

How to draw the directional field:

Assume we have an equation $y' = f(t, y)$.

1st way

Steps:

1. Draw yt -coordinate system, i.e., the horizontal axis is t and the vertical axis is y .
2. Pick some values for pair (t, y) . For example, $(t, y) = (1, 0), (t, y) = (1, 2), (t, y) = (0, 1), (t, y) = (0, 2)$ and etc.
3. Plug in the pairs you picked to function $f(t, y)$. For, example, if $(t, y) = (1, 0)$ then you compute $f(1, 0)$. The number you get is the slope of a line at the picked point. So, at a point $(1, 0)$ you draw a short segment which has a slope $f(1, 0)$, i.e., a piece of tangent line at a point $(1, 0)$, which has an equation

$$y = 0 + f(1, 0)(t - 1).$$

In general, if the point is $(t, y) = (a, b)$ then you compute $f(a, b)$. Afterwards, you draw at a point (a, b) a short segment with a slope $f(a, b)$, i.e., a piece of tangent line at a point (a, b) , which has an equation

$$y = b + f(a, b)(x - a).$$

2nd way

Steps:

1. Draw yt -coordinate system, i.e., the horizontal axis is t and the vertical axis is y .
2. Pick a constant c . For example, $c = 0$. And draw a curve $f(t, y) = c$, which is called isocline. At every point of this curve the direction has slope c . At several points of the curve draw short segments which have slope c .
3. Repeat the previous step for different constants c . For example, take $c = 1, c = -1, c = 4$ and etc.