

Partial Fraction Decomposition

Assume

$$F(s) = \frac{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0},$$

where $n < m$ are natural numbers, $a_n, a_{n-1}, \dots, a_0, b_m, b_{m-1}, \dots, b_0$ are some real numbers and $a_n \neq 0, b_m \neq 0$.

How to use the partial fraction decomposition

- Factor denominator of $F(s)$, i.e. represent it as a product of terms of the form

$s^k, (s + a)^k$, where k – natural number, a – some real number, or

$(s^2 + bs + c)^k$ with $b^2 - 4c < 0$, where k – natural number, b, c – some real numbers.

- Define terms in fraction decomposition using the following table with $A, A_1, A_2, \dots, A_k, B, B_1, B_2, \dots, B_k$ undetermined coefficients.

Factor in denominator	Terms in fraction decomposition
s	$\frac{A}{s}$
s^k	$\frac{A_1}{s} + \frac{A_2}{s^2} + \dots + \frac{A_k}{s^k}$
$s + a$	$\frac{A}{s+a}$
$(s + a)^k$	$\frac{A_1}{s+a} + \frac{A_2}{(s+a)^2} + \dots + \frac{A_k}{(s+a)^k}$
$s^2 + bs + c$	$\frac{As+B}{s^2+bs+c}$
$(s^2 + bs + c)^k$	$\frac{A_1s+B_1}{s^2+bs+c} + \frac{A_2s+B_2}{(s^2+bs+c)^2} + \dots + \frac{A_k s+B_k}{(s^2+bs+c)^k}$

- Find coefficients $A, A_1, A_2, \dots, A_k, B, B_1, B_2, \dots, B_k$ by setting that the expression should be equal to $F(s)$. You need to multiply both sides by the denominator of $F(s)$. Then either plug in several values for s (plug in some numbers instead of s) and find the above coefficients from the equalities you get. The other way is to set equal the corresponding coefficients for every s^l on the left and right sides of the equality and solve the system of linear equations on $A, A_1, A_2, \dots, A_k, B, B_1, B_2, \dots, B_k$.