Integrating factor method for the linear first order equation

Linear first order equation

$$A(t)\frac{dy}{dt} + B(t)y = Q(t) \tag{1}$$

1. Divide both sides of equation (1) by $A(t)$ to get coefficient 1 in front of $\frac{dy}{dt}$

$$\frac{dy}{dt} + \frac{B(t)}{A(t)}y = \frac{Q(t)}{A(t)} \tag{2}$$

2. Multiply both sides of equation (2) by $\mu(t)$

$$\mu(t)\frac{dy}{dt} + \frac{B(t)}{A(t)}\mu(t)y = \frac{Q(t)}{A(t)}\mu(t) \tag{3}$$

3. The integrating factor

$$\mu(t) = e^{\int \frac{B(t)}{A(t)}dt}.$$

   Compute $\mu(t)$.

4. If you use $\mu(t)$ from the previous item, equation (3) becomes

$$\frac{d(\mu(t)y)}{dt} = \frac{Q(t)}{A(t)}\mu(t).$$

5. The solution is

$$y(t) = \frac{\int \frac{Q(t)}{A(t)}\mu(t)dt + C}{\mu(t)},$$

   where C is a constant which depends on initial condition. Compute $y(t)$.

6. If you want such a solution that $y(a) = b$, then plug a instead of t and b instead of y in the expression for $y(t)$. Find the expression for C in terms of a, b.