose of the present paper is to study and develop a MIMO version of the dEWMA controller for non-squared processes, that is, processes where the number of controllable factors exceeds the number of responses.

The process model assumed in [4] is similar to that of Butler and Stefani [1], but extended to the multivariate case:

$$\mathbf{Y}_t = \boldsymbol{\alpha} + \boldsymbol{\beta}\mathbf{U}_{t-1} + \boldsymbol{\delta}\mathbf{t} + \boldsymbol{\epsilon}_t, \tag{1}$$

where $\mathbf{Y}_t$ is a $p \times 1$ vector of the measured quality characteristics or responses measured from run or at time instant t, $\mathbf{U}_{t-1}$ is a $m \times 1$ vector of levels of the controllable factors set at the end of run $(t-1)$, $\boldsymbol{\delta}$ is a diagonal matrix containing the average drift per run for each of the responses, and $\{\boldsymbol{\epsilon}_t\}_{t=1}^{\infty}$ is a multivariate white noise sequence. The vector $\boldsymbol{\alpha}$ contains the offsets of the p responses and the $p \times m$ matrix $\boldsymbol{\beta}$ contains the input-output gains. The above model assumes a deterministic trend (DT) disturbance, $\mathbf{N}_t = \boldsymbol{\delta}\mathbf{t} + \boldsymbol{\epsilon}_t$. For squared process (i.e., when $p = m$), the MIMO dEWMA controller proposed by Del Castillo and Rajagopal [4] is given by:

$$\mathbf{U}_t = (\mathbf{B}'\mathbf{B})^{-1}\mathbf{B}'(\mathbf{T} - \mathbf{A}_t - \mathbf{D}_t) \tag{2}$$

where $\mathbf{B}$ is the estimate of the matrix of parameters $\boldsymbol{\beta}$ (assumed obtained off-line obtained using DOE and regression techniques), $\mathbf{T}$ is the vector of targets for the responses in $\mathbf{Y}$, and $\mathbf{A}_t$ and $\mathbf{D}_t$ are on-line estimates obtained using Multivariate Double EWMA equations at the end of each run:

$$\mathbf{A}_t = \boldsymbol{\Lambda}_1(\mathbf{Y}_t - \mathbf{B}\mathbf{U}_{t-1}) + (\mathbf{I} - \boldsymbol{\Lambda}_1)\mathbf{A}_{t-1}, \tag{3}$$

and

$$\mathbf{D}_t = \boldsymbol{\Lambda}_2(\mathbf{Y}_t - \mathbf{B}\mathbf{U}_{t-1} - \mathbf{A}_{t-1}) + (\mathbf{I} - \boldsymbol{\Lambda}_2)\mathbf{D}_{t-1}. \tag{4}$$

Here, $\boldsymbol{\Lambda}_1$ and $\boldsymbol{\Lambda}_2$ are the two EWMA weight matrices. Although these matrices can have any form, the convenience of selecting them to be diagonal was emphasized in [4], in such a way that the sum $(\mathbf{A}_t + \mathbf{D}_t)$ provides an unbiased estimate of $\boldsymbol{\alpha} + \boldsymbol{\delta}(\mathbf{t} + \mathbf{1})$, (i.e., the estimates provide an asymptotically unbiased one-step-ahead prediction of where the quality characteristics would have drifted in the absence of any control action). Thus, we can represent the weight matrices as $\boldsymbol{\Lambda}_1 = \lambda_1\mathbf{I}$ and $\boldsymbol{\Lambda}_2 = \lambda_2\mathbf{I}$. The stability conditions, derived in [4], are as follows:

$$|1 - 0.5\xi_{jj}(\lambda_1 + \lambda_2) + 0.5z| < 1, \quad \forall j \in \{1, 2, ..., p\} \tag{5}$$

$$|1 - 0.5\xi_{jj}(\lambda_1 + \lambda_2) - 0.5z| < 1, \quad \forall j \in \{1, 2, ...p\} \tag{6}$$

where, $z = \sqrt{\xi_{jj}^2(\lambda_1 + \lambda_2)^2 - 4\lambda_1\lambda_2\xi_{jj}}$, and ξ_{jj} is the j^{th} diagonal element of the $(p \times p)$ matrix $\boldsymbol{\xi} = \boldsymbol{\beta}(\mathbf{B}'\mathbf{B})^{-1}\mathbf{B}'$.

The remainder of the paper is organized as follows. Section 2 proposes two different controllers for the case of non-square systems, where there are unequal number of inputs and outputs $(p \neq m)$. Section 3 provides examples of the performance of these controllers as applied to a Chemical-Mechanical Polishing (CMP) process. The results are summarized in Section 4.

2 MIMO Double EWMA Controllers for Non-Square Systems

Most industrial processes in practice are not squared. A particular practical case is when the number of controllable factors is greater than the number of responses, i.e., when $m > p$. When this occurs, the control equation $\mathbf{U}_t = (\mathbf{B}'\mathbf{B})^{-1}\mathbf{B}'(\mathbf{T} - \mathbf{A}_t - \mathbf{D}_t)$ is not applicable as the matrix $\mathbf{B}'\mathbf{B}$ is not invertible. Hence, modified control laws are required. Two of these are discussed below.

