Scheduling Methods for the Statistical Setup Adjustment Problem

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Revised August, 2002

Keywords: Feedback adjustment, Engineering Process Control, Statistical Process Control.

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Abstract

Feedback control methods have been proposed in recent literature to regulate the quality characteristic of parts or products in a manufacturing process. Depending on the costs involved, adjustments may not be needed at each time instant (i.e., for every part or product). This paper presents scheduling methods to determine the optimal time instants for adjusting a process. The focus is on the setup adjustment problem, in which it is necessary to adjust in order to compensate for an initial offset that occurs due to an incorrect setup operation. The performance of three scheduling methods are compared in terms of the expected manufacturing cost and computational effort of each method. The adjustment methods considered are based on estimates of the process variance and the size of the offset. The robustness of these methods with respect to biased estimates of the process variance and of the setup error or offset are discussed. One simple method, a backward implementation of the Silver-Meal heuristic used for inventory control is recommended based on a performance analysis.

1 Introduction: the setup adjustment problem

In discrete-part manufacturing, the quality of parts produced in batches depends to a great extent on correctly setting-up a machine. A setup error refers to a machine offset that occurs during the startup or maintenance operations. This error or offset may result in quality problems for the whole lot of parts produced after setting up the machine. Although a setup error can be speculated from observing an off-target value of the quality characteristic soon after starting production, the setup error cannot be measured directly due to the inherent variation in the process. In this paper, a sequential adjustment method that uses the sample average value of quality measurements over time to estimate the magnitude of the setup error is presented. A question addressed in this paper is to determine *when* to adjust using this method. We compare several methods for selecting an optimal or close-to-optimal adjustment schedule and provide some practical recommendations for setup adjustment in a short-run manufacturing process. By a *sequential adjustment rule* we will imply a procedure by which an operator makes successive adjustments to a machine. One adjustment is made every time a part is produced, and this can continue for several parts. As discussed below, some parts or time instants may go unadjusted, i.e., it might be beneficial to skip some adjustments in the sequence.

In order to remove a setup error, it will be assumed a controllable variable exists which correlates with the value of the quality characteristic by a gain factor. In traditional manufacturing, e.g., machining operations, such controllable factor is the machine setpoint or the "aimed at" value of a particular dimension. The gain is usually assumed to be one unit in discrete part manufacturing processes and we will follow this assumption. A trivial example of this is in a machining process, where a unit change in the setpoint will correspond to a value very close to an unit change in the machine dimension. The case where the adjustments are imprecise is not considered in this paper (see Trietsch, 1998, for such case). In the presence of uncertainty in the magnitude of the setup error, it is wise to be cautious when performing setup adjustments. The measurements of the quality characteristic collected after each adjustment can be used to determine adjustments later on in time in case they are necessary. Grubbs (1954, 1983) first studied this problem and proposed two sequential adjustment procedures, which we will refer to as Grubbs' harmonic and extended rules.

Recently, sequential adjustment rules have been investigated by Triestch (1998, 2000) and Del Castillo, Pan, and Colosimo (2002a). Triestch (2000) showed that the adjustments can be conducted in a series of arbitrary discrete points of time, i.e., they need not be carried out one following another successively but some adjustment instants may be "skipped". When a certain criterion related to the manufacturing and adjustment costs is considered, an optimal adjustment schedule can be designed. Del Castillo, Pan, and Colosimo (2002b) compared the small-sample performance of Grubbs' harmonic rule and other popular adjustment rules existing in the control engineering literature. They showed that the harmonic rule is very robust in the sense that the process off-target quality cost does not increase too much when the initial estimate of setup error is severely biased compared to the theoretical case when a perfect offset estimate is available.

One should notice that although the sequential adjustment methods discussed herein were introduced for the setup adjustment problem, they can be readily applied for adjusting when shifts and upsets occur within a run. To initiate adjustments, within-run upsets should be detected by a statistical process control (SPC) scheme. Integrating sequential adjustment methods with SPC monitoring schemes was discussed in Pan and Del Castillo (2002).

Using a feedback control scheme for controlling the quality of products has been widely discussed in the quality engineering literature (see, e.g., Vander Weil *et al.* 1992, Tucker *et al.* 1993, etc.). For instance, the "machine tool problem" consists in designing the optimal adjustment time of an autocorrelated process (an IMA(1,1)) based on some economic considerations (Box and Jenkins, 1963). Adams and Woodall (1989) and Box and Kramer (1992) used several types of adjustment and measurement cost models for this problem. Crowder (1992) derived the control limits for adjustments to minimize the total cost for a finite horizon (short-run) manufacturing process. Single-step adjustment methods (i.e., adjusting only once when the process is deemed out-of-control) were used in these papers. An optimal adjustment strategy for the setup error problem was discussed by Sullo and Vandeven (1999). Contrary to the sequential adjustment approach we follow herein, they considered a single adjustment method with a 0-1 type quality cost for conforming or nonconforming manufactured items.

In this paper, a special sequential adjustment strategy is described in which adjustments are scheduled to be carried out at some particular points in time along the time span of the manufacturing process. The cost function associated with the deviations from target will be assumed quadratic, and the adjustment cost will be assumed to be fixed. In the following sections, we first introduce a sample-average adjustment procedure, and show its equivalency to Grubbs' extended rule. Three scheduling methods for sample-average adjustments are then presented. Their performance and robustness on a short-run manufacturing process is studied numerically. Finally, some recommendations and conclusions about the different scheduling adjustment methods are provided.

2 Sample-average adjustment procedure

The model for the setup adjustment problem can be formulated as follows:

$$Y_t = d + X_{t-1} + \varepsilon_t.$$

In this model, $\{\varepsilon_t \sim (0, \sigma_\varepsilon^2)\}$ is a white noise sequence that models uncontrollable process noise; d is the unknown (constant) value of the setup error; and Y_t is the value of the process quality characteristic measured at discrete time t, with target assumed to be 0 without loss of generality. X_t is the level of the controllable factor (or setpoint) that adjusts the process. The problem is to find a series $\{X_t\}$ such that it removes the unknown offset d and drives Y_t to its target. Clearly, for such simple process, X_t should be set as $-\hat{d}_t$, where $\hat{d}_t$ is the "best" estimate of d available at the current time.

The model above can be easily formulated in state-space and $\hat{d}_t$ can then be obtained using a Kalman Filter technique (Del Castillo et al., 2002a). The Kalman Filter estimate of d_t is

$$\hat{d}_t = \hat{d}_{t-1} + \frac{1}{\sigma_\varepsilon^2/P_0 + t} Y_t, \tag{1}$$

where $\hat{d}_0$ is the initial (*a priori*) estimation of d and P_0 is a measure of confidence on this initial estimator. The Kalman Filter estimator is the minimum mean square error linear estimator (Duncan and Horn, 1972), so the "controller" $X_t = -\hat{d}_t$ is optimal in this sense.

