

Linear Homogeneous Equations of the 2nd order with constant coefficients

The general form

$$ay''(t) + by'(t) + cy(t) = 0, \quad (1)$$

where a, b, c - constants.

To find the solution, solve the quadratic equation with unknown r :

$$ar^2 + br + c = 0. \quad (2)$$

Sign of the discriminant $D = b^2 - 4ac$ of the equation (2)	Solutions r_1, r_2 of the quadratic equation (2)	Solution of the ODE (1)
$D > 0$	<p>two different real solutions</p> $r_1 \neq r_2,$ <p>where</p> $r_1 = \frac{-b + \sqrt{D}}{2a}$ <p>and</p> $r_2 = \frac{-b - \sqrt{D}}{2a}$	$y(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t},$ <p>where c_1, c_2 - constants</p>
$D = 0$	<p>one repeated real solution</p> $r_1 = r_2 = r,$ <p>where</p> $r = \frac{-b}{2a}$	$y(t) = e^{rt} \cdot (c_1 + c_2 t) = c_1 e^{rt} + c_2 t e^{rt},$ <p>where c_1, c_2 - constants</p>
$D < 0$	<p>two complex conjugate roots</p> $r_{1,2} = \lambda \pm i\mu,$ <p>where</p> $\lambda = \frac{-b}{2a} \text{ and } \mu = \frac{\sqrt{-D}}{2a}$	$y(t) = e^{\lambda t} \cdot (c_1 \cos(\mu t) + c_2 \sin(\mu t)) =$ $= c_1 e^{\lambda t} \cos(\mu t) + c_2 e^{\lambda t} \sin(\mu t),$ <p>where c_1, c_2 - constants</p>