2.1 Ridge-Solution Controller

The idea behind this controller is to minimize $(E[\mathbf{Y}_t] - \mathbf{T})^2$ subject to the constraint $(\mathbf{U}'\mathbf{U}) < \rho^2$, where ρ is the radius that bounds the values the controllable factors can take. This is analogous to the optimization methods used in Ridge Analysis of Response Surfaces (Myers and Montgomery [6]). If μ is the Lagrange multiplier of the constraint, then the control equation obtained is of the form:

$$\mathbf{U}_t = (\mathbf{B}'\mathbf{B} + \mu\mathbf{I})^{-1}\mathbf{B}'(\mathbf{T} - \mathbf{A}_t - \mathbf{D}_t). \tag{7}$$

The above solution is similar to the control equation for squared systems. The only change here is the $\mu\mathbf{I}$ term which when added to $(\mathbf{B}'\mathbf{B})$ makes it invertible as long as $\mu \neq 0$. As this is a minimization problem, μ is chosen so that the matrix $(\mathbf{B}'\mathbf{B} + \mu\mathbf{I})$ is positive definite. This will happen as long as $\mu > 0$. It is recommended to choose small values of μ that enable to obtain an invertible matrix for computation purposes. All the results for stability and the conditions for oscillations as shown in equations 5 and 6 can be applied to this controller by simply redefining $\boldsymbol{\xi} = \boldsymbol{\beta}(\mathbf{B}'\mathbf{B} + \mu\mathbf{I})^{-1}\mathbf{B}'$. Table 1 presents sample realizations of the adjustments for a linear process that was prescribed by the controller for the first ten runs. The example was simulated for a system with $m = 3$ controllable factors and $p = 2$ responses. The following process parameters were used in the simulation: $\alpha = [10, 30]'$, $\beta = \begin{pmatrix} 23 & 34 & 8 \\ 51 & 16 & 33 \end{pmatrix}$, $\delta = [0.3, -0.4]'$ and $\boldsymbol{\epsilon} \sim N(0, 25)$. The following controller parameters were used: $\mathbf{T} = [200, 300]'$, $\mathbf{B} = \begin{pmatrix} 25 & 35 & 10 \\ 50 & 20 & 35 \end{pmatrix}$, $\mu = 0.001$, $\mathbf{\Lambda_1} = 0.15\mathbf{I}$ and $\mathbf{\Lambda_2} = 0.35\mathbf{I}$. A nd D were initialized at [0,0]'. Thus for example, at the end of run 3, the observed value of the response is [191.3, 298.8]'. Based on this value and on the value of the controllable factor, $\mathbf{U}$, at run 2 which is [3.23, 2.79, 1.81]', the values of $\mathbf{A}$ and $\mathbf{D}$ for run 3 are calculated using equations 3 and 4 as $[0.06, 12.08]'$ and $[-0.73, 8.69]'$ respectively. Using these values of $\mathbf{A}$ and $\mathbf{D}$, the controllable factors for the fourth run of the process are prescribed based on equation 7 and in this case they equal to $[3.19, 2.96, 1.73]'$.

2.2 Right-Inverse Controller

Another controller for non-squared processes is based on the controller that Sachs *et al.* [8] proposed for systems with multiple outputs and a single input. It is obtained by minimizing the adjustments to be made, i.e., $(|\mathbf{U}_t - \mathbf{U}_{t-1}|)$, subject to the constraint that $(\mathbf{Y}_t - \boldsymbol{\alpha} - \boldsymbol{\delta}\mathbf{t} = \mathbf{T})$. More recently, this approach was used by Tseng *et al.* [9] for extending the single EWMA to the multivariate case. In the case of a double EWMA, this controller is given by:

$$\mathbf{U}_t = (\mathbf{I} - \mathbf{B}'(\mathbf{B}\mathbf{B}')^{-1}\mathbf{B})\mathbf{U}_{t-1} + \mathbf{B}'(\mathbf{B}\mathbf{B}')^{-1}(\mathbf{T} - \mathbf{A}_t - \mathbf{D}_t) \tag{8}$$

Table 2 provides a numerical example for the right-inverse controller. The process and controller parameters were chosen to be the same as in the example shown in table 1. The calculations are identical as in the case of the ridge-solution controller except that the controllable factors, $\mathbf{U}$ are calculated using equation (8) instead.

Insert Tables 1 and 2 about here

3 An Application to a Semiconductor Manufacturing Process

In order to test the proposed controllers, a semiconductor manufacturing process called Chemical-Mechanical Polishing (CMP) was simulated based on the equipment models given in Del Castillo and Yeh [5]. There are four controllable factors: plate speed (U_1), back pressure (U_2), polishing downforce (U_3), and the profile of the conditioning system (U_4). The two responses of interest are removal rate (Y_1) and within-wafer non-uniformity (Y_2). The target values for the responses are 2000 and 100 respectively. The equipment equations are:

$$\begin{aligned} Y_1 = 1563.5 &+ 159.3(U_1) - 38.2(U_2) + 178.9(U_3) + 24.9(U_4) \\ &-67.2(U_1U_2) - 46.2(U_1^2) - 19.2(U_2^2) - 28.9(U_3^2) - \\ &12(U_1)(t') + 116(U_4)(t') - 50.4(t') + 20.4(t')^2 + \epsilon_{1,t}, \end{aligned} \tag{9}$$

and

$$\begin{aligned} Y_2 = 254 &+ 32.6(U_1) + 113.2(U_2) + 32.6(U_3) + 37.1(U_4) \\ &-36.8(U_1U_2) + 57.3(U_4)(t') - 2.42(t') + \epsilon_{2,t}, \end{aligned} \tag{10}$$

where, $t' = (t - 53)/52$, $\epsilon_{1,t} \sim N(0, 60^2)$, and $\epsilon_{2,t} \sim N(0, 30^2)$.