This adjustment procedure requires adjustments at every time period, an adjustment policy that may be undesirable if the cost of the adjustments is significant. This implies that it is possible to design an adjustment schedule which skips some time periods between two successive adjustments and maintains the cost optimality of the whole procedure (Triestch, 1998). The idea is rather simple: if there are no adjustments between time i and j, then $\bar{Y}_{ij}$, the average of $Y_{i+1}...Y_j$, is the unbiased MLE (maximum likelihood estimate) of $(d + X_i)$;

therefore, it is intuitive to change the estimate of d to the following equation when the simple adjustment rule, $X_j = -\hat{d}_j$, is applied:

$$\hat{d}_j = \hat{d}_i + \frac{1}{\sigma_\varepsilon^2/P_0 + j} \sum_{t=i+1}^{j} Y_t. \tag{2}$$

In the remaining of this section, we will show that the adjustment procedure based on equation (2) provides the same general expression for Y_{j+1} as a function of the adjustments as given by the procedure based on equation (1). The performance of this adjustment rule will then be studied in subsequent sections.

Suppose Y_i, the value of the quality characteristic at time i, is known. Then, from (1) we have that X_i, which is a function of Y_i, is also known. We first use the procedure based on (1) to derive the function of Y_{j+1}. From the process model and adjustment function, we have

$$Y_{i+1} = d + X_i + \varepsilon_{i+1},$$

$$X_{i+1} = X_i - K_{i+1} Y_{i+1},$$

where $K_{i+1} = 1/(\sigma_\varepsilon^2/P_0 + i + 1)$. So,

$$Y_{i+2} = d + X_{i+1} + \varepsilon_{i+2} = (1 - K_{i+1})(d + X_i) - K_{i+1}\varepsilon_{i+1} + \varepsilon_{i+2}.$$

By substituting $X_{i+2}, X_{i+3}, ...$ into $Y_{i+3}, Y_{t+4}, ...$, we find after some algebra that

$$Y_{j+1} = (d + X_i) \prod_{l=i+1}^{j} (1 - K_l) - \sum_{l=i+1}^{j} K_l \varepsilon_l \prod_{r=l+1}^{j} (1 - K_r) + \varepsilon_{j+1}.$$

This can be simplified to:

$$Y_{j+1} = \frac{i + \sigma_\varepsilon^2/P_0}{j + \sigma_\varepsilon^2/P_0}(d + X_i) - \frac{1}{j + \sigma_\varepsilon^2/P_0} \sum_{l=i+1}^{j} \varepsilon_l + \varepsilon_{j+1}. \tag{3}$$

Now consider using the estimation procedure based on (2) to derive a general expression for Y_{j+1}. We have that

$$Y_l = d + X_i + \varepsilon_l,$$

for $l = i + 1, i + 2, ..., j$. So, the sum of $Y_{i+1}...Y_j$ is

$$\sum_{l=i+1}^{j} Y_l = (j - i)(d + X_i) + \sum_{l=i+1}^{j} \varepsilon_l.$$

Substituting this last result into (2) and taking the result back into $Y_{j+1} = d + X_j + \varepsilon_{j+1} = d - \hat{d}_j + \varepsilon_{j+1}$, we have that

$$Y_{j+1} = d + X_i - \frac{j-i}{j + \sigma_\varepsilon^2/P_0}\left(d + X_i + \frac{1}{j-i}\sum_{l=i+1}^{j} \varepsilon_l\right) + \varepsilon_{j+1}.$$

One can see that this simplifies and equals to (3). Notice that equation (2) can be written as

$$\hat{d}_j = \hat{d}_i + \frac{j-i}{\sigma_\varepsilon^2/P_0 + j}\bar{Y}_{ij}$$

where $\bar{Y}_{ij}$ is the arithmetic average of $Y_{i+1}...Y_j$.

In practice, an operator will set the controllable factor at time (or part number) j, X_j, equal to $-\hat{d}_j$. Thus, we call this adjustment method a “sample-average adjustment”. We now address the question about on which periods j shall the operator utilize this strategy.

3 Algorithmic and heuristic methods for optimal adjustment schedule

The sample-average adjustment procedure provides the opportunity of avoiding adjustment actions between two arbitrary adjusting times. This is especially useful when there are large fixed adjustment costs independent of the magnitude of the adjustments. In this section, we wish to find the best adjustment epochs or time instants such that they minimize the total manufacturing cost which is assumed to include the following components:

– expected off-target quality cost, C_q. This is the expectation of the sum of a quadratic function of Y_t around its target, i.e., $C_q = \sum_{t=1}^{n} E[\Omega(Y_t - T)^2]$, where n is the number of parts that need to be manufactured in the lot and Ω is the quadratic cost per unit. There exists an opportunity for adjusting the controllable factor for each of the n parts.

– adjustment cost, C_a. This is assumed to be fixed and independent of the magnitude of the adjustment, i.e., $C_a = M \times (\sum_{t=1}^{n} \delta(t))$, where $\delta(t)$ equals to 0 when no adjustment is scheduled and is 1 otherwise.

– measurement cost, C_m. This is assumed to be proportional to the number of adjustments, i.e., $C_m = G \times m$, where m is the time of the last adjustment. Obviously, when the last adjustment has been executed and no more adjustments are needed till the end of production, measurements on the following runs are not necessary.

The goal is to find an optimal adjustment schedule that minimizes the total cost when n is given and all of the cost parameters are known. The optimal schedule can be obtained by using dynamic programming. The formulation is analogous to what in the inventory control

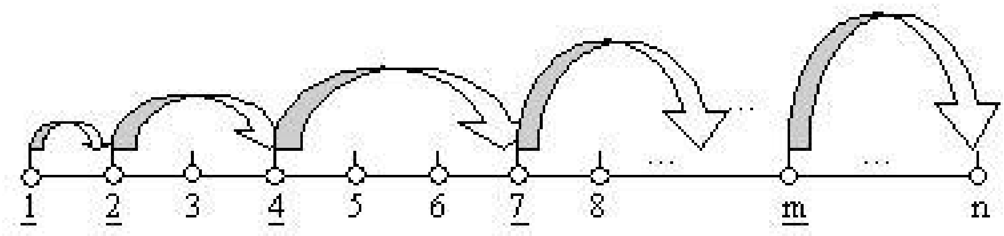

Figure 1: The graphical representation of an adjustment schedule

literature is the well-known Wagner-Whitin (W-W) algorithm (Wagner and Whitin, 1958). In Figure 1, we represent the starting time of manufacturing each part as a node on the production timeline, so the optimal schedule is equivalent to a minimal cost path from node 1 to node n.

