Suggested problems 31

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1. Find all eigenvalues and their corresponding eigenfunctions of each two-point boundary value problem.

   (a) $X'' + \lambda X = 0, \qquad X(0) = 0, \quad X(2\pi) = 0.$

   (b) $X'' + \lambda X = 0, \qquad X'(0) = 0, \quad X'(2\pi) = 0.$

   (c) $X'' - \lambda X = 0, \qquad X'(0) = 0, \quad X(1) = 0.$

2. (* optional) Show that 0 is not an eigenvalue, and that any positive eigenvalue of the boundary value problem

$$X'' + \lambda X = 0, \qquad X(0) - X'(0) = 0, \quad X(L) + 2X'(L) = 0,$$

   must be in the form $\lambda = \sigma^2$, where σ satisfies the equation

$$\cot(\sigma L) = \frac{2\sigma^2 - 1}{3\sigma}.$$