Summary for Lectures 1-3

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We are now working with **double integrals**

$$\int\int_R f(x,y)\ dA$$

which compute the volume under the graph of $f(x,y)$ over the region R. The precise definition is similar to the one-dimensional case. In practice, we compute these integrals by integrating

$$\int_{x_{min}}^{x_{max}} S(x)\ dx$$

where x_{min} is the minimum x-value on R, x_{max} is the maximum x-value on R, and $S(t)$ is the area under the curve obtained by intersecting the graph with the plane $x = t$. Thus,

$$S(x) = \int_{y_1(x)}^{y_2(x)} f(x,y)\ dy$$

where $y_1(x)$ and $y_2(x)$ are the min and max values of y in R when x is fixed. We could do the same by taking slices in y and then integrating over curves with respect to x. In good cases (functions that we will consider), these two integrals are the same and both are equal to $\int\int_R f(x,y)\ dA$.

The technical statement is Fubini's theorem which says that this holds if f is bounded on R, discontinuous on a finite number of smooth curves, and the iterated integrals all exist.

How to compute double integrals using polar coordinates.

Polar coordinates are given by $x = r\cos\theta$ and $y = r\sin\theta$. The important point to remember about polar coordinates is that integration element becomes $rdrd\theta$. To set up the bounds, look at a fixed value of θ and the max/min values of r as in the standard case.

$$\int\int_R f(x,y)dA = \int_{\theta_{min}}^{\theta_{mas}} \int_{r_1(\theta)}^{r_2(\theta)} f(r\cos\theta, r\sin\theta)\ rdrd\theta.$$