As before, the target of Y_t is assumed to be zero, so $C_q = \Omega \sum_{t=1}^{n}(E^2[Y_t] + Var(Y_t))$. From (3), it is easy to derive that

$$\frac{E[Y_t]}{\sigma_\varepsilon} = \frac{d - \hat{d}_0}{P_0/\sigma_\varepsilon^2(t-1)+1}$$

and

$$\frac{Var(Y_t)}{\sigma_\varepsilon^2} = 1 + \frac{t-1}{(\sigma_\varepsilon^2/P_0 + t - 1)^2}.$$

If $\hat{d}_0$ is an unbiased estimation of d (recall that d is an unknown constant), then $E[Y_t] = 0$, i.e., Y_t is also unbiased. Define C_{ij} to be the cost from node $i+1$ to node j. Then, we have that

$$C_{ij} = M + \Omega(j-i)\left[1 + \frac{i}{(\sigma_\varepsilon^2/P_0 + i)^2}\right]\sigma_\varepsilon^2 + (j-i)G. \tag{4}$$

The last item on the right hand side of the equation is dropped when $j = n$. The W-W algorithm requires computation of the cost between pairs of nodes according to the recursion:

$$C^{W-W}(j) = min\{C_{ij} + C^{W-W}(i), j = i+1,\ i+2,\ ...,\ n\} \quad for\ i = n-1, n-2, ..., 1,$$

and

$$C^{W-W}(n) = 0,$$

where $C^{W-W}(i)$ is the minimum cost from node i to j.

The computational effort of the W-W algorithm consists of at least $n(n+1)/2$ calculations for the C_{ij}'s and $n(n-1)/2$ comparisons. Therefore, it has a 2nd-order polynomial complexity. Many heuristic methods have been proposed in the inventory control literature to overcome some of the difficulties of the W-W method (Silver, Pyke and Peterson, 1998).

For example, a simpler method, based on the Silver-Meal (S-M) heuristic which has been proved to have close to optimal performance in inventory control applications (Silver and Meal, 1973, Simpson, 2001), can be applied to this problem. This method searches for the minimum unit cost, $C_{ij}/(j-i)$ by fixing period i and increasing period j until a local minimum is obtained. Obviously, in equation (4), if i is fixed and j increased, the unit cost will decrease consistently, that is, no adjustment is scheduled except for the first period. This result is rather uninteresting. So we work out the procedure in the backward direction: we fix j and decrease i to find the minimum unit cost. We call this searching method the *backward S-M method*. It has a 1st-order polynomial computational complexity.

Illustration of the backward S-M adjustment scheduling method

To illustrate the backward S-M method, consider the case where 20 parts are to be produced, the measurement cost is negligible, and the per unit off-target cost is twice the adjustment cost. Thus we have that $n = 20$, $G = 0$, and let us use $\Omega = 1$ and $M = 0.5$. Furthermore, assume the process variance is known to be $\sigma_\varepsilon = 1$ and let us assume we have a moderate confidence in an initial offset-free setup, so we set $P_0 = 1$. Substituting these values into equation (4), setting $j = 20$ and decreasing i from 19 to 1, it is easy to find that $C_{ij}/(j-i)$ is minimized to 1.1319 when $i = 11$. Then setting $j = 11$ and decreasing i from 10 to 1, it is found that $C_{ij}/(j-i)$ is minimized to 1.2222 when $i = 5$. Finally, after letting $j = 5$ and recalculating $C_{ij}/(j-i)$ for $i < j$, the minimal value is found at $i = 1$. Thus, the adjustment steps given by this method are 1, 5 and 11, instants at we the operator should set $X_j = -\hat{d}_j$. Thus only three adjustments should be performed according to the schedule.

Trietsch (2000) proposed an approximate method for the optimal adjustment schedule, in which the time of the adjustments, t, is treated as a continuous number and the results need to be rounded to the closest integers. To use this method, one needs to solve two complicated equations numerically when the procedure starts and also needs to solve another equation numerically at every subsequent iteration, so the computation effort greatly depends on the initial values selected for these equations. Furthermore, a improper selection of initial values used in this method can cause incorrect results.

4 Comparison of numerical results

In this section, adjustment schedules with different total production runs and different cost coefficients of measurement and adjustment are studied. Notice that the total cost is a linear combination of three components, so we can assume without loss of generality that the off-target quality cost parameter, Ω, is one unit. In this section, we assume that the first adjustment, if it is needed, is always applied at time 1, i.e., before the start of the manufacturing process, based on our previous knowledge of a possible setup error (d is usually assumed to be 0, so in this case no adjustment is needed before manufacturing starts). It is also assumed that the initial estimate of the mean of d, $\hat{d}_0$, is unbiased, so the first adjustment will bring the process to target on average. More importantly, the inherent process variance σ_ε^2 is assumed to be known in this section. Sections 4.1 and 4.2 show further analysis where some of these assumptions are removed.

In Table 1, we contrast results from the three methods mentioned in the last section. The number of production runs varies from 20 to 500, the costs of measurement and adjustment vary from 0 to 2 units and from 0.5 to 2 units, respectively. For those cases in the table, $\sigma_\varepsilon = P_0 = 1$. The W-W algorithm provides the optimal adjustment schedule for each case, but the computation effort of this algorithm is much greater than that of the other methods, especially when the total number of production runs, n, is large. By comparing the S-M and Trietsch's methods with the W-W method, we can make the following remarks.

Remark 1. The cost of measurement and adjustment will greatly affect the total number of adjustments. Generally, when these costs increase, the number of adjustments decreases, and the measurement cost only affects the time of the last adjustment. This occurs because after the last adjustment no more measurements are needed, whereas before that time measurements should be conducted at every period (or part). Therefore, when the production runs are short and significant measurement or adjustment costs exist, it is optimal to adjust just once.

Remark 2. The distance between adjacent adjustments increases steadily in the sequence of adjustments. This can be explained as follows: when the process has been adjusted close but not exactly to target, we need stronger evidence from the process to demonstrate that there is still an offset remaining. Such evidence is only obtained with longer runs of observations between adjustments.

Remark 3. Comparing the backward S-M and Trietsch's methods, the backward S-M method always give fewer or equal number of adjustments than Trietsch's method, and the schedule from S-M is usually closer to the one given by W-W. Thus the backward S-M method is a better heuristic in terms of minimizing total cost.

