

Fee-For-Service Contracts in Pharmaceutical Distribution Supply Chains : Design, Analysis, and Management

Hui Zhao¹, Chuanhui Xiong

Krannert School of Management, Purdue University, West Lafayette 47907

Srinagesh Gavirneni

Johnson Graduate School of Management, Cornell University, Ithaca, NY 14853-6201

Adam Fein

Pembroke Consulting Inc., P.O. Box 58757 1515 Market Street, Philadelphia, PA 19102

September 2011

Fee-For-Service (FFS) contracts, first introduced in 2004, dramatically changed the way the pharmaceutical distribution supply chains are designed, managed, and operated. Investment buying (IB), forward buying in anticipation of drug price increases, used to be the way the distributors made most of their profits! FFS contracts limit the amount of inventory distributors can carry at any time (by imposing an inventory cap) and require inventory information sharing from the distributors to the manufacturers while compensating the distributors with a per-unit fee. In spite of its widespread popularity, FFS model has never been rigorously analyzed and its effectiveness carefully tabulated. In this paper, we formulate the multi-period stochastic inventory problems faced by the manufacturer and the distributor under the FFS and IB models, derive their optimal policies and develop procedures to compute the policy parameters. We show that FFS contracts can improve the total supply chain profit - the manufacturer and distributor are now able to share a larger pie. Thus, there exists a range of the per-unit fees that leads to pareto-improvement. Simulation results show that such improvement is about 1.7% on average and as much as 5.5% and the improvement increases as the inventory cap decreases. Determining the pareto-improving per-unit fees is a source of contention in FFS contract negotiation and we propose a simple, yet effective, heuristic for computing them. Further, supply chain transparency facilitated by the FFS contracts can significantly reduce the manufacturer's supply-demand mismatch costs (by about 3.63% on average and as much as 13.01%) and we show that the manufacturer should take advantage of this transparency especially when the inventory cap and drug price increase are high and demand variance is low. We believe that these results have the potential to improve the efficiency of pharmaceutical distribution supply chains, thus reducing the healthcare costs that are such a big burden on the U.S. economy.

1. Motivation And Introduction

American pharmaceutical industry, with cumulative revenues of more than 315 billion dollars in 2007 (US Pharmaceutical Industry report 2008-2009), is the largest in the world. In the same year, US government spent \$7, 285 per person on healthcare and \$878 of that was spent on drugs (OECD health data 2009). The pharmaceutical industry is one of the most profitable businesses in the U.S. and Bureau of Labor Statistics estimates that pharmaceutical and medicine manufacturing

¹Corresponding author, zhaoh@purdue.edu

alone provided 289,800 jobs in 2008. The pharmaceutical industry plays a vital role in the larger U.S. economy and it is imperative that any and all possible efforts are made to ensure smooth and efficient operation of this industry.

Despite the importance of the pharmaceutical industry, the operations management community has thus far conducted very little research that captures the uniqueness of pharmaceutical supply chains. This is a significant cause for concern given that this industry has recently undergone dramatic changes and faces great challenges over the next few decades, forcing itself to take a deeper look at (and resolve) its supply chain issues. According to a pharmaceutical industry consultant, supply chain management is an area that the industry knows the least about. Recently, Schwarz and Zhao (2011) provided an overview of the industry based on their research of the industry data and interviews of industry executives. As Schwarz and Zhao (2011) indicates, some manufacturers understand the potential of supply chain management to reduce inventories and improve profits, but they do not know how to do it. Following up on their observations, we conduct rigorous analysis of two distribution business models that the pharmaceutical industry has used (Investment Buying and Fee-For-Service) and particularly address the industry challenges of how to best take advantage of the FFS model that is widely used.

The pharmaceutical supply chain, overly simplified, is composed of the manufacturers, the distributors, and the providers or chain retailers who sell to the end users of the pharmaceutical products. While the distributors are highly concentrated (the top three distributors take up more than 90% of the U.S. market), the brand-name manufacturing is more diverse in nature (in the United States, the 10 largest pharmaceutical corporations accounted for about 60% of sales in 2004).

Compared to other consumer product supply chains, pharmaceutical supply chains possess the following unique features:

1. In most consumer supply chains, distributors earn their margin by effectively managing both their upstream and downstream partners. In the pharmaceutical industry, the majority of the distributor margin comes from upstream because: (i) large retail pharmaceutical chains, which dispense the majority of pharmaceuticals, have significant buying power; and (ii) third-party payers such as Medicare, Medicaid, and insurance companies impose significant cost pressure on the distributors (Schwarz and Zhao 2010). Pharmaceutical distributors also generally experience small profit margins (e.g., during the 1995-2008 period, the top-3 distributors had an average aggregate profit margin of 1.2%).
2. Branded drug price continues to increase each year, attracting lots of attention (from the public, the media, and the government) for the pharmaceutical industry. Figure 1 shows the

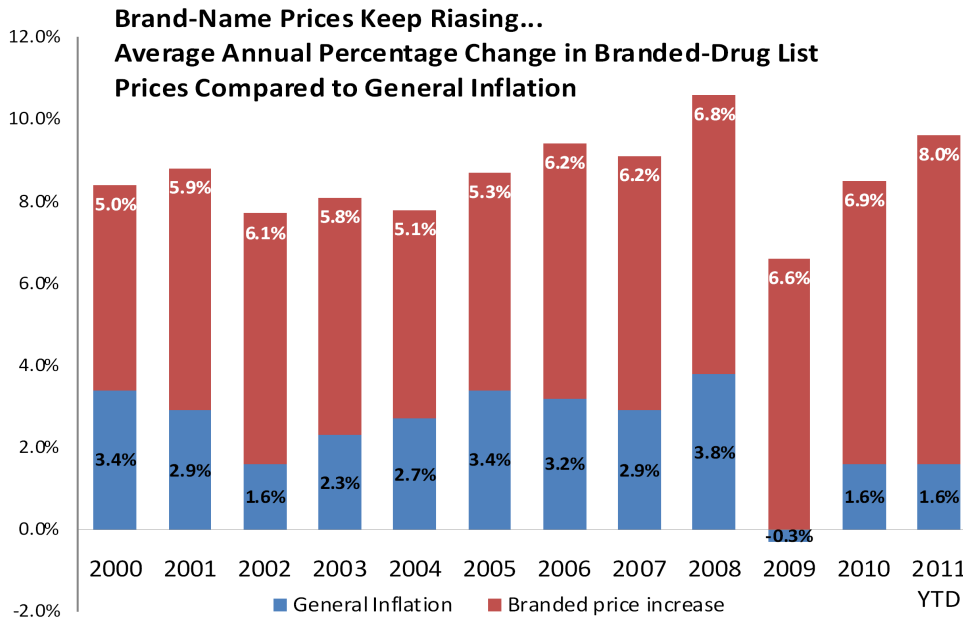
rate of list drug price increases compared to general inflation rates from 2000 to 2011 (year-to-date). For many manufacturers, such price increases are rather predictable. For example, one big brand manufacturer always increases its drug prices on the first business day of each year. Some other manufacturers inform or signal their distributors in advance about price increases of particular drugs in order to, for example, make quarterly sales.

These predictable price increases have profound impact on the business strategies of the manufacturer and the distributor in the industry. In fact, before 2004, drug price increases had been distributors' major revenue source as they buy large stock in speculation of future price increases. This business model is called the Investment Buying (IB) Model. Such practices have led to many disadvantages:

1. High and non-transparent inventory level at the distributors as they try to hide from the manufacturers how much inventory they have in order to buy how much ever they want. This behavior is further exacerbated by the liberal returns policy in this industry.
2. Big fluctuations in the order quantities from the distributors, making production and inventory planning difficult for the manufacturers.
3. Loss of revenue for the manufacturers due to distributors' investment buying.
4. Less stable revenue sources for distributors as they have to rely on speculation instead of services provided to the manufacturers to make profits.

Despite the issues mentioned above, investment buying remained the principal business model for the pharmaceutical distributors until 2004, when an industry-wide switch from IB to Fee-for-Service (FFS) took place. Indeed, this change was fascinating in its scope and speed (all top-3 distributors changed to FFS model with most of their suppliers within one to two years (Fein 2007)). There are many reasons for this switch, including the issues mentioned above, but most importantly, an SEC (Security and Exchange Commission) investigation of a big pharmaceutical manufacturer (Bristol Myers Squibb, BMS) for channel stuffing facilitated by the practice of investment buying.

Although channel stuffing is not, per se, illegal, nor was BMS ever found guilty, BMS reached a settlement with SEC, agreeing to restate its financial reports, pay \$300 Million in fines and payments to investors, and "limit future sales to distributors based on demand or amounts that do not exceed approximately one month of inventory on hand" Fein (2005). Although public attention was focused on BMS, most of the major pharmaceutical manufacturers participated in investment buying, and some were under scrutiny. Following BMS, drug manufacturers were under a lot of



Source: Barclays data from First DataBank on the top 130 branded pharmaceutical products by annual sales.

Figure 1: Branded Drug Price Increases and General Inflation

pressure to reduce the amount of inventory in their distribution channels, leading to the rise of FFS. For more details regarding the switch from IB to FFS, see Schwarz and Zhao (2011).

Under the Fee-For-Service (FFS) model, the distributors draw their revenue mainly from the fees charged to the manufacturer for services provided to them. The services provided include maintaining an inventory level within specified ranges and provide transparency by sharing inventory level and other information (e.g., demand). Specifically, under FFS model, the Inventory Management Agreements (IMA) between the manufacturer and the distributor specifically control the inventory level at the distributors. Distributor's inventory level is required to be high enough to maintain a mutually agreed upon service level to the downstream customers and low enough to control investment buying. It is important to point out that under FFS/IMA, distributors still increase their inventory levels before price increase (a rational action for a decentralized distributor), however, the amount of investment buying is controlled by the maximum level of inventory that the distributor is allowed to carry at any time as specified in the FFS/IMA contracts. Such contracts are easily verifiable due to the aforementioned information sharing. Thus, when appropriately implemented, a FFS/IMA contract reduces investment buying by the distributor, reduces the variability in the demands that the manufacturer faces, and improves transparency throughout the supply chain.

Industry data has shown (Schwarz and Zhao (2011)) that the impact of FFS/IMA on pharmaceutical distributors has been profound as they are forced to be more efficient in achieving the same service level with significantly lower inventory. However, FFS/IMA and particularly information-sharing enabled by FFS/IMA have had little impact on pharmaceutical manufacturers' own inventory-management practices. According to the interviews conducted by Schwarz and Zhao (2011), part of the reason is the lack of decision support mechanism on how to use the information now provided to the manufacturers under FFS. Indeed, although the research on the value of information and how to incorporate information into manufacturers' decision making is extensive (e.g., Gavirneni, et al. 1997 and Chen 2003), little is done to tackle the unique challenges in the pharmaceutical industry. At the same time, the pharmaceutical industry is facing another difficult question - the design of the FFS/IMA contract. Since its inception in 2004, distributors and manufacturers have been trying their own ways to set contract parameters because there have been no studies providing them any guidance in the important contract design problem. Furthermore, the question remains as to who actually benefits from FFS contracts. Is it (i) the distributors because of the fees they receive from the manufacturers, (ii) the manufacturers because of the reduced investment buying by the distributors, or (iii) the consumers because of reduced costs in the supply chain. The answer to this question is not so obvious. Our research shows that the supply chain is indeed better off from this switch. Further, if the fees are determined appropriately, both the manufacturer and the distributor can be better off from switching from an IB model to an FFS model. Thus, it is not a stretch of imagination to say that the pharmaceutical industry, by their implementation of the FFS contract, has found a true win-win situation.

Specifically speaking, the purpose of this research is to (i) characterize the manufacturer's and the distributor's optimal decisions and provide industry-feasible solutions as to how they should operate under the FFS model; (ii) investigate how each FFS parameter affects each player's and supply chain's profit; (iii) by comparing each player's performance under FFS and IB models, provide guidance regarding FFS contract design, and in particular how to set the FFS contract parameters such that each player can be better off under FFS than under IB model; and (iv) capture the value of information in this industry setting. An extensive simulation study verified the analytical results and provides additional managerial insights useful for this industry.

