Solutions and Wronskian

Theorem 1 (Principle of superposition)**.** *If y_1 and y_2 are solutions of the linear homogeneous equation*

$$y'' + p(t)y' + q(t)y = 0,$$

then so is $c_1y_1(t) + c_2y_2(t)$ for arbitrary constants c_1 and c_2.

Theorem 2. *Suppose that y_1 and y_2 are two solutions of*

$$y'' + p(t)y' + q(t)y = 0.$$

Then

$$y(t) = c_1y_1(t) + c_2y_2(t)$$

is the ***general solution*** *of (or the solutions y_1 and y_2 are said to form a* ***fundamental set*** *of solution of)*

$$y'' + p(t)y' + q(t)y = 0$$

if and only if the ***Wronskian*** *of y_1 and y_2, written as $W(y_1, y_2)(t)$, is not a zero function where*

$$W(y_1, y_2)(t) := y_1(t)y_2'(t) - y_1'(t)y_2(t).$$

Theorem 3 (The Existence and Uniqueness Theorem (second order))**.** *Consider the initial value problem*

$$y'' + p(t)y' + q(t)y = g(t), \qquad y(t_0) = y_0, \quad y'(t_0) = y_0',$$

where p, q and g are ***continuous*** *on an* ***open interval*** *I, that contains the point t_0. Also, y_0, y_0' are some constants. Then there is exactly one solution $y = \phi(t)$ of this problem, and the solution exists throughout the interval I.*

Theorem 4 (Abel's Theorem)**.** *If y_1 and y_2 are solutions of*

$$y'' + p(t)y' + q(t)y = 0,$$

where p and q are continuous on an open interval I, then the Wronskian $W(y_1, y_2)(t)$ is given by

$$W(y_1, y_2)(t) = Ce^{-\int p(t)dt},$$

where C is a certain constant that depends on y_1 and y_2 but not on t. Further,

$$W(y_1, y_2)(t)$$

either is zero for all t in I (if $C = 0$) or else is never zero in I (if $C \neq 0$).