Remark 4. The S-M method has the advantage of reducing computing effort significantly comparing to the W-W algorithm and its computing time is consistently less than that of the Trietsch's method. In fact, this method can be easily implemented by using a handheld calculator or a spreadsheet software to support on-line process adjustment decisions.

n	G	M	method	cost	time(sec.)	adjustments	n	G	M	method	cost	time(sec.)	adjustments
20	0	0.5	WW	22.92	~ 0	1-7	100	0	0.5	WW	106.22	0.32	1-7-18-43
			SM	23.02	~ 0	1-5-11				SM	106.52	~ 0	1-5-10-18-33 -58
			Trietsch's	23.36	0.21	1-3-6-11				Trietsch's	106.89	0.33	1-3-7-14-27 -42-65
		1	WW	23.92	~ 0	1-7			1	WW	108.06	0.32	1-10-33
			SM	24.62	~ 0	1-3-9				SM	108.82	0.02	1-4-10-23-49
			Trietsch's	25.36	0.22	1-3-6-11				Trietsch's	109.60	0.28	1-3-8-19-33-58
		2	WW	25.75	~ 0	1			2	WW	111.06	0.32	1-10-33
			SM	25.92	~ 0	1-7				SM	112.28	~ 0	1-5-15-40
			Trietsch's	27.78	0.14	1-2-7				Trietsch's	114.12	0.23	1-2-6-16-40
	1	0.5	WW	24.25	0.02	1		1	0.5	WW	117.67	0.32	1-7
			SM	24.25	~ 0	1				SM	117.92	~ 0	1-3-7
			Trietsch's	25.69	0.13	1-3				Trietsch's	118.02	0.17	1-3-8
		1	WW	24.75	~ 0	1			1	WW	118.67	0.32	1-7
			SM	24.75	~ 0	1				SM	118.67	~ 0	1-7
			Trietsch's	26.25	0.1	1-2				Trietsch's	119.67	0.13	1-2-8
		2	WW	25.75	~ 0	1			2	WW	120.67	0.32	1-7
			SM	25.75	~ 0	1				SM	120.67	~ 0	1-7
			Trietsch's	28.25	0.1	1-2				Trietsch's	122.67	0.13	1-2-8
	2	0.5	WW	24.25	0.02	1		2	0.5	WW	122.11	0.32	1-4
			SM	24.25	~ 0	1				SM	122.11	~ 0	1-4
			Trietsch's	26.25	0.11	1-2				Trietsch's	123.15	0.11	1-2-6
		1	WW	24.75	~ 0	1			1	WW	123.11	0.32	1-4
			SM	24.75	~ 0	1				SM	123.11	~ 0	1-4
			Trietsch's	27.25	0.1	1-2				Trietsch's	124.65	0.13	1-2-6
		2	WW	25.75	~ 0	1			2	WW	125.11	0.32	1-4
			SM	25.75	~ 0	1				SM	125.11	~ 0	1-4
			Trietsch's	29.25	0.11	1-2				Trietsch's	127.65	0.13	1-2-6
50	0	0.5	WW	54.78	0.09	1-7-20	500	0	0.5	WW	509.65	7.93	1-7-18-43-99 -223
			SM	55.08	~ 0	1-4-8-16 -28				SM	510.12	0.08	1-5-10-18-33-57 99-170-29
			Trietsch's	55.46	0.29	1-3-7-14 -22-33				Trietsch's	510.18	0.4	1-3-7-15-31-54 -95-166-288
		1	WW	56.28	0.09	1-7-20			1	WW	512.37	7.92	1-8-25-70-188
			SM	56.86	~ 0	1-4-11-24				SM	513.20	0.07	1-5-13-29-60 123-249
			Trietsch's	57.62	0.26	1-3-7-14 -27				Trietsch's	514.03	0.36	1-3-8-19-44-99 -170-292
		2	WW	58.45	0.08	1-12			2	WW	516.62	7.93	1-11-42-147
			SM	59.29	~ 0	1-7-19				SM	518.80	0.06	1-4-13-34-84 -206
			Trietsch's	61.15	0.18	1-2-7-19				Trietsch's	520.64	0.29	1-2-7-22-63 -126-251
	1	0.5	WW	61.11	0.08	1-4		1	0.5	WW	544.16	7.92	1-7-19
			SM	61.11	~ 0	1-4				SM	544.29	0.04	1-5-10-19
			Trietsch's	61.63	0.17	1-3-5				Trietsch's	544.39	0.21	1-3-8-20
		1	WW	62.11	0.08	1-4			1	WW	545.66	7.99	1-7-19
			SM	62.11	~ 0	1-4				SM	546.37	0.04	1-3-8-19
			Trietsch's	62.25	0.13	1-5				Trietsch's	546.55	0.16	1-2-7-20
		2	WW	63.25	0.08	1			2	WW	548.28	7.94	1-18
			SM	64.11	~ 0	1-4				SM	548.66	0.03	1-7-18
			Trietsch's	64.11	0.1	1-4				Trietsch's	549.35	0.14	1-3-19
	2	0.5	WW	61.75	0.09	1		2	0.5	WW	558.91	7.93	1-6-13
			SM	61.75	~ 0	1				SM	559.21	0.04	1-3-7-13
			Trietsch's	64.11	0.13	1-4				Trietsch's	559.26	0.18	1-3-7-14
		1	WW	62.25	0.08	1			1	WW	560.30	7.93	1-13
			SM	62.25	~ 0	1				SM	560.41	0.04	1-5-13
			Trietsch's	65.11	0.13	1-4				Trietsch's	560.49	0.15	1-5-14
		2	WW	63.25	0.08	1			2	WW	562.30	7.92	1-13
			SM	63.25	~ 0	1				SM	563.49	0.03	1-4-13
			Trietsch's	66.31	0.1	1-3				Trietsch's	563.80	0.13	1-3-14

Table 1: Comparison of costs, time and adjustment schedules of 3 schedule design methods. The numbers in the columns G and M can be viewed as the ratios of the per unit measurement cost to the off-target quality cost (G/Ω), and of the unit adjustment cost to the off-target quality cost (M/Ω).