Before we detail the supply chain setup, the analytical results, and the computational results, we would like to briefly describe our main findings. We formulated the stochastic inventory control problems faced by the manufacturer and the distributor, established that the produce-up-to and order-up-to policies are optimal for them, respectively, and developed procedures to compute the FFS parameters. A comprehensive simulation study demonstrated that FFS contracts improve the total supply chain profit by about 1.7% on average and by as much as 5.5%. Unfortunately,

these profit gains are not universal in the sense that the manufacturer profit increases while the distributor profit decreases. Therefore, it is necessary for the manufacturer to share some of the gains with the distributor and the FFS fees are an ideal mechanism for this purpose. We are able to compute the fee range that results in pareto-improvement for both the players. We further develop a simple heuristic for computing these fee ranges and demonstrate its effectiveness.

The rest of the paper is organized as follows. In §2, we briefly review the literature. In §3, we formally introduce the supply chain model set up. In §4 and §5, we characterize the manufacturer's and distributor's optimal policy under FFS and IB models, respectively. In §6, we investigate the FFS contract design problem through a detailed simulation study. We also derive an easy-to-implement heuristic and demonstrate the effectiveness of this heuristic using a numerical study. In §7, we further establish the robustness of the results and provide further insights through additional numerical analysis. In §8, we conclude the paper with a summary of the managerial insights.

2. Literature Review

Relatively little has been published in the supply chain and operations management literature that is directly on pharmaceutical supply chains. Burns et al. (2002) is the most frequently cited general reference. Schwarz (2010) describes the flow of products, dollars, and information in the supply chains for medical and surgical supplies, pharmaceuticals, and orthotic devices. Most other available resources are website postings and reviews from industry experts, consulting companies, and industry reports. For example, Yost (2005) and Fein (2005, 2007) discuss the change of the business model from Investment Buying to FFS (Fee-For-Service) and the impact of this change on the players in the pharmaceutical supply chain. These readings provide some background information of this industry.

Recently, Schwarz and Zhao (2011) conducted some empirical research focused on the pharmaceutical industry. They provided an overview of the industry, its current status, and the challenges it faces, from the operations perspective. Based on their analysis of the industry data and interviews of industry executives, they also examined the impact of FFS/IMA contracts on the pharmaceutical distributors and manufacturers. They concluded with a discussion of many current issues that are worth the attention of the supply chain/operations management researchers.

Very limited research addresses the pharmaceutical supply chain using analytical models. Recently, Martino et al. (2010) examines the different distribution models for the pharmaceutical supply chain using a mathematical model. Specifically, they compare IB, FFS, and direct-to-pharmacy models for the manufacturer by considering cases where the aggregated demand is deterministic and determine the profit maximizing production-inventory strategy in a multi-period setting for the

manufacturer and distributor under each contractual agreement. They do not consider fee designs or value of information (in fact, the value of information cannot be studied in this case due to the demand being deterministic).

To our knowledge, our paper is the first to analytically study the supply chain, from a multi-stage stochastic inventory perspective, under the IB and FFS business models. By solving the optimal inventory and production policy under each model, we show how players change their behavior under different business models and how to best design FFS/IMA contracts to ensure pareto-improvement.

In addition to the limited research particularly targeted at pharmaceutical industry as discussed above, the research presented in this paper is related to a few areas in the operations area, including production and inventory control, production planning with information sharing, and supply chain management. In the following, we briefly review related literature in these areas.

First, our paper is related to the literature that studies production and inventory decisions in serial supply chains. Graves (1999) reviews production planning and inventory control models for centralized systems with predictable demand. When it comes to multi-stage supply chains with stochastic demands, Anupindi and Akella (1993) and Glasserman and Tayur (1995) were the early papers. Since then, there has been a plethora of research on these systems and most of it has been captured in books compiled by Tayur et al. (1999) and de Kok and Graves (2003).

In modeling the FFS model, one important aspect is to incorporate the distributor's on-hand inventory information into the pharmaceutical manufacturers' production and inventory decisions and analyze the value of such information. A great deal of literature is available (see Chen (2003)) to demonstrate the value of downstream information to upstream suppliers. In particular, this literature shows that with more information about the downstream inventory or demand, manufacturers can improve production planning, better matching their stock levels with the demand, and hence reduce total costs. Gavirneni et al. (1999) is the work that is most closely related to ours. It compares a traditional model (in which downstream orders are the only information available to a manufacturer) with two models, one in which the manufacturer is informed of the downstream replenishment policy and its parameters and another in which the manufacturer is also informed of the downstream on-hand inventory information. Their numerical study shows significant savings (as large as 35%) for the manufacturer due to the modification of his production and inventory policy. Lee et al. (2000), Cachon and Fisher (2000), Aviv and Federgruen (1998), and Simchi-Levi and Zhao (2004) report similar results. For a comprehensive literature review on value of information sharing, see Chen (2003).

None of the studies described above, however, incorporates the unique characteristics of the pharmaceutical supply chain. Price increase, which creates the non-stationary ordering process of

the distributor to the manufacturer, is one such characteristic of brand-name drugs. As Lee et al. (1997) show, price uncertainty is one cause of the bullwhip effect. Although the FFS/IMA business model limits investment buying, distributors still take advantage of price increases within the boundaries of IMA contracts. Therefore, incorporating the impact of price increases is an important aspect when developing models for pharmaceutical decision making. Gavirneni and Morton (1999) study a retailer’s inventory speculation behavior in case of price increase. In contrast, we study a three-level supply chain and show how price increase will change the behavior of the players at various levels in the supply chain.

There has been extensive literature on supply chain contracts in decentralized systems, e.g., wholesale price contracts, revenue sharing contracts, etc. and variations of these contracts. See Cachon (2003) for more detailed review. Most of these contracts are studied under the single-period setting and do not consider the price increase effect which is the key for the pharmaceutical industry. The FFS contract is the pharmaceutical industry’s particular way of limiting investment buying and compensating the distributors. It is essentially a profit/revenue sharing contract between the manufacturer and the distributors.

Investment buying and price fluctuations have been discussed in the operations literature mainly as a cause of demand information distortion, i.e., bullwhip effect (Lee, et al. 1997). Much research discusses various ways to mitigate the bullwhip effect. These include demand information sharing (e.g., Gavirneni, et al. 1999), vendor managed inventory (VMI) (e.g., Aviv and Federgruen 1998), and collaborative planning, forecasting, and replenishment (CPFR) (e.g., Waller, et al. 1999, Aviv 2001). In the pharmaceutical industry, price increase causes investment buying which brings many disadvantages. Directly limiting investment buying using contracts through inventory caps and information sharing are used to reduce the negative impact of investment buying, which are also the main targets of analysis in this paper.

3. Supply Chain Setup

Consider a three-level pharmaceutical supply chain, where the brand-name manufacturer (she) supplies the distributor (he) who sells to a retailer (representing a provider or a chain retailer) over an N -period finite horizon. The downstream retailer faces stochastic demand following *i.i.d.* distribution for each period k with a cdf $\Phi()$ and pdf $\phi()$, which we assume to be known to the manufacturer and the distributor. The manufacturer produces to stock with lead time of one period for regular production. She fills the demand from her stock and any unmet demand is satisfied immediately using overtime or any other possible means reflected in a penalty cost, p . Although this assumption is mainly for simplification of the analytical model, it is not far from

reality since “typically the manufacturer tries all possible means (using overtime or dispatching stocks from different inventory depots) to avoid stockouts”, according to a manager in one of the top-3 distributors. The distributor fills demand from his stock and any unsatisfied demand at the distributor is backordered with a shortage cost, p^d . We assume no lead time for distributor’s orders. The distributor’s lead times may be added which will complicate the model with little additional insights. In each period, the manufacturer determines how much to produce (for the next period) and the distributor determines how much to order.

We consider and compare the FFS model with the IB model over a finite horizon, corresponding to an average length of time within which there is one price increase. For example, if the price of a branded drug is increased every year, the finite horizon is set to be one year, with the price increase occurring in the middle of the horizon. Formally, we assume there is a price increase, δ , in the middle of the horizon at the beginning of period n ($n \sim N/2$). Notice that, in reality, there may be more than one price increase during the drug’s patented years. However, as we will show, after one price increase, each individual party as well as the supply chain settle down to a new steady state. As long as the two price increases are far enough so that their impacts do not interact (which is the case in reality), we can decompose the different price increases and focus, as we do here, on the impact of one price increase on the performance of individual parties and the supply chain. We assume all unsold inventory can be returned to the manufacturer with full refund at the end of the finite horizon (this is consistent with the very liberal returns policy in this industry) and hence we will see stationary policy over the finite horizon. We also assume the manufacturer’s and distributor’s unit holding costs and unit shortage penalty costs do not change after the price increase.

To summarize, in every period, the sequence of events is as follows: (a) the manufacturer receives the units produced in the previous period; (b) the distributor places an order if needed for that period; (c) if the manufacturer does not have enough inventory to fulfill the demand, she gets the product immediately from overtime (or expediting from other depots) at a higher cost; (d) the manufacturer ships the product to the distributor; (e) the manufacturer decides how much to produce (which will be available for next period); (f) distributor’s demand realizes and he satisfies the demand as much as possible from his inventory. Unsatisfied demand at the distributor is backlogged; (g) inventory-related costs (holding costs for the manufacturer and distributor, backorder costs for the distributor and shortage penalty cost for the manufacturer) are tabulated.

Model FFS: Under the FFS model, the manufacturer imposes an upper limit on how much inventory a distributor can hold (inventory position), \bar{y} , referred to as the distributor’s inventory cap. Such an inventory cap effectively controls the amount of investment buying. As a result,

the distributor charges a fee (u) from the manufacturer for each unit the manufacturer distributes through him, as the “compensation” for his loss of revenue for investment buying, or in another term, for his service. The distributor also shares with the manufacturer how much on-hand inventory he has at the beginning of each period. Because of this information, the manufacturer is able to calculate the total customer demand seen by the distributor since he last placed an order to the manufacturer (i.e., the investment buying period) and uses that to more accurately estimate the distributor’s future orders after the price increase. As we will show, in the periods following the price increase, the manufacturer will have a state-dependent base-stock policy with the state defined as the total customer demand seen by the distributor since he last placed an order to the manufacturer (i.e., the investment buying period).

Model IB: Under the IB model, there is no limit of how much the distributor can order and the manufacturer does not have information about distributor’s on-hand inventory. Hence, the manufacturer can only estimate the distributor’s demand by the number of periods since the investment buying period. As we will show, under IB, in the periods following the price increase, the manufacturer will again have a state-dependent policy, with the state defined as the number of periods since the manufacturer’s last demand. It is easy to see that IB model is a special case of FFS model with $\bar{y} = \infty$, $u = 0$, and no on-hand inventory information sharing.

Before we move on to detailed analysis, we first summarize notations used, some mentioned above, others not.

h^d : distributor’s unit holding cost before price increase,

p^d : distributor’s unit penalty cost for backorders before price increase, $p^d > h^d$,

r : distributor’s unit selling price before price increase,

D_k : Stochastic demand seen by the distributor in period k , following *i.i.d.* c.d.f. of $\Phi()$ and p.d.f. of $\phi()$, $1 \leq k \leq N$.

\bar{D} : Mean demand seen by the distributor in each period.

Q_k : Order quantity from the distributor to the manufacturer in each period k , $1 \leq k \leq N$,

c^d : Distributor’s buying price from the manufacturer before price increase,

x_k^d : distributor’s on-hand inventory level at the beginning of period k ,

y_k^d : distributor’s total amount of inventory available to satisfy demand in period k , $x_{k+1}^d = y_k^d - D_k$,

x_k : manufacturer’s on-hand inventory level at the beginning of period k ,

y_k : manufacturer’s total amount of inventory available to satisfy demand in period k , $x_{k+1} = (y_k - Q_k)^+$,

h : manufacturer’s unit holding cost,

p : manufacturer's unit penalty cost if there is shortage, $p > h$,
 c : manufacturer's unit production cost.

4. Fee-for-Service Model

In this section, we analyze each player's optimal policy under FFS. As we mentioned before, under FFS model, there is an inventory cap (\bar{y}) as to how much inventory the distributor may hold in any period and distributor shares his on-hand information with the manufacturer. The distributor charges a fee, u , for each unit that the manufacturer sends to him.

