Economics Lecture 3

More about Pareto, etc, preferences.

We have said that an allocation x' is Pareto preferred to allocation x if there exists a path γ in the manifold M of allocations, with $\gamma(0)=x$, $\gamma(1)=x'$ and $\gamma'(t)$ always in the Pareto cone at $\gamma(t)$. [under reasonable convexity assumptions this condition is the same as saying that $U_\alpha(x') \geq U_\alpha(x)$ for all agents α with strict inequality for at least one.]

If no allocation is preferred to x, then x is Pareto efficient.

The first welfare thm says that efficient allocations can always be produced by the price mechanism.

• Note that Pareto preference is transitive and antisymmetric. • Can also exhibit $S \leq \emptyset$

Imagine a society with two agents, α and β, and one good (money). Originally α has \$10, β has \$20. (call this x)

Now α builds a factory (or something) that enriches him but marginally inconveniences β; say, after constrn, α would have \$20, β \$19. (call this x')

Then x' is not a Pareto improvement on x (nor vice versa). On the other hand, α could pay β \$2 to get into a Pareto-improving situation, and still show a profit...

(We are now in the enlarged manifold $\tilde{M}$ of allocns where we allow for the possibility of "growing the pie").

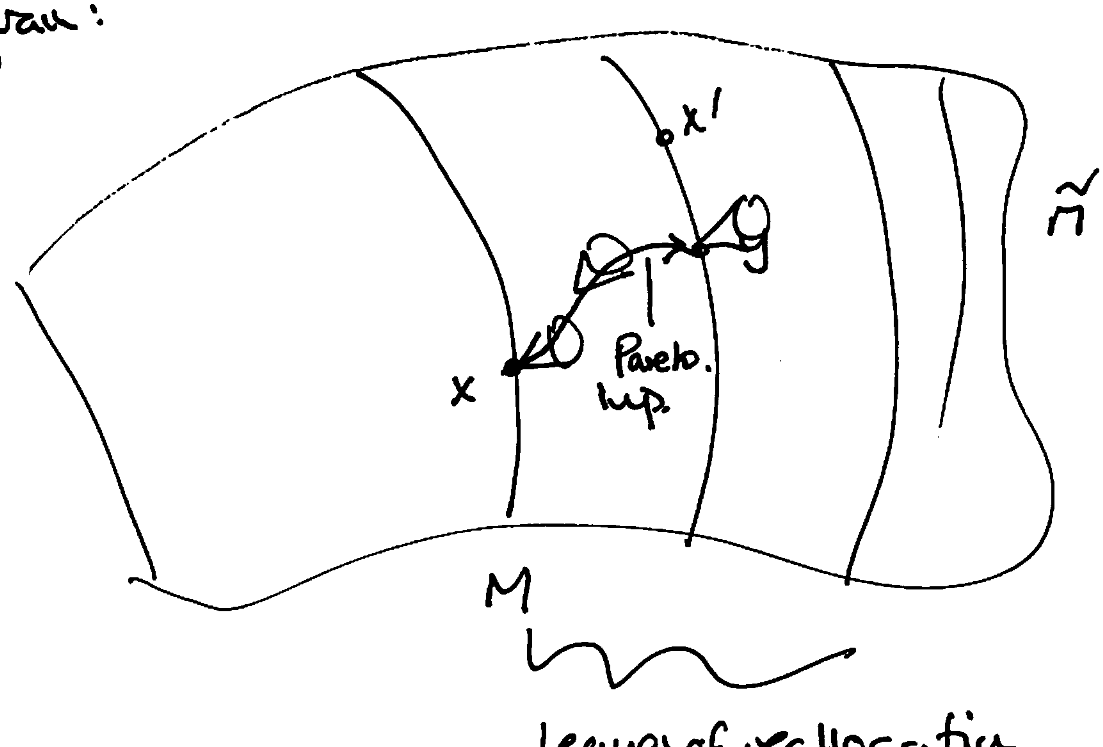
② This leads to the Kaldor criterion.

Defn x′ is a Kaldor improvement over x if ~~one can reach~~ there is a third allocation, y, such that

(a) y is a Pareto improvement on x.

(b) x′ can be reached from y via a reallocation.

See diagram:



This relation is neither transitive nor antisymmetric in general. (see figure). The Kaldor-Hicks criterion forces antisymmetry: one says that x′ is a KH improvement of x if x′ is a Kaldor imp. of x and x is NOT a Kaldor imp. of x′. (interp. in terms of bribery). This is still not transitive — there is a further elaboration due to

Samuelson that forces that.