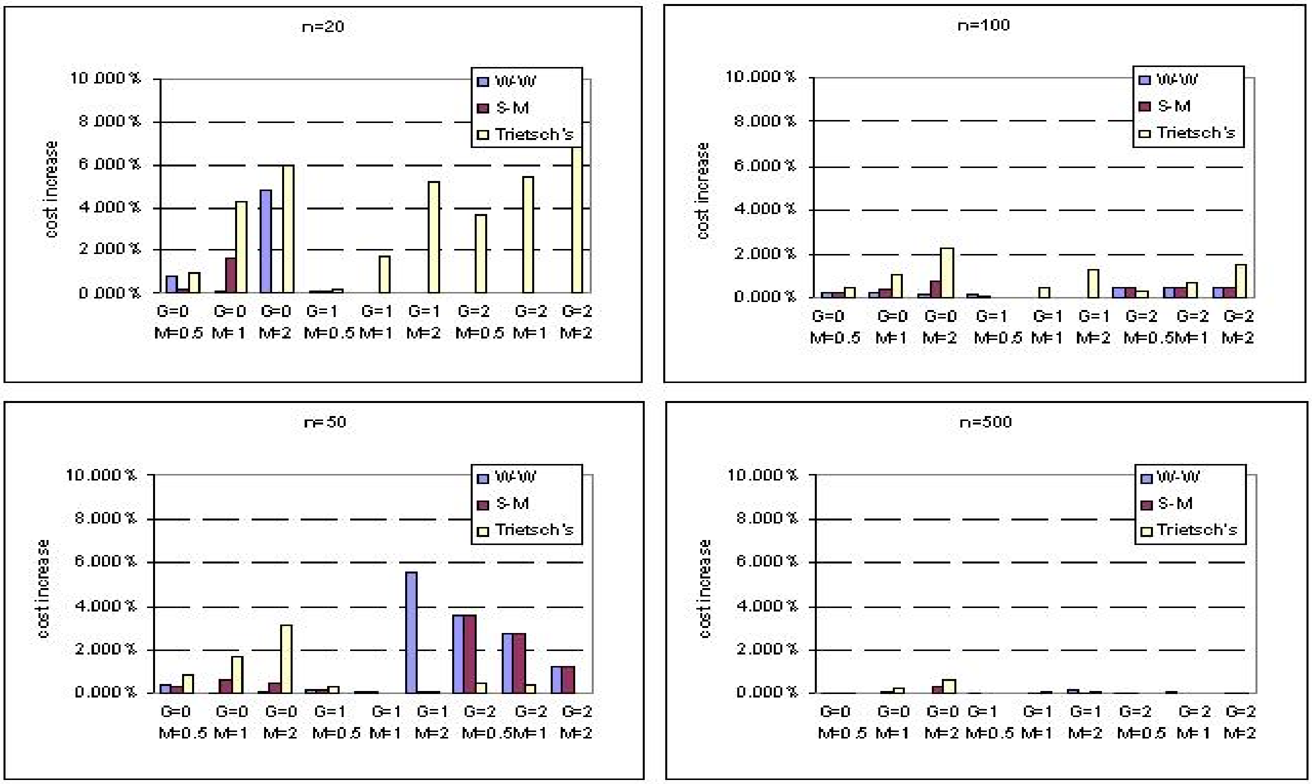

Figure 2: Case when σ_ε is over-estimated ($\sigma_\varepsilon = 0.8$, $\hat{\sigma}_\varepsilon = 1$). All of the cases presented in Table 1 are investigated and compared.

4.1 The case when the process variance (σ_ε) is unknown

In the following discussion, we vary the true value of σ_ε from 1.0 to 0.8 and 1.2, that is, the estimate $\hat{\sigma}_\varepsilon$, which was assumed to be 1 in the previous discussion and was used in the three methods to obtain the optimal or near-optimal adjustment schedules is now an over- or under-estimate, respectively, of the true σ_ε. The minimum cost and the optimal adjustment schedule can be obtained by introducing the true value of σ_ε into the W-W algorithm. This was used as a benchmark to compare cost increments due to a poorly estimated σ_ε. The cost increments induced by over- or under-estimating σ_ε are investigated and presented in Figures 2 and 3.

Remark 5. When the production run, n, is large ($n = 100$, or 500), the cost increments by introducing over- or under-estimated σ_ε of using the three methods are very small and can be ignored.

Remark 6. The backward S-M method generally performs better than Trietsch's method, except in some particular cases if σ_ε is over-estimated (see, e.g., in Figure 2, when $n = 50$, $G = 2$, $M = 0.5$, 1 and 2). The W-W method still performs well when $G = 0$ (no

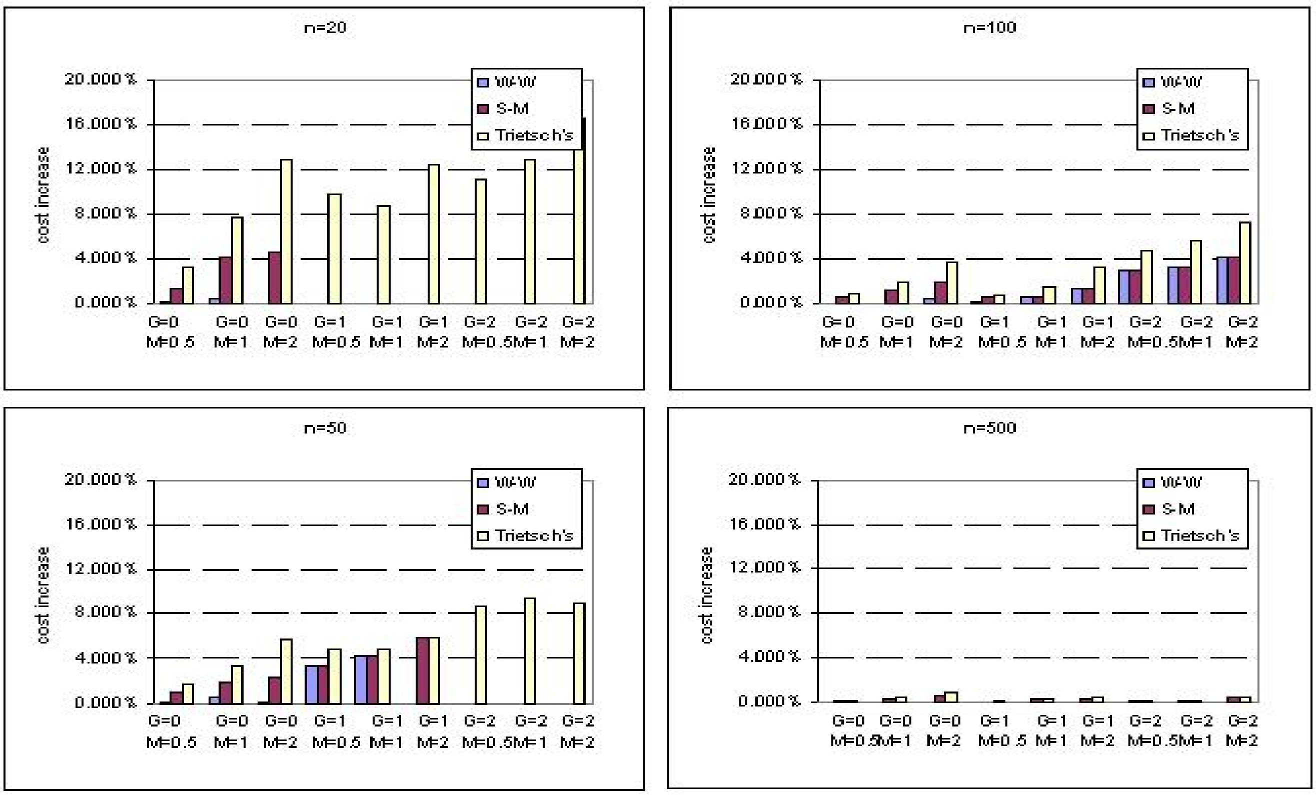

Figure 3: Case when σ_ε is under-estimated ($\sigma_\varepsilon = 1.2$, $\hat{\sigma}_\varepsilon = 1$) .

