Question: A Rocket ship leaves earth at a speed of \((3/5)c\). The rocket was measured to be 100 meters long before its launch.

The rocket has two lights, one on its front end, and one on its rear end.

When a clock on the Rocket indicates that 1 hour had elapsed since the launch, the two lights (front and rear) flash simultaneously (for the person in the rocket), sending the light signals back to Earth.

(1) According to the Earth Clock, how long after the launch were the two Light signals sent?

(2) According to the Earth Clock, how far apart in space were the two lights when they flashed?

(3) According to Earth clock, what time interval apart did the two Light signals from the Rocket arrive back on Earth?

(4) From the answer in (3), how long does the rocket look to the Earth observer?

(5) According to the Rocket Clock, what time interval apart did the two Light signals arrive back on Earth?

(6) According to the Rocket observer, what distance apart were the two light signals received on earth? (This is how long a rocket observer thinks the earth observer measured as the length of the rocket).

If after receiving each of the two light signals, Earth observer reflects them both back to the Rocket Ship,

(7) According to the Rocket Clock, what is the time interval between receiving the two light signals from earth.

(8) According to the Rocket observer, what distance apart will the two light signals be received?
A Rocket ship leaves earth at a speed of \((3/5)c\). The rocket was measured to be 100 meters long before its launch.

\[
\text{Tanh}(\beta) = \frac{v}{c} = \frac{3K}{5K}
\]

Indicates Right angle

\(K\) is a constant to be determined in the next slide.
When a clock on the Rocket indicates that 1 hour had elapsed since the launch, the two lights (front and rear) flash *simultaneously* (for the person in the rocket), sending the light signals back to Earth.

Hyperbolic Pythagorean rule

\[(5K)^2 - (3K)^2 = (c \times 1\text{hr})^2\]

Hence, \(K = 0.27 \times 10^{12}\text{ m}\)

\(5K = 1.35 \times 10^{12}\text{ m} \equiv 1.25\text{ hrs}\)

\(3K = 0.81 \times 10^{12}\text{ m}\)
(1) According to the Earth Clock, how long after the launch were the two Light signals sent?

According to Earth Clock:
Rear flash sent: 5K meters = 1.25hrs after the launch
Front flash sent: 5K + 100 sinh(β) meters = 1.35x10^{12} m + 75 m = 1.25 hrs + 250 ns
This is called Time Dilation. The flying clock runs slower.

\[\cosh^2 \beta = \frac{1}{1-\tanh^2 \beta} = \left(\frac{5}{4}\right)^2\]
\[\sinh^2 \beta = \cosh^2 \beta - 1 = \left(\frac{3}{4}\right)^2\]
(2) According to the Earth Observer, how far apart in space were the two lights when they flashed?

According to Earth Observer (EO):
Rear flash is 3K meters = 0.81 x 10^{12} \text{m} from the Launch point
Front flash is 3K+100\cosh(\beta) \text{ meters} = 0.81 \times 10^{12} \text{m} + 125 \text{ m} from the launch point

*Note however that they are not simultaneous measurements, since the flashes occur at different times for the EO. Also see Slide 11 for Problem 2(a).
(3) According to Earth clock, what time interval apart did the two Light signals from the Rocket arrive back on Earth?
(4) From the answer in (3), how long does the rocket look to the Earth observer?

According to Earth Observer (EO):
Time apart when the flashes were received = 100 \cosh(\beta) + 100 \sinh(\beta) = 200 \text{ m} = 666.66 \text{ nsec}
The Earth Observer would deduce that one light flash arrives 666.66 nsc after the other, so it travelled 200m more. Or that the two flashes of light were 200 m apart.

\[ \cosh^2 \beta = \frac{1}{(1-\tanh^2 \beta)} = \left(\frac{5}{4}\right)^2 \]
\[ \sinh^2 \beta = \cosh^2 \beta - 1 = \left(\frac{3}{4}\right)^2 \]
5) According to the Rocket Clock, what time interval apart did the two Light signals arrive back on Earth?

According to Rocket Clock:
Time when rear flash reached Earth = $8K \cosh(\beta) = 2.5$ hrs
Time when front flash reaches the Earth = $(8K + 100 \cosh(\beta) + 100 \sinh(\beta)) \cosh(\beta)$
$= 2.5$ hrs + $833.33$ nsec

Time Interval between the two events = $666.66$ ns x $\frac{5}{4} = \frac{2500}{3}$ nsec = $833.33$ nsec

$c^2 \beta = \frac{1}{1-\tanh^2 \beta} = \left(\frac{5}{4}\right)^2$

$\sinh^2 \beta = \cosh^2 \beta - 1 = \left(\frac{3}{4}\right)^2$

Indicates Right angle
(6) According to the Rocket observer, what distance apart were the two light signals received on earth? (This is how long a rocket observer thinks the earth observer measured as the length of the rocket).

According to Rocket Man:
Distance between the events when the two Light signals were received by the Earth =
\[(100 \cosh(\beta) + 100 \sinh(\beta)) \sinh(\beta)\]
\[= 200\text{m} \times \frac{3}{4} = 150\text{m}.\]

\[
\cosh \beta = \frac{5}{4} \\
\sinh \beta = \frac{3}{4}
\]
If after receiving each of the two light signals, Earth observer reflects them both back to the Rocket Ship,

\[ (7) \text{ According to the Rocket Clock, what is the time interval between receiving the two light signals from earth.} \]

\[
\cosh \beta = \frac{5}{4} \\
\sinh \beta = \frac{3}{4}
\]

According to Rocket Man:
Time interval between when the two Light flashes bounced back by Earth was received at the Rocket =

\[
200 \text{m} \cosh(\beta) + 200 \text{m} \sinh(\beta) = 200 \text{m} \times 2 = 400 \text{m} = 4000/3 \text{ nsec} = 1333.33 \text{ nsec}
\]
According to the Rocket observer, what distance apart will the two light signals be received?

\[
\begin{align*}
\cosh \beta &= \frac{5}{4} \\
\sinh \beta &= \frac{3}{4}
\end{align*}
\]

This is trivial:

Since the rocket observer stands in one place and receives both the Light signals back, his Rocket coordinate has not changed. So distance between receiving the two signals is zero in rocket coordinates.

Of course, it is not the same in Earth observer’s coordinates. It will be \(400 \text{m} \sinh(\beta) = 300\text{m}\).
EXTRA PROBLEM: goes with (2), so let's call it (2a):
If the two lights on the Rocket keep blinking from before the launch and after, what is the distance between two simultaneous blinks for the Earth observer before launch and after the rocket is travelling at \((3/5)c\) ?

\[
\cosh \beta = \frac{5}{4} \\
\sinh \beta = \frac{3}{4}
\]

According to Earth Observer (EO):
Before launch, the lights blinked 100m apart
After launch, the distance between two simultaneous blinks = \(100m / \cosh(\beta) = 80\) m
This is called Length Contraction.