In order to test the MIMO dEWMA controllers, let us assume that from designed experiments and regression analysis the following approximate linear models were fit:

$$\widehat{Y}_1 = 1600 + 150(U_1) - 40(U_2) + 180(U_3) + 25(U_4) - 0.9t, \tag{11}$$

and

$$\widehat{Y}_2 = 250 + 30(U_1) + 100(U_2) + 30(U_3) + 35(U_4) + 0.05t. \tag{12}$$

Equations (11) and (12) provide the off-line estimate of the matrix of gains, $\mathbf{B}$.

In order to test the controllers under realistic scenarios, two sets of simulations were conducted. The first set of simulations was carried out assuming that the true equipment equations were not (9) and (10) but instead that the models were linear with respect to the controllable factors and the drift according to:

$$Y_1 = 1563.5 + 159.3(U_1) - 38.2(U_2) + 178.9(U_3) + 24.9(U_4) - 0.9t + \epsilon_{1,t}, \tag{13}$$

and

$$Y_2 = 254 + 32.6(U_1) + 113.2(U_2) + 32.6(U_3) + 37.1(U_4) + 0.05t + \epsilon_{2,t}. \tag{14}$$

The initial value of $\mathbf{A_t}$ was set at $[1600, 250]'$ based on equations 11 and 12, and $\mathbf{D_t}$ was initialized at $[0, 0]'$. The gain matrix, $\mathbf{B}$ was taken from Equations (11) and (12). Targets were set at 2000 for Y_1 and 100 for Y_2. The Lagrange multiplier μ was set at 0.001 in case of the ridge-solution controller. The process was simulated for 5000 runs with EWMA weights $\mathbf{\Lambda}_1 = (0.15)\mathbf{I}$ and $\mathbf{\Lambda}_2 = (0.35)\mathbf{I}$ for both of the controllers (ridge solution and right inverse controllers).

The results of the simulation are plotted in figures 1 and 2 for the ridge-solution controller and figures 3 and 4 for the right-inverse controller. Figures 1 and 3 show the plots of the two responses over the 5000 runs under the proposed controllers. The means for the responses were obtained as $[2000.0, 100.0]$ and $[2000.0, 100.0]$ for the ridge-solution and the right-inverse controllers respectively, suggesting that the process is accurately on target. The standard deviation of the responses for the respective controllers were $[76.3, 36.1]$ and $[72.4, 35.8]$, and is mostly contributed by the inherent and uncontrollable noise in the process which was simulated as $\epsilon_1 \sim N(0, 60^2)$, and $\epsilon_t \sim N(0, 30^2)$.

The simulation of 5000 runs was repeated 100 times, and in each case the mean and standard deviation of the responses were calculated. The standard errors (standard deviation of the means over the 100 trials) were obtained as $[0.0133, 0.007]$ and $[0.0149, 0.0078]$ respectively for the ridge-solution and the right-inverse controllers. The average of the standard deviations over the 100 trials was calculated as $[70.77, 35.96]$ and $[70.88, 35.99]$ respectively for the ridge-solution and the right-inverse controllers. The controllable factors either increase or decrease linearly (figures 2 and 4) as the run number increases in order to compensate for the drift in the process. The value of $\mathbf{A}_t + \mathbf{D}_t$ was observed to be close to $\boldsymbol{\alpha} + \boldsymbol{\delta}(\mathbf{t+1})$, with a value of $[-2892.1, 442.3]$ for the ridge-solution controller and a value of [-2914.1, 464.0] for the right-inverse controller after 5000 runs. It was also noted that higher values of μ introduce instability in the responses, and hence it is important that its value is set considerably lower compared to the magnitude of the values in the gain matrix, $\mathbf{B}$. Overall, the MIMO dEWMA controllers seem effective in controlling non-square linear systems.

Insert Figures 1, 2, 3 and 4 about here

The second set of simulations were carried out using the non-linear equipment equations (Equations 9 and 10) as the true process description. The length of the time a CMP process operates without interruption is typically shorter than 100 batches of wafers, after which the polishing pads are replaced. Thus, the simulation was run for only 100 time units. The objective of this simulation is to understand the performance of the MIMO dEWMA controller for a realistic non-linear process. In practice, many processes have some element of non-linearity in them, and it is of importance to see the implications of regulating them using the MIMO dEWMA controller which is based on the assumption that the process is linear. The controller parameters were kept same as before except for the EWMA weights. As larger weights caused instability, smaller weight matrices

$\mathbf{\Lambda}_1 = \mathbf{\Lambda}_2 = (0.15)\mathbf{I}$ were used.