measurement cost). But when measurement costs are considered, the W-W method is not better than the backward S-M method in general.

Remark 7. Under-estimating σ_ε ($\sigma_\varepsilon = 1.2$) will induce larger increases in cost than when over-estimating σ_ε for all rules. Normally when the true value of σ_ε is larger than expected, longer runs of parts between adjustments are needed since these provide more measurements for estimating the setup error magnitude; therefore, the total number of adjustments will decrease. Since Trietsch's method always provides more adjustments than the other two methods, it will incur in a higher cost. Conversely, when σ_ε is over-estimated, we find that Trietsch's method has an advantage in three cases: $n = 50$, $G = 2$, and $M = 0.5$, 1 and 2.

4.2 The case when a biased initial estimate of the offset is used

In the previous calculations, we assume the initial estimate, $\hat{d}_0$, equals to d. This assumption is hardly realistic since if d were exactly known, a one-step adjustment (or calibration) would be enough for removing the offset. A biased initial estimation of d will lead to an increase in cost in the adjustment schedule calculated by each method. Similarly as in Section 4.1, the robustness of the three methods under such situation are compared and presented in Figures 4 and 5. We varied $|d - \hat{d}_0|$ from zero to 1 and 2.

Remark 8. Similarly as in remark 5, when the production run is large, the cost increments caused by the biased initial estimate of d are insignificant.

Remark 9. When n is small, Trietsch's method is generally the best one and the W-W method is worst. We observe in some cases when $\hat{d}_0$ is close to d the backward S-M method can out-perform the Trietsch's method, e.g., see the cases of $|d - \hat{d}_0| = 1\sigma_\varepsilon$, $n = 50$ and $G = 0$. But when $\hat{d}_0$ is strongly biased from d or when the measurement cost is high, the S-M method may lead to a high cost increase, e.g., see the cases of $|\hat{d}_0 - d| = 2\sigma_\varepsilon$, $n = 50$ and $G = 2$.

Remark 10. When $|d - \hat{d}_0|$ is large, the cost increments are more severe and Trietsch's method is more robust compared to other methods since it adjusts more often than the other two methods.

It was found that when $|d - \hat{d}_0| \neq 0$, more adjustments are required by the optimal schedule, especially in the first few periods. Since Trietsch's method happens to provide more adjustments, it results in the lowest cost increments.

To enhance the performance of the backward S-M method when $|d - \hat{d}_0| \neq 0$, we suggest to add one more adjustment at the beginning of the production run. The comparison of small-sample ($n = 20$ and $n = 50$) performance of the modified S-M heuristic and other methods is presented in Figure 6 for some inaccurate initial estimate of the offset. Our results show that the adjustment schedule given by this heuristic method has a similar cost

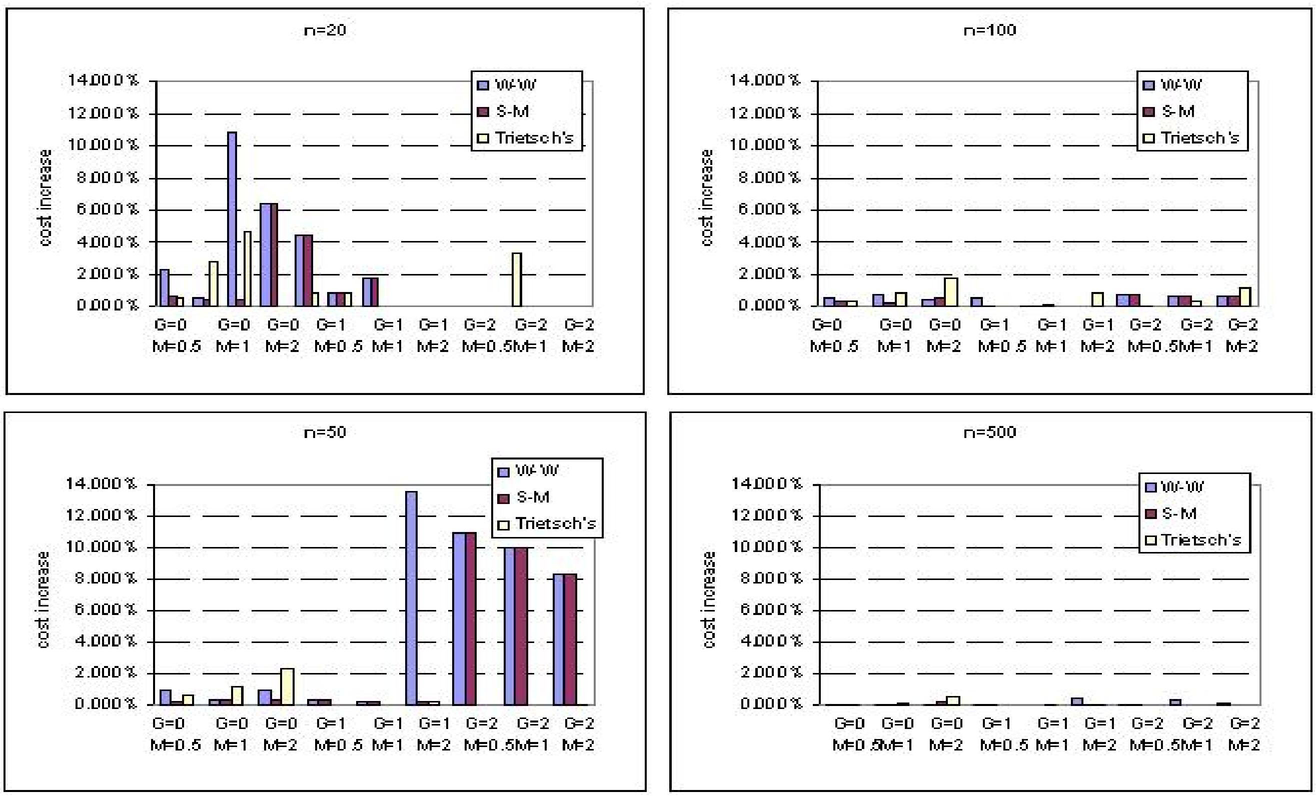

Figure 4: Case when $|d - \hat{d}_0| = 1\sigma_\varepsilon$.