4.1 Distributor's Optimal Policy

Due to backorders, distributor's expected revenue/sales is not affected by his inventory decisions. Hence, his optimal ordering policy is obtained by minimizing his expected cost. Let $V_k^d(x)$ represent distributor's expected optimal cost from period k to the end of the horizon, given he starts this period with an on-hand inventory of x . We have $V_k^d(x) = \min_{x \leq y_k^d \leq \bar{y}} \{G_k^d(y_k^d) - (c_k^d - u)x\}$, where

$$\begin{aligned} G_k^d(y_k^d) &= L_k^d(y_k^d) + (c_k^d - u)y_k^d + \int_0^\infty V_{k+1}^d(y_k^d - D)\phi(D)dD \\ L_k^d(y) &= h_k^d \int_0^y (y - D)\phi(D)dD + p_k^d \int_y^\infty (D - y)\phi(D)dD. \end{aligned}$$

Note that before the price increase, i.e., $1 \leq k \leq n - 1$, $c_k^d = c^d$ and after price increase, i.e., $k \geq n$, $c_k^d = c^d + \delta$. As mentioned, we assume the distributor's terminal cost function as:

$$V_{N+1}^d(x_{N+1}^d) = -(c^d + \delta - u)x_{N+1}^d,$$

which means at the end of last period, the distributor can return all leftover units with full refund. Moreover, in FFS/IMA contracts, there is an end-customer fill-rate requirement from the manufacturer to the distributor, γ , corresponding to a minimum stock level, $\underline{y} = \Phi^{-1}(\gamma)$ at the distributor. Due to the high penalty cost for not filling a demand immediately, we assume that all newsvendor quantities satisfy the minimum stock level, \underline{y} . Following the standard backward induction, we obtain the distributor's optimal policy, summarized in the following theorem.

Theorem 1 Define $y^* = \Phi^{-1}(\frac{p^d}{p^d + h^d})$. The distributor's optimal policy is a non-stationary base-stock policy with the base-stock level equal to $\min(\bar{y}, y_k^{d*})$, where $y_k^{d*} = y^*$ for any period other than $n - 1$ and $y_k^{d*} = y_{n-1}^{d*}$ for period $n - 1$ with $y_{n-1}^{d*} \geq y^*$.

Theorem 1 shows that the distributor maintains his newsvendor base-stock level in all the periods except in the period right before price increase (period $n - 1$) in which he will do investment buying as much as allowed by the inventory cap, i.e., $y_{n-1}^{d*} \geq y^*$, $\forall k \neq n - 1$.

4.2 Manufacturer's Optimal Policy

Because all distributor orders are satisfied from the manufacturer (either from her inventory or from overtime production), the manufacturer's sales is equal to distributor demand which is not related to her production and inventory decision. Hence, the manufacturer's production decision could also be solved to minimize her relevant expected cost. For tractability, we assume the manufacturer's terminal cost to be $V_{N+1}(x) = -cx$.

To capture the manufacturer's expected cost, we need to characterize her demand process. Since the distributor has a stationary base-stock policy for periods $1 \leq k \leq n - 2$, $Q_k = D_{k-1}$, the manufacturer will see *i.i.d.* demand from the distributor with a pdf of $\phi()$ before period $n - 1$. Because of investment buying in period $n - 1$ before price increase, there may be one or more period(s) starting from period n (price increase period) during which the manufacturer does not see demand. We refer to these periods as transitional periods, $k = n, n + 1, \dots$. After the distributor resumes ordering, he will have stationary base-stock levels again (Theorem 1), hence, the manufacturer will again see *i.i.d.* demand following a pdf of $\phi()$. However, the length of the transitional periods is uncertain and the first positive demand seen by the manufacturer once the distributor resumes ordering also follows a different distribution depending on the number of transitional periods. Since the distributor shares on-hand inventory with the manufacturer, she can use this information to help estimate the distributor's demand process for $k \geq n$.

Specifically, define the *state* of the manufacturer as follows: (1) Before the price increase ($k \leq n - 1$), the manufacturer is in state 0^- ; after the distributor resumes his ordering, she is in state 0^+ . (2) During the transitional periods, the manufacturer's state, $i = 0, 1, \dots, \bar{y} - y^* - 1$, represents the total downstream demand realized at the distributor since his investment buying in period $n - 1$. Now define p_i , $0 \leq i \leq \bar{y} - y^* - 1$ (i.e., p_i is defined only for the transitional periods), as the probability that manufacturer sees a positive demand in state i , hence, $p_i = Prob\{i + D > \bar{y} - y^* | i < \bar{y} - y^*\}$, where D is the random demand seen by the distributor in one period. If the manufacturer sees a demand from the distributor in state i , the demand, η_i , equal to $i + D - (\bar{y} - y^*) = D - (\bar{y} - y^* - i)$ in this case, follows a distribution with a cdf of $\Theta_i(\eta_i)$. It is easy to verify that $p_i \leq p_{i+1}$ and $\Theta_i(\cdot) \leq_{st} \Theta_{i+1}(\cdot)$, for $i < \bar{y} - y^* - 1$, i.e., the chance and quantity of distributor demand realization increase as time goes along after the price increase.

Let $V_k^i(x)$ be the manufacturer's minimum expected cost² from period k to the end of the horizon when she is in state i at period k with a starting inventory of x . We have $V_k^i(x) = \min_{y \geq x} \{G_k^i(y) - cx\}$, where

² $V_k^i(x)$ does not include the manufacturer's fees to the distributor since they are only affected by the downstream demands, not by her production decisions.

$$G_k^i(y) = \begin{cases} cy + L(y) + \int_0^\infty V_{k+1}^i(y-D)^+ \phi(D) dD & \text{if } i = 0^- \text{ or } 0^+ \text{ and } k \neq n-1 \\ cy + L_{n-1}(y) + \int_0^\infty V_n^0(y - (D + \bar{y} - y^*))^+ \phi(D) dD & \text{if } k = n-1 \\ cy + p_i \left[L^i(y) + \int_0^\infty V_{k+1}^{0+}(y - \eta_i)^+ \theta_i(\eta_i) d\eta_i \right] \\ \quad + (1 - p_i) \left[hy^+ + \int_0^\infty V_{k+1}^{i+D}(y) \phi(D) dD \right] & \text{otherwise,} \end{cases}$$

where

$$\begin{aligned} L(y) &= h \int_0^y (y-D) \phi(D) dD + p \int_y^\infty (D-y) \phi(D) dD, \\ L_{n-1}(y) &= h \int_0^{(y-\bar{y}+y^*)^+} (y - (D + \bar{y} - y^*)) \phi(D) dD + p \int_{(y-\bar{y}+y^*)^+}^\infty (D + \bar{y} - y^* - y) \phi(D) dD, \\ L^i(y) &= h \int_0^y (y - \eta_i) \theta_i(\eta_i) d\eta_i + p \int_y^\infty (\eta_i - y) \theta_i(\eta_i) d\eta_i. \end{aligned}$$

The following theorem shows the optimal production and stocking policy for the manufacturer.

Theorem 2 *Under the FFS model,*

- *A produce-up-to policy is optimal for the manufacturer. The policy is characterized by a series of produce-up-to levels, y_k^i , for period k in state i . Particularly, before the price increase, the produce-up-to levels are y_k^{0-} , $k < n$. After price increase ($k \geq n$), the produce-up-to levels are uniquely defined by their state, referred to as y^i , $i = 0, 1, 2, \dots, \bar{y} - y^* - 1, 0^+$.*
- *The optimal produce-up-to levels in the transitional periods have the following relationships: $y^0 \leq y^1 \leq y^2 \dots \leq y^{\bar{y}-y^*-1} \leq y^{0^+} = \Phi^{-1}\left(\frac{p}{p+h}\right)$.*

The above theorem shows that because of price increase, the manufacturer's optimal stock level drops to a low level in the period of price increase (n), increasing during the transitional periods until the distributor resumes ordering, from which time it remains at the newsvendor quantity through the end of the horizon. The following corollary shows that as \bar{y} increases, the manufacturer's optimal stock levels will decrease in the transitional periods and remain at the newsvendor quantity after the distributor resumes ordering.

Corollary 1 *Given $\bar{y}_1 \leq \bar{y}_2$, $y^i(\bar{y}_1) \geq y^i(\bar{y}_2)$ for $i \geq 0$, and $y^{0^+}(\bar{y}_1) = y^{0^+}(\bar{y}_2)$.*

5. Investment Buying Model

As mentioned above, the IB model is a special case of the FFS model with (1) $\bar{y} = \infty$, (2) $u = 0$, and (3) no sharing of distributor's on-hand inventory to the manufacturer. Therefore, the distributor's policy is the same as that under FFS with $\bar{y} = \infty$ and $u = 0$.

As for the manufacturer's policy, since the distributor does not share his on-hand inventory information, during the transitional periods, she can only estimate distributor's demand based on how many periods have elapsed since the distributor's last order (investment buying in period $n - 1$). Hence, while the manufacturer is still in state 0^- before the price increase and in state 0^+ after the distributor resumes ordering after the price increase (as defined in FFS model), during the transitional periods, state i is re-defined as the number of periods that have elapsed since the distributor's investment buying in period $n - 1$, $0 < i \leq N - n + 1$, $i \in I^+$ (positive integers).

During the transitional periods, still define p_i , $0 < i \leq N - n + 1$, as the probability of manufacturer seeing the distributor resuming ordering in state i .

$p_i = Prob\left(\sum_{k=n-1}^{i+n-1} D_k > y_{n-1}^{d^*} - y^* \mid \sum_{k=n-1}^{i+n-2} D_k < y_{n-1}^{d^*} - y^*\right)$. We also define the distributor's demand to the manufacturer (η_i), if realized in state i , follows a distribution with a cdf of $\Psi_i(\eta_i)$. We assume that $p_i \leq p_{i+1}$ and $\Psi_i(\cdot) \leq_{st} \Psi_{i+1}(\cdot)$, i.e., the chance and quantity of distributor demand realization increase as time goes along after the price increase. It is easy to show that both assumptions are true if the end-item retailer demand has an increasing failure rate (IFR) distribution (Normal, uniform, or Erlang distributions).

Let $V_k^i(x)$ be the manufacturer's minimum expected cost from period k to the end of the horizon when she is in state i at period k with inventory x . We have $V_k^i(x) = \min_{y \geq x} \{G_k^i(y) - cx\}$, where

$$G_k^i(y) = \begin{cases} cy + L(y) + \int_0^\infty V_{k+1}^i(y - D)^+ \phi(D) dD & \text{if } i = 0^- \text{ or } 0^+ \text{ and } k \neq n - 1 \\ cy + \tilde{L}_{n-1}(y) + \int_0^\infty V_n^1(y - (D + y_{n-1}^{d^*} - y^*))^+ \phi(D) dD & \text{if } k = n - 1 \\ cy + p_i[\tilde{L}^i(y) + \int_0^\infty [V_{k+1}^{0^+}(y - \eta_i)^+] \psi_i(\eta_i) d\eta_i \\ \quad + (1 - p_i)[hy^+ + \int_0^\infty V_{k+1}^{i+1}(y) \phi(D) dD] & \text{otherwise,} \end{cases}$$

where

$$\begin{aligned} \tilde{L}_{n-1}(y) &= h \int_0^{(y - y_{n-1}^{d^*} + y^*)^+} (y - (D + y_{n-1}^{d^*} - y^*)) \phi(D) dD + p \int_{(y - y_{n-1}^{d^*} + y^*)^+}^\infty (D + y_{n-1}^{d^*} - y^* - y) \phi(D) dD, \\ \tilde{L}^i(y) &= h \int_0^y (y - \eta_i) \psi_i(\eta_i) d\eta_i + p \int_y^\infty (\eta_i - y) \psi_i(\eta_i) d\eta_i. \end{aligned}$$

With minor adjustment, the proofs for FFS model apply to IB model as well. Hence we briefly summarize the results: (1) The produce-up-to policy is the optimal policy for the manufacturer. (2) The relationship of the optimal stocking levels in the transitional periods (Theorem 2) still holds. Further, the following theorem shows that as the amount of price increase increases, the manufacturer's optimal stock levels in the transitional periods decrease.