The results of the simulation are given in figures 5 and 6 for the two controllers we have presented. The mean and standard deviation of the responses over 100 runs was found to be $[1999.2, 100.7]$ and $[60.6, 34.4]$, respectively for the ridge-solution controller, and $[1999.1, 99.7]$ and $[87.6, 40.2]$ for the right-inverse controller. This suggests that the nonlinear process is on target with most of the variation explained by the inherent noise in the process which were set at $\epsilon_1 \sim N(0, 60^2)$, and $\epsilon_t \sim N(0, 30^2)$. The standard errors over 100 such trials (standard deviation of the means) were obtained as $[1.0053, 0.5819]$ and $[1.0274, 0.5544]$ respectively for the ridge-solution and the right-inverse controllers. The average of the standard deviations over the 100 trials was calculated as $[66.93, 34.68]$ and $[66.65, 34.45]$ respectively for the ridge-solution and the right-inverse controllers.

It was also noted for both the controllers that the process becomes unstable after extended periods of time, and that this time depends on the EWMA weights chosen. When larger values are chosen the process becomes unstable within a few hundred runs, and for relatively smaller values like the ones used in the simulation the controller becomes unstable after a few thousand runs. This is because the EWMA equations (Equations 3 and 4) used in the controller could not predict the process parameters, $\boldsymbol{\alpha}$ and $\boldsymbol{\delta}$, owing to the non-linear nature of the process, i.e., $\mathbf{A}_t + \mathbf{D}_t$ does not converge to $\boldsymbol{\alpha} + \boldsymbol{\delta}(\mathbf{t}+\mathbf{1})$. This poor prediction eventually causes the process to diverge. Rajagopal [7] presents a plot of the controllable factors prescribed over the 100 runs.

INSERT FIGURES 5 AND 6 ABOUT HERE

These results suggest that the MIMO dEWMA controller may be used for processes with small degrees of non-linearity as long as the number of runs is not large, such as in this CMP process.

4 Conclusions

MIMO control laws proposed in [4] have been modified so that they can be applied to non-squared systems. Two different control laws were presented and applied to a CMP process as a case study. The study included the cases of both linear and nonlinear processes. The proposed MIMO dEWMA controllers were seen to perform well for non-square systems. The following summarizes the results:

- In case of a linear process, both MIMO dEWMA controllers gave unbiased responses. The standard deviations of the responses were almost the same for both the controllers, and reflected mostly the variance of the inherent noise in the process.

- The adjustments prescribed by both controllers for the linear case was also observed to be almost identical, as long as the μ parameter of the ridge-solution controller is small.

- In the case of a non-linear CMP process equation, both controllers were able to give stable performance for lower values of the EWMA weights, but were unstable when higher values are chosen. The EWMA statistics were not good predictors of the actual process because of the non-linearity involved. This causes the process to eventually become unstable even when low weights are used. However, these controllers are still useful for processes such as CMP whose length is less than 100 runs, typically, after which the wearing parts (in this case, the polishing pads) are replaced.

- In case of the the ridge-solution controller, there is an additional parameter that has to be chosen, namely, μ. Choosing this is quite crucial as large values lead to an unstable response, and values very close to zero make the $(\mathbf{B}'\mathbf{B} + \mu\mathbf{I})$ matrix non-invertible. This problem is not encountered in the Right Inverse controller.

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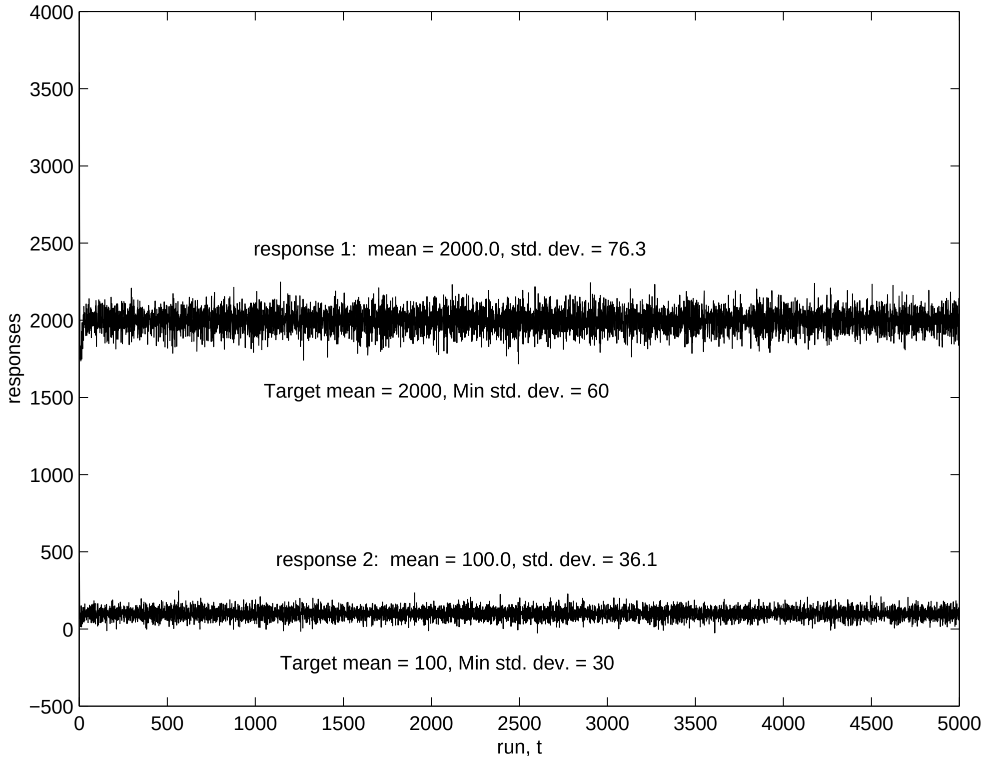