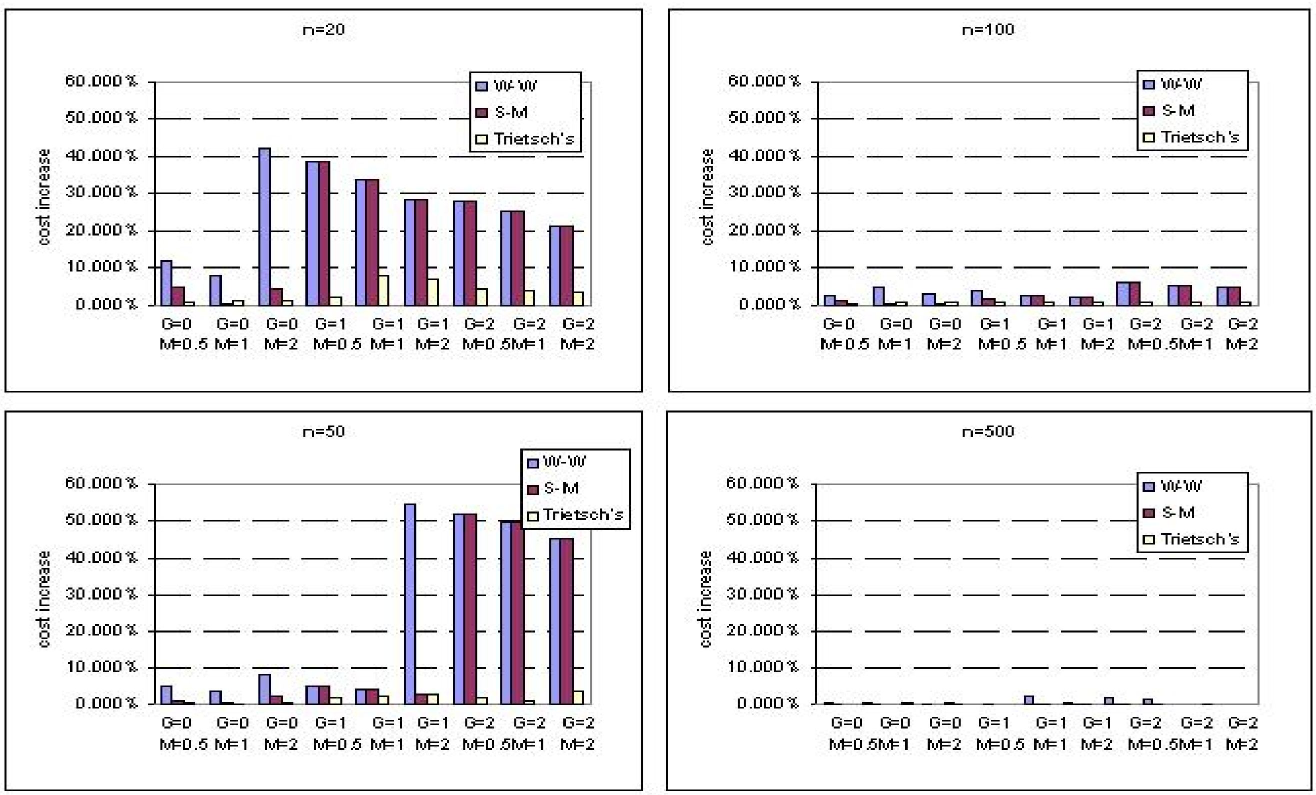

Figure 5: Case when $|d - \hat{d}_0| = 2\sigma_\varepsilon$.