Theorem 3 *If price increase amount $\delta_1 \leq \delta_2$, then $y^i(\delta_1) \geq y^i(\delta_2)$ for $i \geq 0$, and $y^{0^+}(\delta_1) = y^{0^+}(\delta_2)$.*

6. FFS Contract Parameters - Numerical Study

In this section, we conduct an extensive numerical study to determine how the FFS contract parameters affect supply chain performance. Further, we provide managerial guidelines on how these parameters should be set in order to achieve pareto-improvement. Specifically speaking, the objective of the computational study is to answer the following questions:

1. How does the inventory cap, \bar{y} , affect the manufacturer's, the distributor's, and the supply chain's profit? As \bar{y} decreases, we know that the distributor's profit decreases due to less investment buying. However, we do not know how the manufacturer's profit will change, because on the one hand, she enjoys less profit loss from investment buying, on the other hand, she will have less benefits from information sharing (the information is more useful to her under more investment buying). A priori, we also do not know what will happen to the total supply chain profit. We will use the computational study to better understand this behavior.
2. The per-unit fee (u) does not affect the supply chain profit; it only serves as a means to divide the supply chain profit between the manufacturer and the distributor. What is the range of fees that will make the manufacturer and the distributor better off by shifting from IB to FFS? How are the fees affected by the various supply chain parameters?
3. Given that the calculation of the fees depend on solution to multi-period stochastic models, are there any heuristics that managers can use to determine the pareto-improving fee values? How effective are these heuristics?
4. What is the value of distributor's on-hand inventory information to the manufacturer? How do the various supply chain parameters affect this value?

To achieve our objective of quantifying and studying the value of information, we introduce an "intermediate" model between FFS and IB. Referred to as FFS-NI, this model is the same as the FFS model except that the manufacturer does not use the on-hand information in her decision making. Under this model, the distributor makes the same decisions as under FFS, whereas the manufacturer makes her decisions without using the on-hand inventory information from the distributor. According to Schwarz and Zhao (2011), pharmaceutical manufacturers have largely foregone the benefits of information shared with them by the distributor. Hence, studying the FFS-NI model and comparing to the FFS model will enable us to characterize the value of information that the manufacturer is foregoing in this process.

To answer these questions, we designed a comprehensive test bed of numerical experiments with the following parameter values. In the pharmaceutical industry, all prices (between manufacturer

and the distributor and even also the selling prices to the retailers) are set in terms of a reference price, called Wholesale Acquisition Cost (WAC), published to all players. It is interesting that WAC is never the true price between any two players, but nevertheless all the prices (For example, c^d, r, u) are set as a percentage of WAC.

- Almost all distributors obtain 2% off WAC as prompt pay discount, which makes $c^d = 98\%WAC$.
- On average, distributors sell to the retailers at 2.7% off WAC, i.e., $r = 97.3\%WAC$. Note that the distributors are selling at a price lower than their buying price. However, the distributors obtain positive margin because of the service fees.
- Manufacturer's production cost is typically very low, about 10% of WAC, i.e., $c = 10\%WAC$.
- Holding cost at the distributor is more significant than at the manufacturer because of distributor's low margin and higher requirement for cash flow. Typically, annual holding cost at the distributor and at the manufacturer is 20% of WAC and 10% of c , i.e., $h^d = 20/24\%WAC$ and $h = 10/24\%c$ (we look at a one-year horizon with 24 periods and h^d and h are holding costs per period).
- Penalty costs are set to assure a 98% service level at both the distributor and the manufacturer using news-vendor quantity, i.e., $p^d = 49h^d$ and $p = 49h$.

To summarize, we use the following parameters as discussed above: $c^d = 98\%WAC$, $r = 97.3\%WAC$, $c = 10\%WAC$, $h^d = 20/24\%WAC$, $h = 10/24\%c$, $p^d = 49h^d$, and $p = 49h$. In addition to the above parameter setup that is quite typical (and validated by our industry contacts) in the industry, we investigate the impact of a few other very important parameters that vary a lot in the industry and study their impact on the supply chain profit, the fees that should be used, and the value of information. These parameters are: WAC^3 , δ (the annual price increase percentage), the demand variance, and the inventory cap. Specifically, we set 5 levels of $WAC = 200, 650, 1100, 1550, 2000$ and 5 levels of $\delta = 5\%, 7.5\%, 10\%, 12.5\%, 15\%$. As for demand, we choose Erlang distribution, which may resemble normal distribution but does not have the concerns of negative demand and hence has more flexibility of the variance change. We choose 4 levels of variances by having Erlang with $(\mu, k) = (30, 1), (15, 2), (10, 3), (7.5, 4)$. These four sets of Erlang share the same average demand with decreasing variance level. For each of the above 100 sets of parameters, we also solve the problem under different levels of inventory caps. Let \bar{D} be the expected value of the i.i.d random demand for one period. The inventory cap \bar{y} is chosen at its

³This value is included to answer the current industry question regarding whether the same fees should be charged for cheaper drugs compared to the much-higher-valued bio-pharmaceuticals.

newsvendor amount (y^*) plus additional periods of average demand, i.e., \bar{y} is chosen at y^* , $y^* + \bar{D}$, $y^* + 2\bar{D}, \dots$, and increase by an incremental of \bar{D} until it hits y_{n-1}^{d*} , the distributor's stock level with full investment buying under IB. Beyond y_{n-1}^{d*} , \bar{y} does not limit investment buying any more. This usually includes at least 8 levels of inventory cap (depending on the price increase level). These settings are evaluated using a 24-period timeline designed to capture a one-year horizon with each period equivalent to half a month.

For each of 1337 cases described above, we use simulation with IPA (see Glasserman and Tayur (1995) for details) to solve for, under IB, FFS, and FFS-NI, the optimal production levels at the manufacturer, optimal stocking levels at the distributor, profits for each player and the supply chain, pareto-improving fee ranges, as well as the value of information measured by the percentage savings in the manufacturer's supply-demand mismatch (inventory holding and penalty) cost. In the following, we describe our findings on the supply chain and each player's profit, the pareto-improving fees, and the value of information. For each of these, we explore how their behavior is impacted by the inventory caps, price increase, and demand variance.

6.1 Impact of FFS on Supply Chain Performance

Figure 2 shows how the manufacturer's, the distributor's, and the supply chain's average profit per period change with the inventory cap for a system with $WAC = 200$, $\delta = 10\%$, demand following Erlang (15.0, 2) distribution, and fee $u = 2\%WAC$. Inventory caps are plotted in additional periods of demand. For example, an inventory cap of 0 and 2 in the figure corresponds to $\bar{y} = y^*$ and $\bar{y} = y^* + 2\bar{D}$, respectively. Observe that as \bar{y} decreases, the distributor's profit decreases and the manufacturer's profit increases. This is along the lines of the arguments we presented earlier. It is pleasant to see that as \bar{y} decreases, supply chain profit increases. What this means is that a transition from IB to FFS will increase the supply chain efficiency, resulting in increased overall profit. Hence, a transition from IB to FFS is beneficial to the industry as a whole and the players are able to share a bigger pie. A summary analysis over all 1337 simulation cases reveals that the increase in system profit is 1.7% on average and is as large as 5.5%. For this specific example, a transition from IB to FFS has the potential to increase the system profit from \$5,982 to \$6,131 per period. This is a result of the manufacturer's profit increasing from \$3076 to \$3369 per period while the distributor's profit decreases from \$2906 to \$2762. Thus, if the manufacturer were willing to transfer about \$144 per period to distributor (in addition to the 2% fee she currently pays), then the distributor should be ambivalent to the transition and the manufacturer will be able to keep the additional \$149 resulting from this transition. Ideally some of these additional profits will be shared with the end-consumer as well (in the form of lower prices) and thus, as a result of the transition from IB to FFS, all the parties involved would be better off.

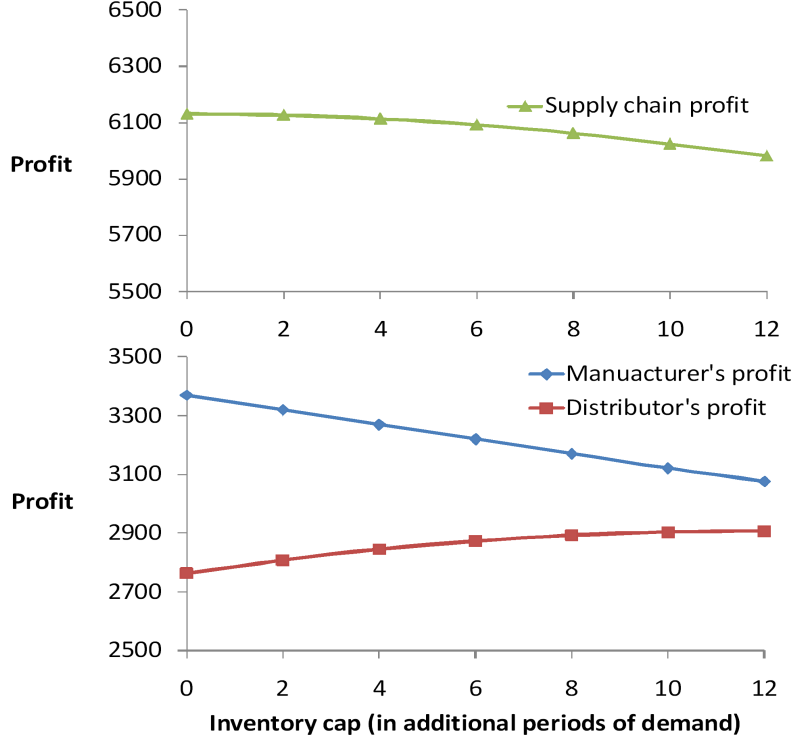


Figure 2: Individual player's and supply chain's profit

The per-unit fee-for-service, u , used in the FFS contracts serves the aforementioned role of transferring profit from the manufacturer to the distributor. Since the benefit realized by the manufacturer is greater than the profit loss seen by the distributor, we can define upper and lower limits for the fees for pareto-improvement as follows.

Define \bar{u} as the value of the fee under FFS at which the manufacturer's profit is the same as that under the IB model and \underline{u} as the value of the fee at which the distributor's profit is the same as that under the IB model. We have

$$\bar{u} = \frac{\Delta\pi_m(\bar{y})}{ND}, \quad (1)$$

and

$$\underline{u} = \frac{\Delta\pi_d(\bar{y})}{ND}, \quad (2)$$

where $\Delta\pi_m(\bar{y})$ is the manufacturer's expected profit gain when switching from IB model to a FFS model with an inventory cap of \bar{y} , and $\Delta\pi_d(\bar{y})$ is the distributor's expected profit loss when switching from IB model to a FFS model with an inventory cap of \bar{y} .

Clearly, since the increase of the manufacturer's profit is higher than the decrease of the distributor's profit (supply chain profit increases), $\underline{u} \leq \bar{u}$. When the fee is set anywhere between these two values, both the manufacturer and the distributor are better off than they were in the IB model, thus achieving pareto-improvement. Within these pareto-improving ranges, the lower the

fee is, the more profit is obtained by the manufacturer. In real-world contracts, where the fee lies depends on the negotiation power of the players. However, it is easy to see these two values of the fee are very useful in designing and managing FFS contracts effectively and our study is designed to estimate these values, using both optimal and heuristic procedures, and understand their behavior. For simplicity, we shall refer to \underline{u} and \bar{u} as the lower fee and the upper fee, respectively.

In closing this section, we would like to summarize our main observation in the form of guidelines to the various stakeholders in this industry:

Manufacturers: The manufacturers of brand-name drugs should be at the fore-front of this transition from IB to FFS as they have the most to gain from this transition. However, without appropriate incentives, the distributors in the supply chain may not follow and thus they should ensure that the distributors be appropriately compensated if they are to see a successful implementation of these contracts.

Distributors: While FFS provides more stable revenue than investment buying, they should realize that if the fees are not set appropriately, they could be worse off than they used to be under IB model. Fortunately, with the implementation of FFS contracts, there is a larger pie to share, so they should work with the manufacturers to ensure that they get their fair share of this bigger pie.

Policymakers: Policymakers should acknowledge that the FFS contracts are better for the pharmaceutical industry and the economy as a whole and ensure that they enact policies that encourage the implementation of these contracts. They should also realize that some of the improved efficiencies or increased profits should potentially be shared with the end-consumers and they should urge the manufacturers and distributors to do so.

End-consumers: End-consumers are the ones that eventually bear the burden of the supply chain cost and they should realize that their burden is smaller under the FFS contract. In addition, they should demand a share of the increased profits.