Figure 1: Responses of a Linear CMP Process controlled by the Ridge-Solution Controller

Run, t	$\mathbf{Y_t}$	$\mathbf{A_t}$	$\mathbf{D_t}$	$\mathbf{U_t}$
1	$\begin{pmatrix}206.8\\318.4\end{pmatrix}$	$\begin{pmatrix}1.71\\4.61\end{pmatrix}$	$\begin{pmatrix}1.71\\4.61\end{pmatrix}$	$\begin{pmatrix}3.38\\2.64\\1.98\end{pmatrix}$
2	$\begin{pmatrix}198.9\\317.5\end{pmatrix}$	$\begin{pmatrix}1.88\\10.12\end{pmatrix}$	$\begin{pmatrix}1.45\\8.97\end{pmatrix}$	$\begin{pmatrix}3.23\\2.79\\1.81\end{pmatrix}$
3	$\begin{pmatrix}191.3\\298.8\end{pmatrix}$	$\begin{pmatrix}0.06\\12.08\end{pmatrix}$	$\begin{pmatrix}-0.73\\8.69\end{pmatrix}$	$\begin{pmatrix}3.19\\2.96\\1.73\end{pmatrix}$
4	$\begin{pmatrix}198.5\\292.1\end{pmatrix}$	$\begin{pmatrix}-0.49\\12.28\end{pmatrix}$	$\begin{pmatrix}-1.09\\6.71\end{pmatrix}$	$\begin{pmatrix}3.21\\2.97\\1.74\end{pmatrix}$
5	$\begin{pmatrix}199.0\\303.3\end{pmatrix}$	$\begin{pmatrix}-1.01\\14.78\end{pmatrix}$	$\begin{pmatrix}-1.34\\7.53\end{pmatrix}$	$\begin{pmatrix}3.16\\3.04\\1.68\end{pmatrix}$
6	$\begin{pmatrix}198.0\\297.7\end{pmatrix}$	$\begin{pmatrix}-1.84\\16.09\end{pmatrix}$	$\begin{pmatrix}-1.84\\6.96\end{pmatrix}$	$\begin{pmatrix}3.14\\3.10\\1.65\end{pmatrix}$
7	$\begin{pmatrix}205.2\\292.8\end{pmatrix}$	$\begin{pmatrix}-0.99\\16.01\end{pmatrix}$	$\begin{pmatrix}-0.53\\5.15\end{pmatrix}$	$\begin{pmatrix}3.18\\2.99\\1.71\end{pmatrix}$
8	$\begin{pmatrix}202.4\\298.2\end{pmatrix}$	$\begin{pmatrix}-0.52\\16.85\end{pmatrix}$	$\begin{pmatrix}0.07\\4.69\end{pmatrix}$	$\begin{pmatrix}3.18\\2.96\\1.72\end{pmatrix}$
9	$\begin{pmatrix}194.7\\289.2\end{pmatrix}$	$\begin{pmatrix}-1.83\\15.31\end{pmatrix}$	$\begin{pmatrix}-1.25\\1.98\end{pmatrix}$	$\begin{pmatrix}3.23\\2.99\\1.75\end{pmatrix}$
10	$\begin{pmatrix}193.2\\291.7\end{pmatrix}$	$\begin{pmatrix}-3.84\\13.73\end{pmatrix}$	$\begin{pmatrix}-2.95\\-0.09\end{pmatrix}$	$\begin{pmatrix}3.27\\3.07\\1.76\end{pmatrix}$

