

## Suggested problems 19

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1. For each problem, determine whether the system is under-,over-, or critically damped.

(a)

$$u'' + 6u' + 9u = 0, \quad u(0) = 1, \quad u'(0) = 1.$$

(b)

$$u'' + 4u' + 3u = 0, \quad u(0) = 0, \quad u'(0) = -4.$$

(c)

$$u'' + 6u' + 10u = 0, \quad u(0) = -2, \quad u'(0) = 9.$$

2. A mass-spring system is described by the equation

$$5u'' + \gamma u' + ku = F(t),$$

- (a) Suppose the mass originally stretched the spring 2 meters to reach its equilibrium position. What is the spring constant  $k$ ? (Assume  $g = 10m/s^2$  to be the gravitational constant.)
- (b) Suppose  $k = 20$ . For what value(s) of  $\gamma$  would this system be **critically damped**?
- (c) Suppose  $\gamma = 0$  and  $k = 45$ . What is the natural frequency of this system?
- (d) True or false: Suppose  $\gamma = 0, k = 5$ , and  $F(t) = 7 \sin t$ , then the mass-spring system is undergoing resonance?
- (e) Suppose  $\gamma = 3, k = 1, F(t) = 0$ . What is the quasi-frequency of the system?