6.2 Behavior of the Pareto-Improving Fee Levels

We compute the pareto-improving fees for all cases and investigate how different parameters impact the fee ranges. As is the industry norm, all fees are reported as percentages of WAC. Figure 3(a) shows the average pareto-improving fee ranges over all cases under different inventory caps, \bar{y} . Notice that as \bar{y} increases, the fees, both \underline{u} and \bar{u} , decrease. The manufacturer's breakeven fee, \bar{u} ,

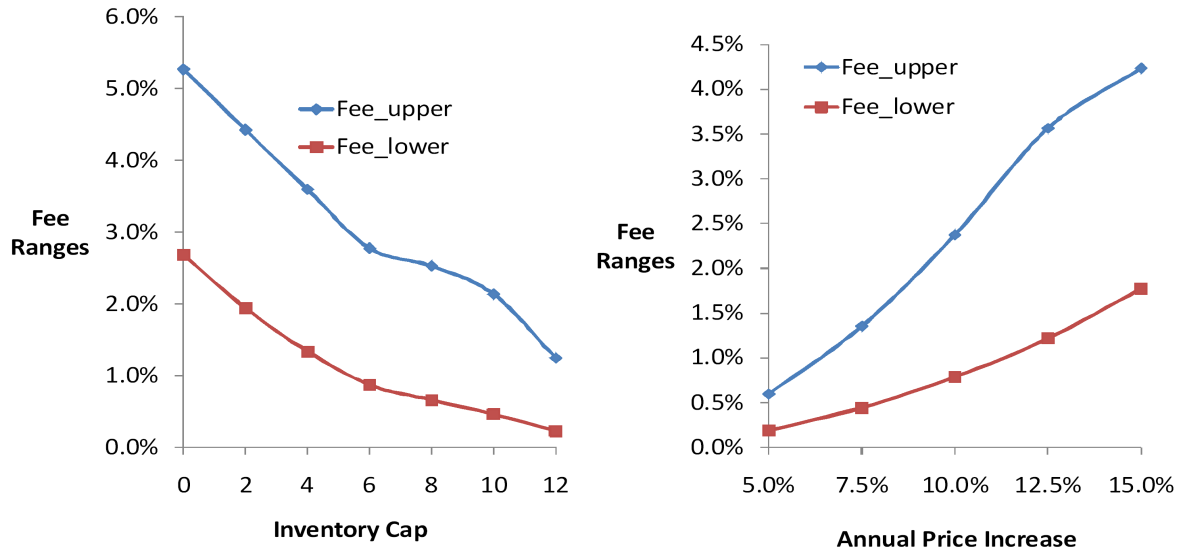


Figure 3: The Change of Pareto-Improving Fee Ranges with Inventory Cap (a) and Annual price increases (b)

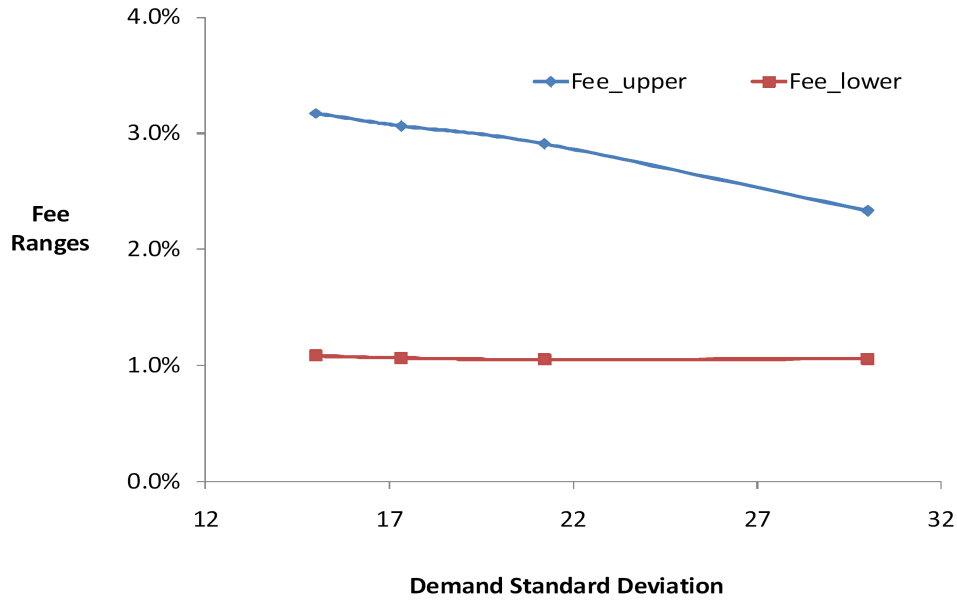


Figure 4: The change of Pareto-improving Fee Ranges with Downstream demand variance

decreases from 5.3% to 1.2% while the distributor's breakeven fee, \underline{u} , decreases from 2.7% to 0.2%. The reasons behind this behavior are as follows. As \bar{y} increases, the manufacturer's profit decreases thus urging her to provide a smaller fee. On the other hand, as \bar{y} increases, the distributor's profit increases from more investment buying and thus he requires a smaller fee to break even. The magnitude of the fee interval (i.e. $\bar{u} - \underline{u}$) also decreases as \bar{y} increases because the improvement of the supply chain profit decreases as \bar{y} increases (Figure 2).

Figure 3(b) shows the average pareto-improving fee ranges (over all the instances with that value of δ) under different annual price increase rate, δ . It can be seen that as δ increases, the pareto-improving fees (both \underline{u} and \bar{u}) increase and the magnitude of the fee interval also increases. When δ was at 5%, the fee range was from 0.2% to 0.6% and when δ was at 15%, the fee range was from 1.8% to 4.2%. This is because a higher value of δ means more profit from investment buying and hence the manufacturer needs to pay more under FFS to compensate for the profit loss the distributor sees from the restriction on investment buying.

Figure 4 shows the impact of downstream demand variance on the fees. The figure reveals that the lower bound of the pareto-improving fee ranges (\underline{u}) is not affected much by demand variance because the distributor is mostly concerned about the change in the magnitude of investment buying necessitated by the imposition of \bar{y} . On the other hand, as the demand variance increases, \bar{u} decreases. This is because the manufacturer is impacted both by the change in investment buying and by the use of information shared from the distributor. As the demand variance increases, the value of information decreases (discussed in the next subsection) resulting in a slight decrease in the fees that she is willing to pay to the distributor.

In addition to these observations, our numerical study also confirms that fees, as a percentage of WAC, are not affected by WAC since all parameters and fees are represented as percentages of WAC.

In closing this section, we would like to summarize our findings in the form of the following managerial guidelines:

1. When the inventory cap is small, the manufacturers should offer a larger per-unit service fee and the distributors should expect a larger fee. Both the distributor and the manufacturer have a little more leeway in deciding the pareto-improving fees when \bar{y} is small whereas when \bar{y} is large, they should be very careful in determining the fees because there is increased possibility that they could arrive at a value that is not pareto-improving.
2. When the price increase (as a percentage of WAC) is small (high), the manufacturers should offer a lower (higher) fee and the distributors should expect a smaller (larger) per-unit fee. The determination of the pareto-improving per-unit fees should be done more carefully at

lower values of δ as the magnitude of the effective interval is smaller.

3. The minimum fee a distributor expects should not be impacted by the variance of the end-customer demand whereas the manufacturer should offer a smaller fee when the end-consumer demand has a higher variance.

6.3 Value of Information

Here, we study the manufacturer’s performance under FFS and FFS-NI, whose difference reflects the value of the on-hand inventory information shared by the distributor with the manufacturer. We measure value of information as the percentage savings of the manufacturer’s supply-demand mismatch (inventory holding and penalty) cost when using the distributor’s on-hand inventory information. We observed that, over all the instances in our study, the average value of information is 3.63% and this value can be as high as 13.01%.

Figures 5 show the average value of information under different inventory caps and different levels of price increase, respectively. Figure 5(a) shows that this value increases as the inventory cap increases. This is because, as \bar{y} increases, the distributor will have more investment buying quantity leading to longer periods (following investment buying) in which the distributor does not place an order (longer transitional periods). It is during these periods that the information shared from the distributor to the manufacturer is useful and thus the average value of information increases with \bar{y} .

Figure 5(b) shows that as price increase (δ) increases, the value of information will first increase and then flatten out (after δ reaches 10%). This is because, as δ increases, the investment buying quantity and hence the number of transitional periods increases. Thus the value of information increases. However, there is a limit as to how much investment buying takes place and hence the value of information flattens out.

Our experiments also show that the value of information slightly decreases as demand variance increases. The average value of information is 3.36% when the demand has the Erlang (7.5, 4) distribution while it is 2.86% when with demand has the Erlang (30, 1) distribution. This is because when the variability is very large, the information provided by the distributor is not enough to resolve a significant portion of the uncertainty. As the demand variance decreases, the information provided by the distributor resolves a larger percentage of the uncertainty, thus making the information relatively more valuable.

In summary, the manufacturers should ensure that they make their best efforts to incorporate the information (they receive from the distributor) into their decision making process when the inventory cap is high, price increase is large, and the end-customer demand variance is not too

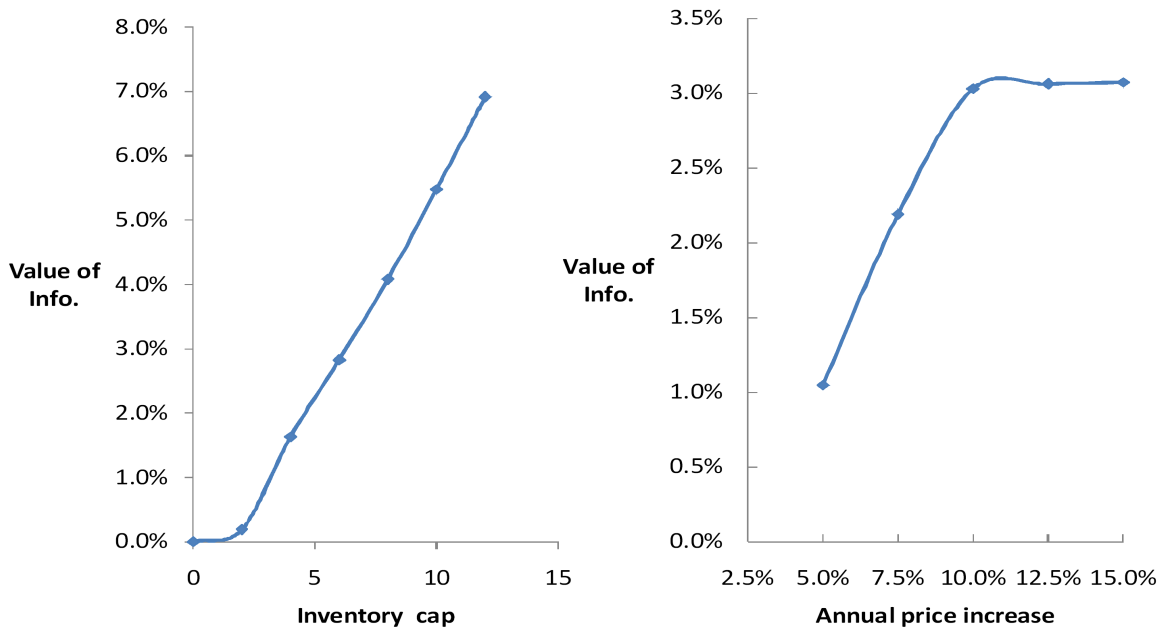


Figure 5: The change of value of information with inventory cap (a) and annual price increase (b)

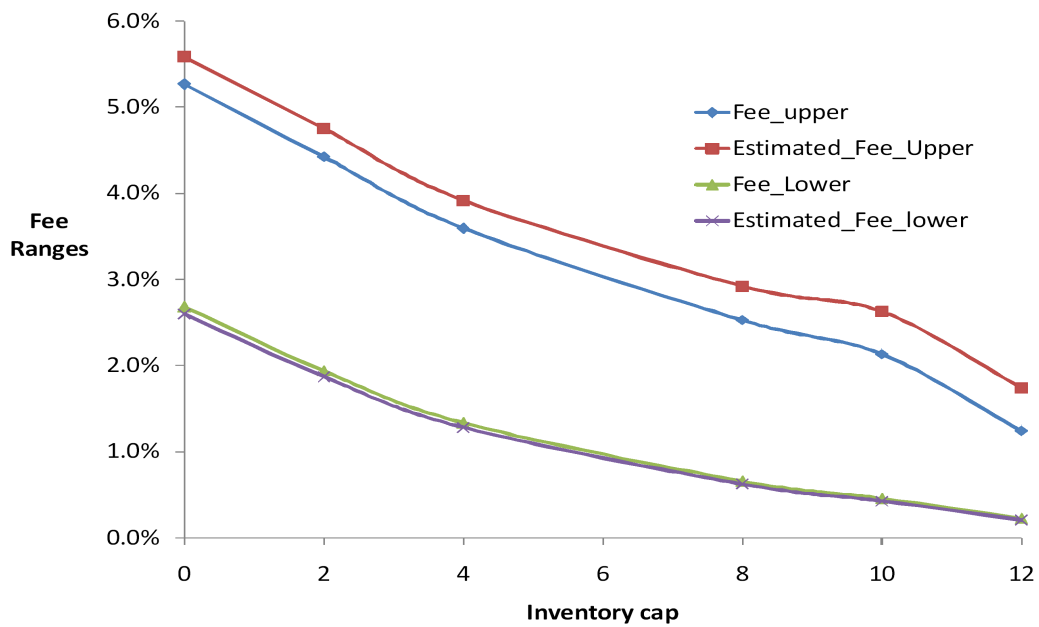


Figure 6: The performance of the heuristic in estimating the pareto-improving fee ranges

large. If they do not use the information, they may lose an opportunity to reduce supply-demand mismatch costs by an average of 3.63% and as much as 13.01%.

