Summary for Lectures 4-5

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We also see that there are applications of the double integral similar to those of the standard integral. For example, the area of a region R is given by

$$\text{area}(R) = \iint_R 1 \, dA$$

and the average value of $f(x.y)$ over a region R is given by

$$\frac{1}{\text{area}(R)} \iint_R f(x,y) \, dA.$$

Finally, the mass of a flat object with mass density $\rho(x,y)$ has total mass

$$m = \iint_R \rho(x,y) \, dA.$$

Following this theme, there is a slight generalization of average value which is the weighted average value of f with respect to a weight $\rho(x,y)$ given by

$$\bar{f} = \frac{\iint f(x,y)\rho(x,y) \, dA}{\iint \rho(x,y) \, dA}.$$

A special case of this is the center of mass of a flat lamina with mass density $\rho(x,y)$. This is given by the point $(\bar{x},\bar{y})$ where

$$\bar{x} = \frac{1}{m} \iint x\rho(x,y) \, dA \qquad \text{and} \qquad \bar{y} = \frac{1}{m} \iint y\rho(x,y) \, dA$$

are the average values of x, y and $m = \int\int \rho(x,y) \, dA$ is the mass. We also have the moment of inertia of an object of mass density $\rho(x,y)$ about an axis is given by

$$I = \int\int d^2(x,y)\rho(x,y) \, dA$$

where $d(x,y)$ is the distance from (x,y) to the axis of rotation. In the specific case when the axis is just the origin, then $d^2 = x^2 + y^2$. Finally, we can describe probability by a probability density function $\rho(x,y)$ as a function of the random variables x, y. Then, the probability over any region is the integral of $\rho(x,y)$ over that region. Note that the integral over all values should be 1 and ρ should be a non-negative function. The expected value of x is $E(x) = \bar{x}$ using the formula above with $m = 1$ (the same holds for y).

Applications of double integrals in Calculus, 7th Edition, by James Stewart, are in Section 15.5