Table 1: Numerical illustration of the ridge-solution controller

Run, t	$\mathbf{Y_t}$	$\mathbf{A_t}$	$\mathbf{D_t}$	$\mathbf{U_t}$
1	$\begin{pmatrix}192.9\\317.5\end{pmatrix}$	$\begin{pmatrix}-1.77\\4.36\end{pmatrix}$	$\begin{pmatrix}-1.77\\4.36\end{pmatrix}$	$\begin{pmatrix}3.35\\2.88\\1.89\end{pmatrix}$
2	$\begin{pmatrix}188.6\\312.7\end{pmatrix}$	$\begin{pmatrix}-5.06\\8.61\end{pmatrix}$	$\begin{pmatrix}-4.62\\7.52\end{pmatrix}$	$\begin{pmatrix}3.22\\3.21\\1.68\end{pmatrix}$
3	$\begin{pmatrix}207.1\\300.9\end{pmatrix}$	$\begin{pmatrix}-4.43\\10.73\end{pmatrix}$	$\begin{pmatrix}-2.83\\7.76\end{pmatrix}$	$\begin{pmatrix}3.19\\3.16\\1.67\end{pmatrix}$
4	$\begin{pmatrix}212.4\\291.7\end{pmatrix}$	$\begin{pmatrix}-2.04\\10.60\end{pmatrix}$	$\begin{pmatrix}0.27\\5.69\end{pmatrix}$	$\begin{pmatrix}3.25\\2.93\\1.79\end{pmatrix}$
5	$\begin{pmatrix}210.4\\299.8\end{pmatrix}$	$\begin{pmatrix}0.62\\11.96\end{pmatrix}$	$\begin{pmatrix}2.87\\5.63\end{pmatrix}$	$\begin{pmatrix}3.25\\2.76\\1.84\end{pmatrix}$
6	$\begin{pmatrix}198.9\\302.1\end{pmatrix}$	$\begin{pmatrix}1.06\\13.89\end{pmatrix}$	$\begin{pmatrix}2.58\\6.15\end{pmatrix}$	$\begin{pmatrix}3.22\\2.80\\1.80\end{pmatrix}$
7	$\begin{pmatrix}191.6\\301.9\end{pmatrix}$	$\begin{pmatrix}-0.40\\15.90\end{pmatrix}$	$\begin{pmatrix}0.48\\6.63\end{pmatrix}$	$\begin{pmatrix}3.17\\2.96\\1.71\end{pmatrix}$
8	$\begin{pmatrix}199.3\\292.0\end{pmatrix}$	$\begin{pmatrix}-0.44\\15.57\end{pmatrix}$	$\begin{pmatrix}0.32\\4.64\end{pmatrix}$	$\begin{pmatrix}3.20\\2.93\\1.75\end{pmatrix}$
9	$\begin{pmatrix}199.8\\294.3\end{pmatrix}$	$\begin{pmatrix}-0.40\\15.21\end{pmatrix}$	$\begin{pmatrix}0.27\\3.21\end{pmatrix}$	$\begin{pmatrix}3.23\\2.91\\1.77\end{pmatrix}$
10	$\begin{pmatrix}211.8\\294.4\end{pmatrix}$	$\begin{pmatrix}2.61\\14.72\end{pmatrix}$	$\begin{pmatrix}3.22\\1.82\end{pmatrix}$	$\begin{pmatrix}3.28\\2.66\\1.89\end{pmatrix}$

Table 2: Numerical illustration of the right-inverse controller

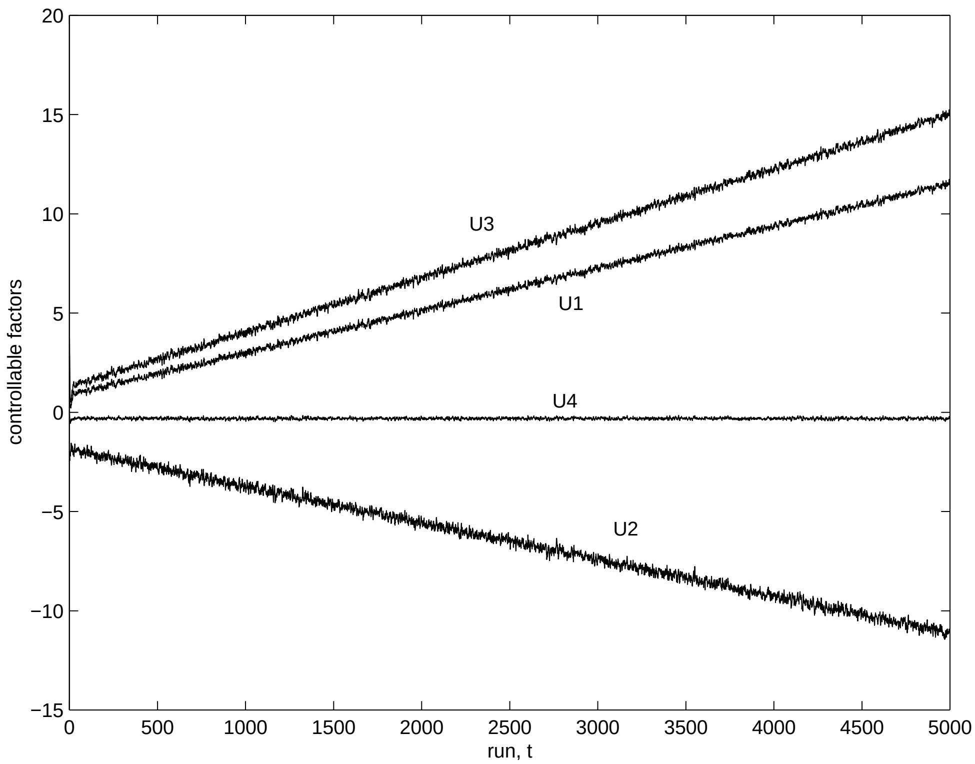

Figure 2: Controllable Factors of a Linear CMP Process controlled by the Ridge-Solution Controller