6.4 A Simple Heuristic to Estimate the Pareto-Improving Fee Range

Given that the optimal solution involves solving a multi-period model and is possibly beyond the scope of a decision maker in the real-world, in this subsection, we develop a simple heuristic that can be used to estimate the pareto-improving fee ranges given a specific inventory cap and other parameters. To develop this heuristic, we first need to understand how to obtain the upper and lower bounds of the pareto-improving fee ranges, \bar{u} and \underline{u} , respectively.

Consider an N -period horizon. Based on (1) and (2), the key to estimate \bar{u} and \underline{u} is to estimate $\Delta\pi_m$ and $\Delta\pi_d$. In the following, we develop a simple heuristic to estimate these two values.

In estimation of $\Delta\pi_m$, recall y_{n-1}^{d*} is the optimal stock level for the distributor under IB model and \bar{y} is the maximum stock level the distributor can reach in period $n - 1$ under FFS. Changing from IB to FFS, although losing profit due to less investment buying, the distributor saves on holding cost. Since the service level is quite high, we will omit the loss of penalty cost in the estimation and estimate the distributor's holding cost savings.

To do this, note that if there were no price increase, $y^* = \Phi(\frac{p^d}{p^d+h^d})$ for all periods. Next we estimate the change of the distributor's holding cost when switching from the no-price-increase case to IB model or to FFS model.

For the IB model, let $k_{IB} = \frac{y_{n-1}^{d*} - y^*}{D}$ and $[k_{IB}]$ represent the integer part of k_{IB} . The distributor's estimated holding cost increase under IB model compared to the no-price-increase case is calculated in this way: there is roughly \bar{D} units of overstock sold in period n (the period of price increase) that cost extra holding cost for one period (period n), $h^d\bar{D}$, and there is roughly \bar{D} units sold in period $n + 1$ that cost extra holding cost for 2 periods (periods $n - 1$ and n), $2h^d\bar{D}$, and so on. Finally, there are $(k_{IB} - [k_{IB}])\bar{D}$ units sold in period $n + [k_{IB}]$ that cost extra holding cost, $([k_{IB}] + 1)(k_{IB} - [k_{IB}])h^d\bar{D}$. So the estimated increased holding cost is:

$$\begin{aligned} & h^d\bar{D} + 2h^d\bar{D} + \dots + [k_{IB}]h^d\bar{D} + ([k_{IB}] + 1)(k_{IB} - [k_{IB}])h^d\bar{D} \\ &= \left(\sum_{i=1}^{[k_{IB}]} h^d\bar{D} \right) + ([k_{IB}] + 1)(k_{IB} - [k_{IB}])h^d\bar{D} \\ &= h^d\bar{D}([k_{IB}] + 1)(k_{IB} - \frac{[k_{IB}]}{2}) \end{aligned} \quad (3)$$

Similarly, for the FFS model, let $k_{FFS} = \frac{\bar{y} - y^*}{D}$ and define $[k_{FFS}]$ to be the integer part of k_{FFS} . Following the same logic, we estimate the distributor's increase under FFS model compared to the no-price-increase model to be:

$$h^d\bar{D}([k_{FFS}] + 1)(k_{FFS} - \frac{[k_{FFS}]}{2}) \quad (4)$$

Then the decreased cost switching from IB to FFS model is (3) –(4). We also know the distributor’s reduced revenue when switching from IB to FFS model is $\delta(y_{n-1}^{d*} - \bar{y})$.

So the distributor’s total profit loss after switching from IB to FFS model can be estimated as:

$$\Delta\pi_d \sim \delta(y_{n-1}^d - \bar{y}) - h^d \bar{D}[(\lfloor k_{IB} \rfloor + 1)(k_{IB} - \frac{\lfloor k_{IB} \rfloor}{2}) - (\lfloor k_{FFS} \rfloor + 1)(k_{FFS} - \frac{\lfloor k_{FFS} \rfloor}{2})] \quad (5)$$

and $\underline{u} = \frac{\Delta\pi_d}{N*D}$.

Next, we estimate the manufacturer’s profit gain ($\Delta\pi_m$) to calculate \bar{u} . When switching from IB to FFS, the manufacturer gains a revenue of $\delta(y_{n-1}^d - \bar{y})$ because of the limited investment buying. The cost should not change too much because the manufacturer basically carries enough stock for periods that he predicts that he will not see demand. So we can estimate $\Delta\pi_m$ as $\delta(y_{n-1}^d - \bar{y})$ and the upper bound of the fee range, \bar{u} , can be estimated as $\frac{\delta(y_{n-1}^d - \bar{y})}{N*D}$.

Figure 6 reflects the performance of the heuristic by showing the optimal pareto-improving fee ranges \bar{u} and \underline{u} and their estimates calculated from the heuristic under different inventory caps. As seen from the figure, the heuristic provides very accurate estimation of the \underline{u} and a slightly (and consistently) higher estimate of \bar{u} (by about 0.3 – 0.4%). Given that the manufacturer is usually the leader in setting the per-unit fees, she is most interested in identifying \underline{u} , the fee value that motivates the distributor just enough to participate in the FFS contract. Given the simplicity of the proposed heuristic and its effectiveness (almost perfect estimation of the lower fee), it can be an invaluable asset to the manufacturer in determining the per-unit fees she should offer the distributor in FFS contract negotiation.

7. Extensions

In this section, we investigate four additional issues and their impact on the supply chain profits, pareto-improving fee ranges, and value of information. These extensions provide us more insights into the industry practice and also help us establish the robustness of our results.

7.1 Impact of the Mean Demand

Different products may face different volumes of demand. To separate from the effect of demand variance (which was already shown in section 6), we expanded the computational study by using the demand distributions Erlang (15, 2), Erlang (15, 2)+5, Erlang (15, 2)+10, Erlang (15, 2)+15..., hence keeping the demand variance the same, but capturing mean demand per period to be 35, 40, 45,...through 75 in addition to the 30 that was already in our study. We do this for the case with $\delta = 10\%$ as cases with other δ values have similar insights.

Figures 7(a) and 7(b) show how the average percentage of supply chain profit increase of FFS over IB and the pareto-improving fee ranges change with mean demand. Observe that: (1) As the

average demand increases, the percentage of improvement of FFS over IB decreases, indicating that, the benefits of FFS model is bigger for products with smaller average demand. This is because the supply chain transparency enabled by FFS is more valuable at lower mean demands as the demand variance then plays a larger role in defining supply chain efficiency. (2) As the mean demand increases, the upper fee decreases, but the lower fee does not change much. This is because the lower fee is related to the break-even compensation to the distributor for not using investment buying, which is directly connected to the mean demand. Thus, as the mean demand increases, the break-even compensation also increases, leaving the *per-unit* lower fee roughly unchanged. On the other hand, the upper fee captures the change in the manufacturer profit which consists of (i) the reduction in investment buying; and (ii) the value of information. The reduction in investment buying is directly related to the mean demand (as mentioned above) while the value of information is a function of demand variance. Thus, as the mean demand increases, the first quantity increases while the second quantity remains the same. This leads to a decreasing trend of the *per-unit* upper fee.

To summarize, given similar demand variance, for products with lower mean demand (potentially more expensive products like bio-pharmaceuticals), the benefit of FFS over IB is higher, distributor could potentially charge a higher fee, and the value of information is roughly the same (figure omitted).

7.2 Impact of fixed ordering Cost

Generally speaking, in the pharmaceutical industry, the cost of a distributor placing an order is not significant. Nearly all transactions between a manufacturer and a distributor are conducted using Electronic Data Interchange (EDI). In addition, all pharmaceutical manufacturers use bar codes at the pallet, case, shelf-pack, and individual item levels. This enables very high levels of efficiency in order receipt and handling by distributors. According to HDMA (Healthcare Distribution and Management Association), 100% of distributors use warehouse management systems (WMS) and the distributor operating expenses average only 1.12% of revenues, i.e., very small. Therefore, we do not specifically model ordering cost. But the periodic review set-up still helps us to capture the impact of fixed ordering cost. Specifically, in our periodic review model, an ordering cycle is defined as one period. In practice, the ordering cycle is usually one or two weeks, which is adopted with the balance of holding and penalty costs and the ordering/transaction cost. Thus, given a specific finite horizon, say one year (as used in our numerical study), a decrease in the number of periods in the horizon indicates an increase in the length of an ordering cycle, corresponding to an increase of the ordering cost. Thus, by investigating how the number of periods in the one-year

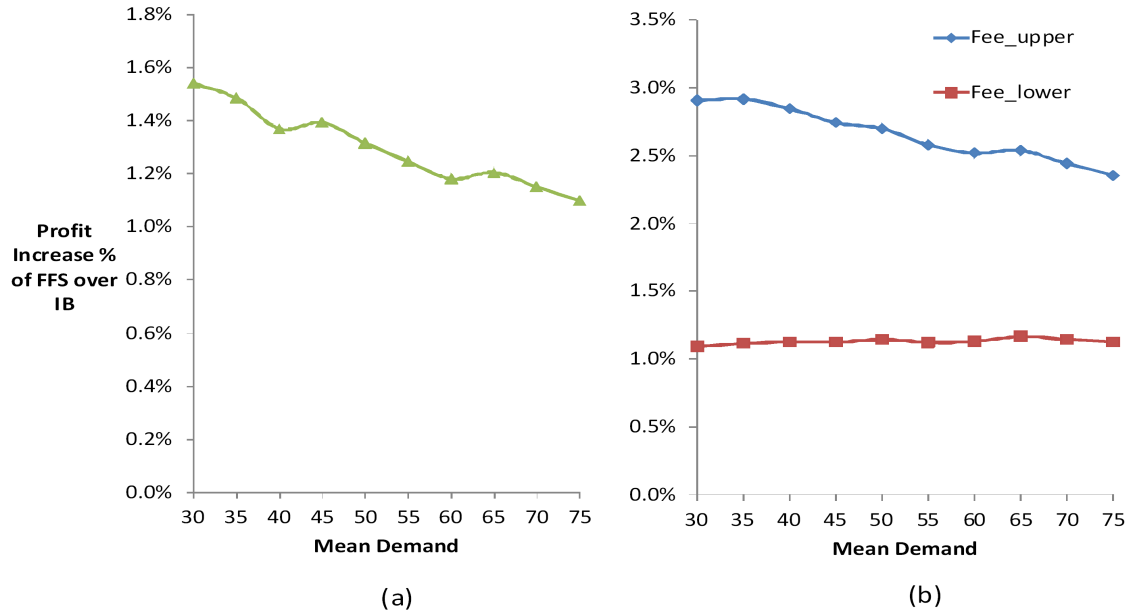


Figure 7: The impact of mean demand on the profit increase of FFS over IB (a) and the pareto-improving fee ranges (b)

finite horizon affects the supply chain profits, fee ranges, and the value of information, we can see the impact of ordering cost.