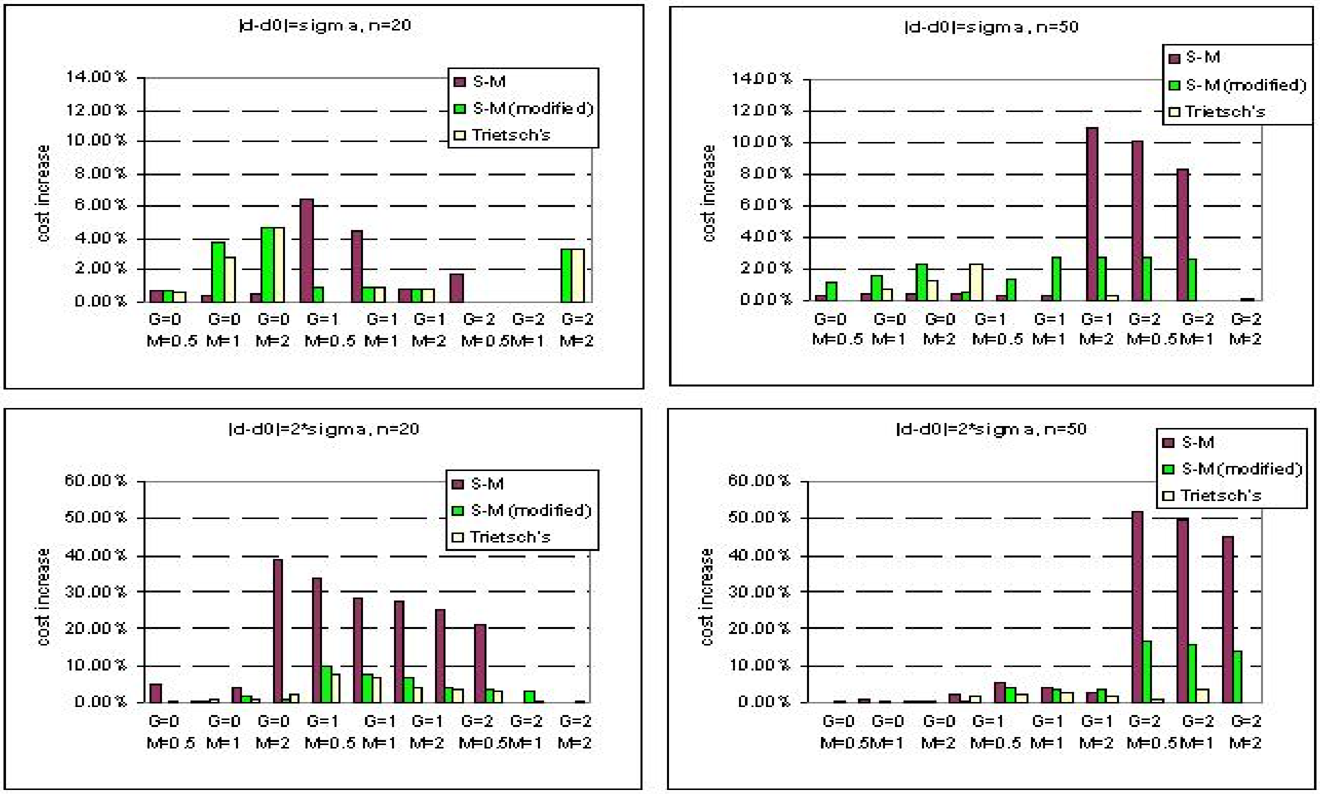

Figure 6: Comparison of performance of the modified S-M heuristic and other methods when $|d - \hat{d}_0| = 1\sigma_\varepsilon$ or $|d - \hat{d}_0| = 2\sigma_\varepsilon$ and $n = 20$ or $n = 50$.

as that given by Trietsch's method. However, Trietsch's method seems to still have a slight cost advantage but this is not enough, in our view, to justify its more complex computations over the modified S-M method.

Illustration of the modified backward S-M heuristic

To illustrate the modified S-M heuristic, consider the case when $n = 50$ parts are to be produced, with measurement cost $G = 2$ and adjustment cost $M = 0.5$ ($\Omega = 1$). In this case, the backward S-M method (without the proposed modification) suggests a single adjustment at node 1. This schedule will incur in more than a 50% cost increase compared to the minimal cost if the initial estimate of the setup error is two units from the true value, i.e., if $|d - \hat{d}_0| = 2$. The modified backward S-M method calls for inserting one more adjustment after the first one, i.e., changing the adjustment steps to 1 and 2 in this example. In these two time instants the operator should set $X_j = -\hat{d}_j$. With such schedule, the cost increment reduces to only 14% over the minimum.

5 Conclusion

In this paper, a simple adjustment procedure for adjusting a machine for setup errors was introduced. Three adjustment scheduling methods were compared which can achieve optimal or near optimal expected total manufacturing cost: the Wagner-Whitin method, a backward implementation of the Silver-Meal method and a method due to Trietsch (2000). It was found that when the production runs are long (i.e., large lots of product), there is not significant difference between the performance of three methods. For a short-run manufacturing process, the proposed backward S-M method has the advantage of providing a close-to-optimal solution with small computation effort (for example, the method can be implemented easily in a spreadhseet software), even when the process variance estimate is biased. A problem found with this method is that when there exists a significant bias on the initial estimate of the setup error and when the adjustment or the measurement costs are high, the schedule provided by the backward S-M method may incur in a much higher cost increase than Trietsch's method. As a solution to this drawback, it was demonstrated that simply adding one more adjustment close to the beginning of the schedule enhances the robustness of the backward S-M method.

Acknowledgements

This research was partially funded by NSF grant DMI 0200056.

References

ADAMS, B.M. and WOODALL, W.H., (1989) "An Analysis of Taguchi's On-line Process Control Procedure under a Random-Walk Model", *Technometrics*, **34**: 401-413.

BOX, G.E.P. and KRAMER, T., (1992) "Statistical Process Monitoring and Feedback Adjustment - a Discussion", *Technometrics*, **34**: 401-413.

BOX, G.E.P. and JENKINS, G.M., (1963) "Further contributions to adaptive quality control: simultaneous estimation of dynamics: non-zero costs", *Bulletin of the International Statistical Institute*, **34**: 943-974.

CROWDER, S.V., (1992) "An SPC Model for Short Production Run: Minimizing Expected Costs", *Technometrics*, **34**: 64-73.

DEL CASTILLO, E. PAN, R., and COLOSIMO, B. (2002a) “An Unifying View of Some Process Adjustment Methods”, to appear in *Journal of Quality Technology*.

DEL CASTILLO, E. PAN, R., and COLOSIMO, B. (2002b) “Small Sample Performance of Some Statistical Adjustment Methods”, to appear in *Communications in Statistics, Simulation and Computation*.

DUNCAN, D.B. and HORN, S.D., (1972) “Linear Dynamic Recursive Estimation from the Viewpoint of Regression Analysis”, *Journal of the American Statistical Association*, **67**: 815-821.

GRUBBS, F.E., (1954). “An Optimum Procedure for Setting Machines or Adjusting Processes,” *Industrial Quality Control*, July, reprinted in *Journal of Quality Technology*, 1983, **15**(4): 186-189.

PAN, R. and DEL CASTILLO, E., (2002) “Integration of Sequential Adjustments and Process Monitoring Techniques”, Technical Report, Engineering Statistics Laboratory, Industrial Engineering Dept., Penn State University.

SILVER, E.A., PYKE, D.F. and PETERSON, R., *Inventory Management and Production Planning and Scheduling*, 3rd edition, 1998, John Wiley and Sons, Inc.

SILVER, E.A. and MEAL, H.C., (1973) “A Heuristic for Selecting Lot Size Quantities for the Case of a Deterministic Time-varying Demand Rate and Discrete Opportunities for Replenishment”, *Production and Inventory Management*, **10**(4): 64-74.

SIMPSON, N.C., (2001) “Questioning the Relative Virtues of Dynamic Lot Sizing Rules”, *Computers and Operations Research*, **28**: 899-914.

SULLO, P. and VANDERVEN, M., (1999) “Optimal Adjustment Strategies for a Process with Run-to-run Variation and 0-1 Quality Loss”, *IIE Transactions*, **31**: 1135-1145.

TRIETSCH, D., (1998). “The Harmonic Rule for Process Setup Adjustment with Quadratic Loss”, *Journal of Quality Technology*, **30**(1): 75-84.

TRIETSCH, D., (2000). “Process Setup Adjustment with Quadratic Loss”, *IIE Transactions*, **32**(4): 299-307.

TUCKER, W.T., FALTIN, F.W. and VANDER WEIL, S.A., (1993) “Algorithm Statistical Process Control: an Elaboration”, *Technometrics*, **35**(4): 363-375.

VANDER WEIL, S.A., TUCKER, W.T., FALTIN, F.W. and DOGANAKSOY, N., (1992) "Algorithmic Statistical Process Control: Concepts and an Application", *Technometrics*, **35**: 139-151.

WAGNER, H., and WHITIN, T.M., (1958) "Dynamic Version of the Economic Lot Size Model", *Management Science*, **5**(1): 89-96.