To find eigenvalues and eigenvectors of a matrix

Let A be a matrix 2×2, whose eigenvalues and eigenvectors we would like to find.

Find eigenvalues:

1. Compute matrix $A - rI$, where $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and r is an unknown number.

2. Compute $\det(A - rI)$ in terms of r.

3. Solve the equation $\det(A - rI) = 0$ for r. The solutions are exactly the eigenvalues of the matrix A.

Find eigenvectors:

Let $r = r_1$ be an eigenvalue for A.

1. Compute $A - r_1 I$.

2. Compute the product $(A - r_1 I) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$.

3. Find a <u>nonzero</u> solution of the linear system

$$(A - r_1 I) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

   i.e. find nonzero pair of numbers x_1, x_2.

4. A vector $\bar{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ is an eigenvector for the matrix A for the eigenvalue $r = r_1$.

If you have <u>several eigenvalues</u>, then repeat the procedure of finding eigenvectors for <u>every eigenvalue</u>.