Specifically, we re-ran the computational study with $N=36$, 48, and 60, compared to $N=24$ in the original study. These correspond to ordering every 1.5, 1.0, and 0.8 weeks (6 days) as compared to every 2 weeks ($N=24$) that was in the study. We choose these numbers because in practice, the ordering cycle almost never exceeds two weeks.

Fig 8(a) shows that as N decreases (ordering cost increases), supply chain profit increase of FFS over IB increases, i.e., the benefit of switching from IB to FFS increases. This is because the supply chain is able to better recover from the shock created by the price increase when ordering more frequently, hence the transition to FFS is not that beneficial. Moreover, Fig 8(b) shows that as the ordering cost increases (N decreases), the upper fee increases and the lower fee decreases, leading to a larger pareto-improving fee range. This is because when ordering cost increases, the distributor's ordering pattern under FFS is closer to that under the IB system. As a result, he needs a smaller fee to compensate him for switching from IB to FFS (i.e. lower fee decreases). Similarly, this effect would lead the manufacturer to offering a smaller fee to the retailer when switching from IB to FFS. On the other hand, as N decreases, the value of information to the manufacturer increases (see Fig 8c), enabling the manufacturer to offer a larger fee to the distributor. This latter effect outweighs

the former effect, hence leading to an increase in the upper fee when ordering cost increases (N decreases).

In summary, for companies with a higher fixed ordering cost, there is a bigger benefit switching from IB to FFS. In addition, the upper fee will be higher while the lower fee will be lower, leading to a larger pareto-improving fee range. Finally, the company would forego a bigger benefit when not using the inventory information shared by the distributor.

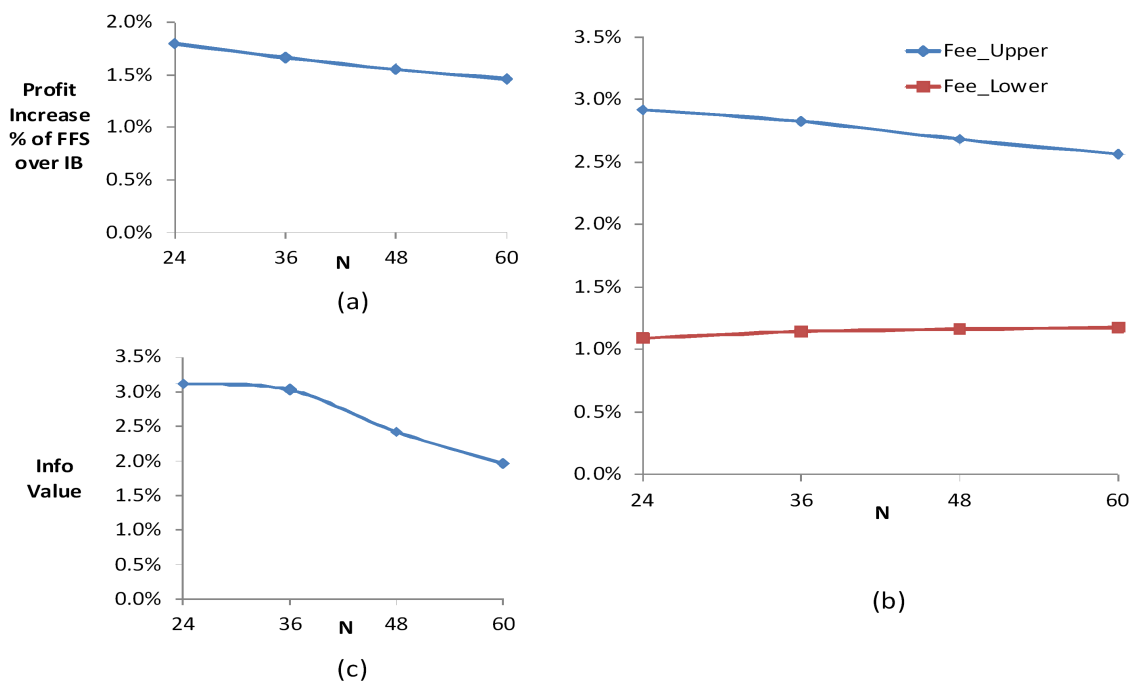


Figure 8: The impact of fixed cost on the profit increase of FFS over IB (a), the pareto-improving fee ranges (b), and the value of information (c)

7.3 Impact of Holding and Penalty Cost Increase after price increase

In the previous sections, we have assumed that holding and penalty costs are not changed after price increase. In reality, these costs will be affected by price increase. In this section, we ran further numerical experiments on the impact of holding and penalty cost change after price increase. The change of the holding and penalty costs for both the manufacturer and the distributor are based on the price change δ and their relationship to WAC. For example, when $\delta = 15\%$, h, p, h_d, p_d all increase 15% after price increase. Theoretically, holding costs should only increase for products purchased at higher price, yet tracking products based on purchase prices is very difficult. Hence, in the computational study, holding and penalty costs are increased for all products after price increase in period n . By doing this, we overestimate the impact of the holding cost hence our results provide an upper-bound of such impact. We chose to run experiments for the cases with

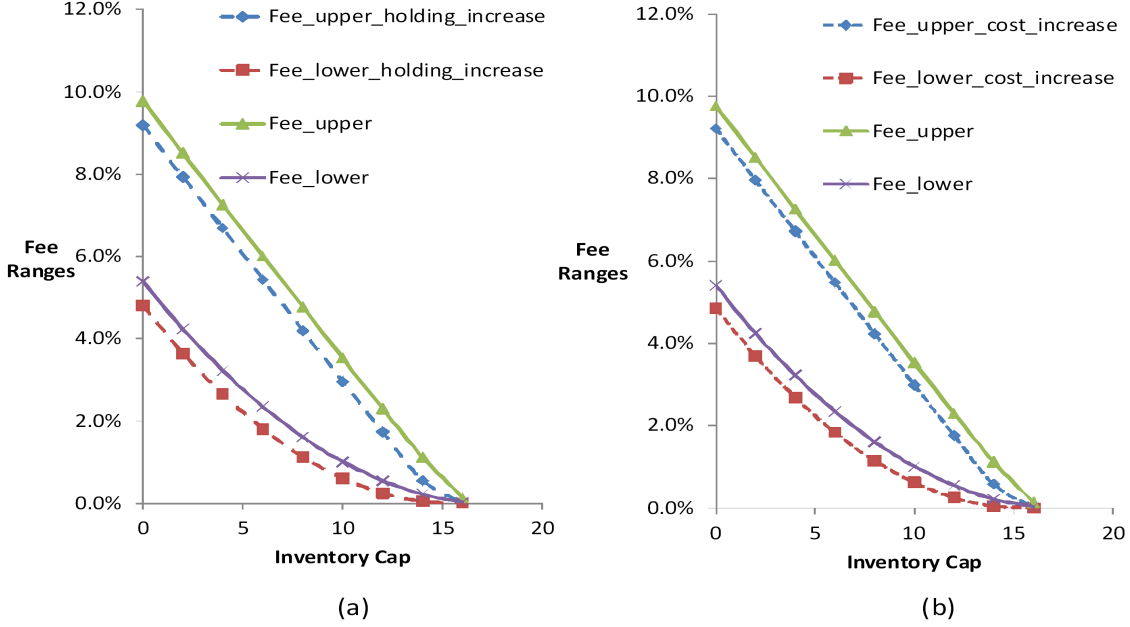


Figure 9: Pareto-improving fee ranges with only holding cost increase (a) and with holding and penalty cost increases (b)

$\delta = 15\%$ since these are the cases with the maximum change to the holding and penalty costs in the study, hence would again provide an upper bound of this impact.

To see the impact of holding cost and penalty cost, we re-ran the experiments first changing only the holding cost after price increase (penalty cost remained the same). Fig 9(a) shows that when considering only holding cost increase, both the upper fee and lower fee decrease slightly (by at most 0.6%). This is because distributors will order less in investment buying anticipating holding cost increases after price increase. Hence, the manufacturer needs to compensate the distributor less for switching from IB to FFS (upper and lower fees decrease). On the contrary, when penalty cost increases after the price increase, distributors will order more in investment buying (due to the greater penalty during the transitional periods and the following periods if they stock out), leading the upper and lower fees to increase, opposite to the effect of holding cost. Since there is only a small chance of incurring penalty cost (due to the high service level), the effect of holding cost outweighs that of the penalty cost, hence the overall effect will have similar trend as the holding cost. Fig 9(b) confirms this, showing that considering both the holding and penalty cost increase, the upper and lower fees decrease slightly (by most 0.5%).

In summary, given these results provide an upper bound, the impact of the increase of holding and penalty costs after price increase seems to be not very significant (fees decrease by at most

0.5% and the value of information remains almost the same (figure not shown)).

7.4 Different Demand Distributions

In this section, we investigate whether our results hold true for distributions other than the Erlang and Exponential distributions. Specifically, we expand our study to include uniform and normal demands. For uniform distribution, we ran experiments on demand following $U[10, 50]$ and $U[20, 40]$; for normal distribution, we ran experiments on demand following $N(30, 5)$ and $N(30, 10)$. All distributions have a mean demand of 30 and we ran all experiments with $WAC = 200$, $\delta = 10\%$, and \bar{y} levels starting from y^* until it reaches y_{n-1}^{d*} . Our results show that supply chain profit increases when switching from IB to FFS for all the distributions and the average profit increase of FFS over IB is comparable for all three distributions that we tested, i.e., 1.48% for normal distribution, 1.46% for the uniform distribution, and 1.53% for the Erlang distributions in the original study. This result is not surprising since imposing an inventory cap increases the efficiency of the supply chain by reducing excessive inventory without sacrificing the service level. Hence, we expect our results also hold for other demand distributions.

8. Conclusion

In this paper, we address some pressing issues facing the current pharmaceutical distribution industry. By comparing investment buying (IB) models and fee-for-service (FFS) models, we clarify the industry's doubts regarding which model is better, provide guidelines in how the manufacturer and the distributor operate under FFS model (a model adopted by most distributors today), and how to set the FFS contract parameters which lead to pareto improvement for both the manufacturer and the distributor. Here, we list the main managerial insights obtained from our analysis.

- Transition from IB to FFS is beneficial to the supply chain as a whole and certainly is advantageous to the manufacturer. While this limits the investment buying opportunity to the distributor, an appropriate per-unit service fee can overcome this deficiency. The improved efficiency of the supply chain under FFS ensures that there exists a range of fees that guarantee the manufacturer and the distributor achieve pareto-improvement.
- Under the FFS contract, given that the distributor is able to satisfy the service level target, as the inventory cap decreases, the manufacturer's and the supply chain's profits increase and the distributor's profit decreases.
- The values (and the range) of pareto-improving per-unit fees decrease as the inventory cap increases. Thus at lower values of \bar{y} , the manufacturer and the distributor have more flexibility

in choosing the fees while they should be a lot more careful in setting the fees when the \bar{y} is larger.

- Manufacturers may forego significant (on average 3.63% and as much as 13.01%) supply-demand mis-match cost savings when they do not take advantage of the distributor's inventory information that is provided to them under FFS for production planning. The value of information decreases as they impose tighter inventory cap on the distributor, when the price increase is smaller, or the end-consumer demand has a high variance.
- Given similar demand variance, transition from IB to FFS brings more benefits for products with lower mean demand (potentially more expensive products like bio-pharmaceuticals) and distributor could potentially charge a higher fee for these products.
- For companies with a higher fixed ordering cost (i.e., the ordering cycle being longer), we see a bigger benefit switching from IB to FFS. In addition, the upper fees should be higher while the lower fees should be lower, leading to a larger pareto fee range. Finally, the manufacturer would forego a bigger benefit when not using the distributor's on-hand inventory information.
- Considering the change of holding and penalty costs after price increase, both the upper and lower limits of the pareto-improving fee ranges decrease (by at most 0.5%).
- A simple, effective, and useful heuristic is developed that could be very useful for real-world FFS contract negotiations by decision makers that are not adept at solving stochastic inventory control problems.