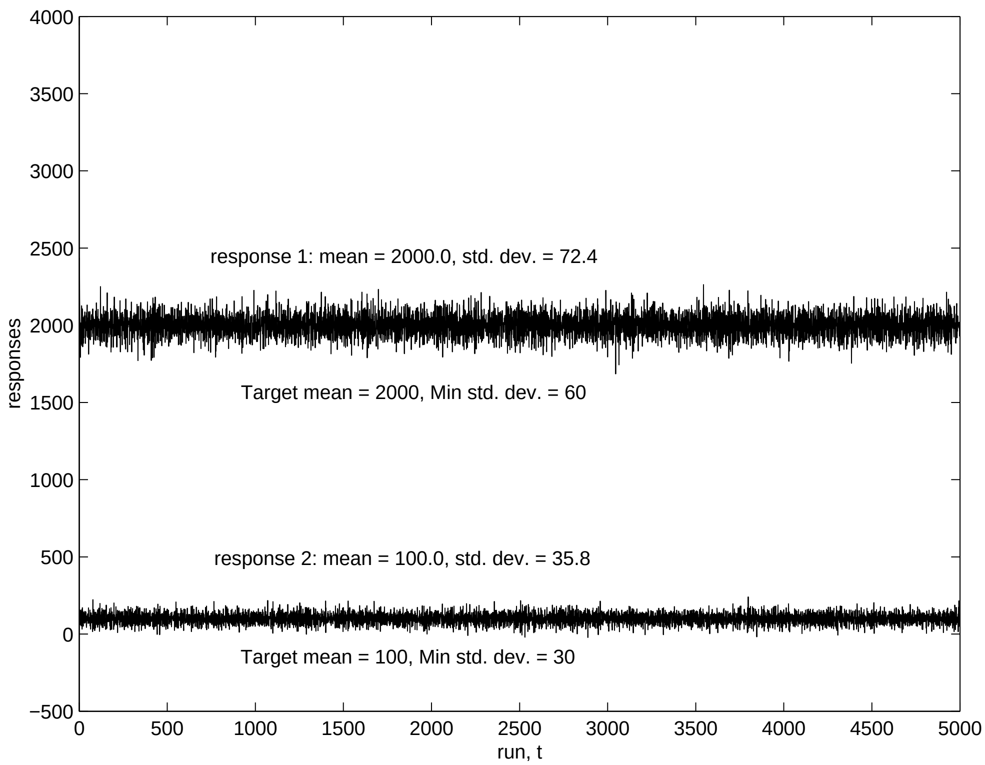

Figure 3: Responses of a Linear CMP Process controlled by the Right-Inverse Controller

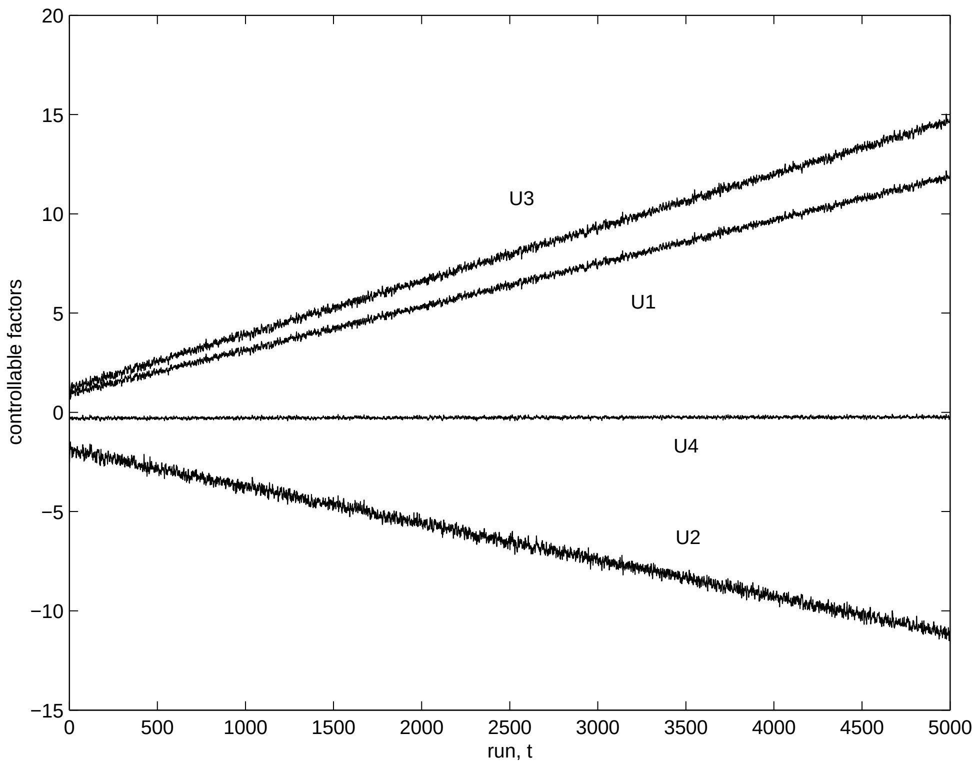

Figure 4: Controllable Factors of a Linear CMP Process controlled by the Right-Inverse Controller

