Suggested problems 32

Instructor: Alena Erchenko

1. Determine whether the given function is periodic. If so, find its fundamental period.

   (a) $\sin(5x)$.

   (b)
   $$f(x) = \begin{cases} 0, & 2n-1 \leqslant x < 2n, \\ 1, & 2n \leqslant x < 2n+1, \end{cases}$$
   where $n = 0, \pm 1, \pm 2, \cdots$.

2. Find the Fourier series representation of each periodic function.

   (a) $f(x) = 5, \qquad -1 < x < 1, \qquad f(x+2) = f(x);$

   (b) $f(x) = (\cos x + \sin x)^2, \qquad -\pi < x < \pi, \qquad f(x+2\pi) = f(x);$

   (c) $f(x) = x^2, \qquad 0 \leqslant x < \pi, \qquad f(x+\pi) = f(x).$

3. (*optional) Suppose that g is an integrable periodic function with period T.

   (a) If $0 \leqslant a \leqslant T$, show that
   $$\int_0^T g(x)dx = \int_a^{a+T} g(x)dx.$$
   Hint: Show first that $\int_0^a g(x)dx = \int_T^{a+T} g(x)dx$. In the second integral, consider the change of variable $s = x - T$.

   (b) Show that for any value of a, not necessarily in $0 \leqslant a \leqslant T$,
   $$\int_0^T g(x)dx = \int_a^{a+T} g(x)dx.$$

   (c) Show that for any values of a and b,
   $$\int_a^{a+T} g(x)dx = \int_b^{b+T} g(x)dx.$$

4. (*optional) If f is differentiable and is periodic with period T, show that f' is also periodic with period T. Determine whether
   $$F(x) = \int_0^x f(t)dt$$
   is always periodic.