References:

- Anupindi, R. and R. Akella. 1993. Diversification Under Supply Uncertainty. *Management Science*, 39(8). 944-963.
- Aviv, Y. 2001. The effect of collaborative forecasting on supply chain performance. *Management Sci.* 47(10). 1326-1343.
- Aviv, Y., A. Federgruen. 1998. The operational benefits of information sharing and vendor managed inventory (VMI) programs. <http://apps.olin.wustl.edu/faculty/aviv/papers/vmr.pdf>.
- Burns, L. R., et al. 2002. *The Health Care Value Chain: Producers, Purchasers, and Providers*, Jossey-Bass, San Francisco.
- Cachon, G., M. Fisher. 2000. Supply chain inventory management and the value of shared information. *Management Sci.* 46(8) 1032-1048.
- Cachon, G. (2003). Supply chain coordination with contracts. *Handbooks in Oper. Res. and Man. Sci.: Supply Chain Management*, edited by Steve Graves and Ton de Kok. North-Holland.

- Chen, F. 2003. Information Sharing and Supply Chain Coordination. In the Handbooks in Operations Research and Management Science, Vol. 11, Supply Chain Management: Design, Coordination, and Operation, edited by A.G. de Kok and S.C. Graves, Elsevier, Amsterdam.
- De Kok, A.G., S.C. Graves. 2003. Supply Chain Management: Design, coordination & Operations. Elsevier, Amsterdam.
- Fein, A. 2005. Drive the Right Supply Chain Behaviors. Harvard Business Review Supply Chain Strategy, August.
- Fein, A. 2007. Managing Wholesaler Agreements in the Fee-for-Service Era. American Journal of Health-System Pharmacy.
- Gavirneni, S., R. Kapuscinski, S. Tayur. 1999. Value of information in capacitated supply chains. Management Sci. 45(1) 16-24.
- Graves, S. C. (1999). Manufacturing Planning and Control. Working Paper, Massachusetts Institute of Technology, Boston, MA.
- Glasserman, P., S. Tayur. 1995. Sensitivity Analysis for Base-Stock Levels in Multiechelon Production-Inventory Systems. Management Science, Vol. 41, No. 2, 263-281.
- Lee H., P. Padmanabhan and S. Whang, (1997). Information distortion in a supply chain: the bullwhip effect. Management Science, 43(4) 546-558.
- Lee, H., K. So, C. Tang. 2000. The value of information sharing in a two-level supply chain. Management Sci. 46(5) 626-643.
- Gavirneni, S., and T.E. Moron. 1999. "Inventory Control under Speculation: Myopic Heuristics and Exact Procedures" with Thomas Morton. European Journal of Operational Research, Vol. 117, No. 2, pp. 211-221.
- Martino, K., Y. Zhao, A. Fein. 2010. Resell versus direct models in brand drug distribution. Working paper, Department of supply chain management and marketing sciences, Rutgers Business School, Newark, NJ.
- OECD Health Data 2009. Organization for Economic Co-operation and Development.
<http://www.researchinchina.com/Htmls/Report/2009/5688.html> Accessed August 2011.
- Schwarz, L. B. 2010. Healthcare-product supply Chains: Medical-surgical supplies, pharmaceuticals, and orthopedic devices. Y. Yi, ed. Handbook of Healthcare-Delivery Systems, CRC Press, Boca Raton, FL.
- Schwarz, L. and H. Zhao. 2011. The Unexpected Impact of Information-Sharing on US Pharmaceutical Supply-Chains. Interfaces. Jul./Aug. 2011 41:354-364.
- Simchi-Levi, D., Y. Zhao. 2004. The Value of Information Sharing in a Two-Stage Supply Chain with Production Capacity Constraint: The Infinite Horizon Case. Probability in the Engineering and Informational Sciences, 18. 247-274.

- Tayur, S., M. Magazine, R. Ganeshan. 1999. Quantitative models for supply chain management. Kluwer Academic Publishers, Norwell, MA.
- US Pharmaceutical Industry report 2008-2009. <http://www.pr-inside.com/us-pharmaceutical-industry-report-r1291427.htm>. Accessed August 2011.
- Yost, R. 2005. New economics of the pharmaceutical supply chain. *Am J Health-Syst Pharm*, 62:525-6.
- Zhu, W., S Gavirneni, R. Kapuscinski. 2010. Periodic flexibility, information sharing, and supply chain performance. *IIE Transactions*. 42(3). 173-187.

Technical Appendices

A-1. Proof for Theorem 1

Use backward induction. Since the distributor's terminal cost function is $V_{N+1}^d(x_d) = -(c^d + \delta - u)x_d$ with a slope of $-(c^d + \delta - u)$, based on Theorem 4.3 of (Porteus (2002)), a base-stock policy with base-stock level $y^* = \Phi^{-1}(\frac{p^d}{p^d + h^d})$ is optimal for period k with $n \leq k \leq N$ since $c_k^d = c^d + \delta$ for any $k \geq n$. Consider period $n - 1$, we have $G_{n-1}^{d'}(y^*) = L_{n-1}^{d'}(y^*) + (c^d - u) - (c^d + \delta - u) \leq 0$ since $L_{n-1}^{d'}(y^*) = 0$, which implies $y_{n-1}^{d*} \geq y^*$ is the optimal stock level for period $n - 1$. Next consider period $n - 2$, we have $G_{n-2}^{d'}(y_{n-1}^{d*}) = L_{n-2}^{d'}(y_{n-1}^{d*}) + c^d - c^d \geq 0$ since $L_{n-2}^{d'}(y_{n-1}^{d*}) \geq L_{n-2}^{d'}(y^*) = 0$ which implies $y_{n-2}^{d*} \leq y_{n-1}^{d*}$. By solving $G_{n-2}^{d'}(y_{n-2}^{d*}) = 0$ we get $y_{n-2}^{d*} = y^* = \Phi^{-1}(\frac{p^d}{p^d + h^d})$. Then by the backward induction, we can get $y_k^{d*} = \phi^{-1}(\frac{p^d}{p^d + h^d}) = y^*$ for $1 \leq k \leq n - 2$. ■

A-2. Proof for Theorem 2

Since the total production cost through the finite horizon is $c \sum_{k=1}^N (y_k - x_k) - cx_{N+1} = c \sum_{k=1}^N Q_k$, which is not dependent on the stock level, to show the property of optimal stock level we don't need to consider the production cost $-c(y - x)$ in each period. Thus, we rewrite $V_k^i(x) = \min_{y \geq x} \{G_k^i(y)\}$, where

$$G_k^i(y) = \begin{cases} L(y) + \int_0^\infty V_{k+1}^i(y - D)^+ \phi(D) dD & \text{if } i = 0^- \text{ or } 0^+ \text{ and } k \neq n - 1 \\ L_{n-1}(y) + \int_0^\infty V_n^0(y - (D + \bar{y} - y^*))^+ \phi(D) dD & \text{if } k = n - 1 \\ p_i \left[L^i(y) + \int_0^\infty V_{k+1}^{0+}(y - \eta_i)^+ \theta_i(\eta_i) d\eta_i \right] \\ \quad + (1 - p_i) \left[hy^+ + \int_0^\infty V_{k+1}^{i+D}(y) \phi(D) dD \right] & \text{otherwise.} \end{cases}$$

To prove the optimality of the produce-up-to policy, we will use backward induction again. It can be easily shown that $L(y)$, $L_{n-1}(y)$ and $L^i(y)$ are convex by definition. Since we know $V_{N+1}^i(x)$ is convex in x for any state i , for $G_N^i(y)$ whether $i = 0^+$ or not, we know $G_N^i(y)$ is convex in y by the convexity of V_{N+1}^i . Based on the fact that if $f(y)$ is convex then $g(x) = \min_{y \geq x} f(y)$ is also convex, we know $V_N^i(x) = \min_{y \geq x} \{G_N^i(y)\}$ is also convex in x for any i . Now assume in period k , $G_k^i(y)$ and $V_k^i(x)$ are convex for any i . We want to prove then $G_{k-1}^i(y)$ and $V_{k-1}^i(x)$ are convex for any i . Consider the situation where $i = 0^-$ or 0^+ and $k \neq n - 1$, we have $G_{k-1}^i(y) = L(y) + \int_0^\infty V_k^i(y - D)^+ \phi(D) dD$, then

$$\begin{aligned} G_{k-1}^i(y) &= L(y) + \left(\int_0^\infty V_k^i(y - D)^+ \phi(D) dD \right)' \\ &= L(y) + \left(\int_0^y V_k^i(y - D) \phi(D) dD \right)' + \left(\int_y^\infty V_k^i(0) \phi(D) dD \right)' \\ &= L(y) + \int_0^y V_k^i(y - D) \phi(D) dD \end{aligned}$$

$$\begin{aligned}
G_{k-1}''(y) &= L''(y) + \left(\int_0^y V_k''(y-D)\phi(D)dD \right)' \\
&= L''(y) + \int_0^y V_k''(y-D)\phi(D)dD + V_k''(0)\phi(y) \\
&\geq V_k''(0)\phi(y)
\end{aligned}$$

Since $V_k''(0) \geq \max\{G_k''(x), 0\} \geq 0$, we get $G_{k-1}''(y) \geq 0$ and prove that $G_{k-1}^i(y)$ is convex. By use the similar method we can prove for other state i , $G_{k-1}^i(y)$ is convex. Then we get $V_{k-1}^i(x) = \min_{y \geq x} \{G_{k-1}^i(y)\}$ is also convex. So we prove that G_{k-1}^i, V_{k-1}^i are convex for all i . By backward induction, we get G_k^i, V_k^i are convex for all i, k which shows the produce-up-to policy is the optimal policy.

Next to prove $y^0 \leq y^1 \leq y^2 \dots \leq y^{\bar{y}-y^*-1} \leq y^{0+}$. It is easy to show after state is changed to $i = 0^+$, the stationary base stock level $y^{0+} = \Phi^{-1}(\frac{p}{p+h})$ is the optimal produce-up-to level by applying Theorem 4.3 of (Porteus (2002)). Since $\Theta_0(\cdot) \leq_{st} \Theta_1(\cdot) \leq \dots \leq \Theta_{\bar{y}-y^*-1}(\cdot) \leq_{st} \Phi(\cdot)$, the demand from periods n to the ending period N is monotonic stochastically increasing. So the myopic policy is optimal which means for any period $k \geq n$ with state $i = 0, 1, \dots, \bar{y} - y^* - 1$, the optimal y^i must have $L^i(y^i) + (hy^+)' = 0$. With the property that $\Theta_i \leq_{st} \Theta_{i+1}$, we can solve $L^i(y^i) + (hy^+)' = 0$ to get that $y^0 \leq y^1 \leq y^2 \dots \leq y^{\bar{y}-y^*-1} \leq y^{0+}$. ■

A-3. Proof for Corollary 1

From Theorem 2, we know for $y^{0+}(\bar{y}_1) = y^{0+}(\bar{y}_2) = \Phi^{-1}(\frac{p}{p+h})$. For the transitional periods, we know $p_i(\bar{y}_1) \geq p_i'(\bar{y}_2)$ and $\Theta_i(\cdot|\bar{y}_1) \geq_{st} \Theta_i(\cdot|\bar{y}_2)$ given $\bar{y}_1 \leq \bar{y}_2$ (details omitted). From Theorem 2 we know during the transitional periods, myopic policy is optimal. By using the property that $\Theta_i(\cdot|\bar{y}_1) \geq_{st} \Theta_i(\cdot|\bar{y}_2)$, we get $y^i(\bar{y}_1) \geq y^i(\bar{y}_2)$ for $i \geq 0$. ■

A-4. Proof for Theorem 3

If $\delta_1 \leq \delta_2$, we have $y_{n-1}^{d*}(\delta_1) \leq y_{n-1}^{d*}(\delta_2)$. We know $y^{0+}(\delta_1) = y^{0+}(\delta_2) = \Phi^{-1}(\frac{p}{p+h})$. For the transitional periods, we know $p_i(\delta_1) \geq p_i(\delta_2)$ and $\Psi_i(\cdot|\delta_1) \geq_{st} \Psi_i(\cdot|\delta_2)$ by knowing $y_{n-1}^{d*}(\delta_1) \leq y_{n-1}^{d*}(\delta_2)$. In the Investment Buying model, we have similar result as Theorem 2 that during the transitional periods, the myopic policy is optimal. By using the property that $\Psi_i(\cdot|\delta_1) \geq_{st} \Psi_i(\cdot|\delta_2)$, we get $y^i(\delta_1) \geq y^i(\delta_2)$ for any $i \geq 0$. ■