

Even and odd functions

1. An **even** function is any function f such that

$$f(-x) = f(x)$$

for all x in its domain.

2. An **odd** function is any function f such that

$$f(-x) = -f(x)$$

for all x in its domain.

3. $f(x) = 0$ is the only function that is both even and odd.

The Fourier Cosine Series

If f is an even periodic function with period $2L$, then its Fourier series $F(x)$ is a cosine series

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L},$$

where

$$a_0 = \frac{2}{L} \int_0^L f(x) dx$$
$$a_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi x}{L}, \quad \text{for } n = 1, 2, 3, \dots$$

All $b_n = 0$ for $n = 1, 2, \dots$

The Fourier Sine Series

If f is an odd periodic function with period $2L$, then its Fourier series $F(x)$ is a sine series

$$F(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L},$$

where

$$b_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L}, \quad \text{for } n = 1, 2, 3, \dots$$

All $a_n = 0$ for $n = 0, 1, 2, \dots$