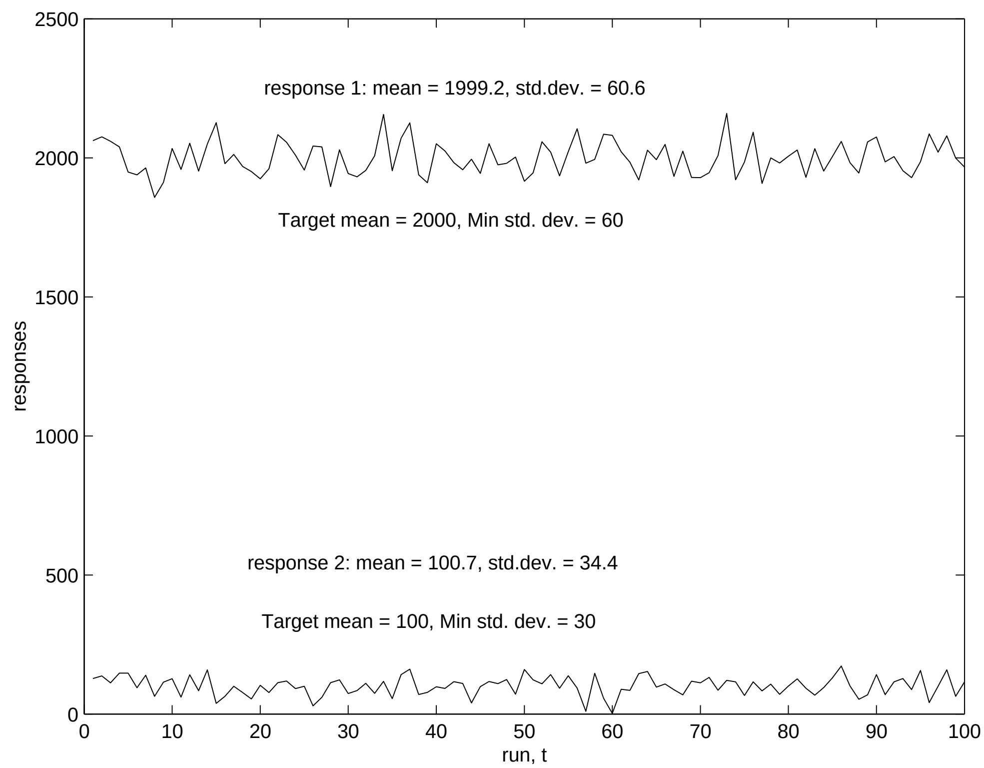

Figure 5: Responses for a Non-linear CMP Process controlled using the Ridge-Solution Controller

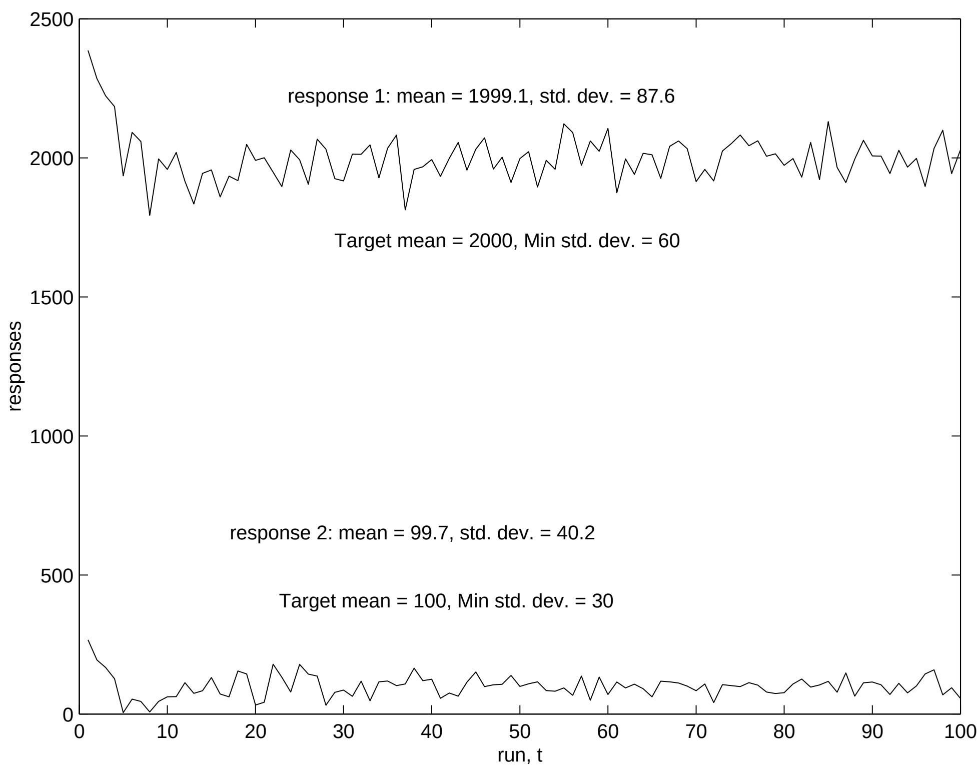

Figure 6: Responses for a Non-linear CMP Process controlled using the Right-Inverse Controller