the n-th order equation:
$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \dots + a_1 y' + a_0 y = g(t)$$

How to transform nth order equation to a system of n first order linear equations:

1. Introduce n new variables $x_1, x_2, \dots, x_n$ defined by
$$x_1 = y, \quad x_2 = y', \quad \dots, \quad x_n = y^{(n-1)}.$$
2. Then it follows that
$$x_1' = x_2, \quad x_2' = x_3, \quad \dots, \quad x_{n-1}' = x_n$$
are $n-1$ first order linear equations.
3. Rewrite the given equation in the following form,
$$y^{(n)} = -\frac{a_{n-1}}{a_n} y^{(n-1)} - \dots - \frac{a_1}{a_n} y' - \frac{a_0}{a_n} y + \frac{g(t)}{a_n}.$$
Then change the equations to new variables introduced in step 1 to receive
$$x_n' = -\frac{a_{n-1}}{a_n} x_n - \dots - \frac{a_1}{a_n} x_2 - \frac{a_0}{a_n} x_1 + \frac{g(t)}{a_n}.$$
4. Equations from step 2 and 3 form a system of n first order linear equations.

The answer:
$$\begin{cases} x_1' = x_2 \\ x_2' = x_3 \\ \vdots \\ x_{n-1}' = x_n \\ x_n' = -\frac{a_0}{a_n} x_1 - \frac{a_1}{a_n} x_2 - \dots - \frac{a_{n-1}}{a_n} x_n + \frac{g(t)}{a_n} \end{cases}$$

The reverse is also true. Given a system of n 1st order linear equations, it can be rewritten into a single nth order linear equation. We take a look here for the special case when $n = 2$.
Let
$$x_1' = a x_1 + b x_2$$
$$x_2' = c x_1 + d x_2.$$

$\boxed{\text{Assume } b \neq 0}$

be a given system of 1st order linear equations.

How to transform a system of two 1st order linear equations to a 2nd order equation:

1. From the first equation, solve for x_2:
$$x_2 = \frac{x_1'}{b} - \frac{a x_1}{b}.$$
2. Then substitute x_2 we get from step 1 to the second equation.