3. It is not necessary that the reallocations actually take place, merely that they could have taken place. Nevertheless, the Kaldor-Hicks criterion is often used as an instrument of social policy, e.g. in a decision on a large construction project. Some will win, some will lose, but if the winners could compensate the losers, go ahead.

Obvious ethical questions here!

Cost-benefit analysis

It might seem hopeless to try to apply the Kaldor criterion in practice since it relies on hypothetical redistribution of all the goods in the economy. Fortunately (or maybe not) there is a simple-minded reduction.

We imagine that we are in a competitive economy with price vector p. We are interested in evaluating some new potential project from the Kaldor-Hicks point of view. We assume that there is a subset $S \subseteq A$ of agents affected by the project ~~~~ — the rest are completely unaffected.

Key assumption S is small enough (relative to A) that members of S may freely trade with ~~all~~ at prices p.

<u>Theorem</u> Under the key assumption, an allocation x' (to members of S) is Kaldor preferred to the given (competitive) allocation x iff $p_*(x'_{|S}) > p_*(x_{|S})$ (p_* being the "total value" fn induced by p.)

i.e. nothing matters but the bottom line!

<u>Proof</u>: Suppose that $p_*(x'_{|S}) > p_*(x_{|S})$. Then we can reallocate within S so each $\alpha \in S$ has greater net worth under x' than under x. By the free trade assumption, each such α may freely trade at prices p to reach his/her maximum utility subj. to budget. Call the state thus reached y. Then y is obtained from x' via reallocation, and each $\alpha \in S$ prefers y to x b/c utility (for an individual) increases w. budget constraint.

Conversely, if $p_*(x'_{|S}) < p_*(x_{|S})$ then at least one $\alpha \in S$ has a lower budget under ~~x'~~ any realloc.^{n} of x' than under x. Since ~~x~~ α's alloc. under x is optimal (assumption of a competitive alloc^n), y cannot be Pareto pref. x. ∎

⇒ "Cost benefit analysis"

with $A \cap B = \emptyset$ and B open: then $\exists\ \phi \in W^*$ and a scalar $\lambda \in \mathbb{R}$ s.t.

$$\phi(a) \leq \lambda < \phi(b) \qquad \forall\, a \in A,\ b \in B.$$

<u>Addendum</u> (obvious) suppose that A contains an affine subspace of the form $a + \mathcal{U}$, $\mathcal{U} \leq W$; then $\mathcal{U} \subseteq \ker \phi$.

Let now $W = V^N$ be the space of all possible allocations to the agents $\mathcal{A}$, including those that don't match the initial endowment. Let $w \in W$ be the initial endowment. Let $\mathcal{U}\ (\leq W) = \{t_\alpha \mid \sum_{\alpha \in \mathcal{A}} t_\alpha = 0\}$ be the subspace of possible trades, and $\mathcal{U}^- = \{t_\alpha \mid \sum t_\alpha \leq 0\}$ — wasteful trades. Then $A = w + \mathcal{U}^-$ is the space of (achievable) allocations, allowing waste, and $x \in A$. Let

$$B = \{(y_\alpha) \in W \mid \mathcal{U}_\alpha(y_\alpha) > \mathcal{U}_\alpha(x_\alpha)\}$$

be the open set of alloc^{n}s strictly Pareto preferred by everyone. By assumption, $A \cap B = \emptyset$, and B is convex.

This is so crass, something must be wrong. Possibilities

- positional goods
- b) no market in many contributions to utility/happiness
- c) incommeasurability / tragedy.
- d) externalities / property rights

~~[illegible]~~

The second fundamental theorem

In our exchange economy context the second fundamental th can be stated as follows.

Theorem Consider an exchange economy as described above. Let x be any Pareto-efficient allocation. Then there exist

.... a price vector p, and

.... a list s_α of lump sum redistributions ($\sum s_\alpha = 0$)

such that the allocation x is competitive at prices p, if agent α is initially transferred s_α dollars.

Proof This is just the Hahn-Banach theorem in disguise. The required version of the HBT is the following: let W be a TVS (over $\mathbb{R}$), A, B convex subsets

(7) Thus by Hahn Banach $\exists\ \phi$ s.t. $\phi(a) \le \lambda < \phi(b)$ $\forall a \in A, b \in B$. By the addendum, $\ker\phi \supseteq U$, so ϕ is really a linear fctl on $W/U \cong V$, i.e., it is a price vector p. (Why is p positive? Because U^- is a half space...)

Define the redistributions

$$s_\alpha = p(x_\alpha - w_\alpha).$$

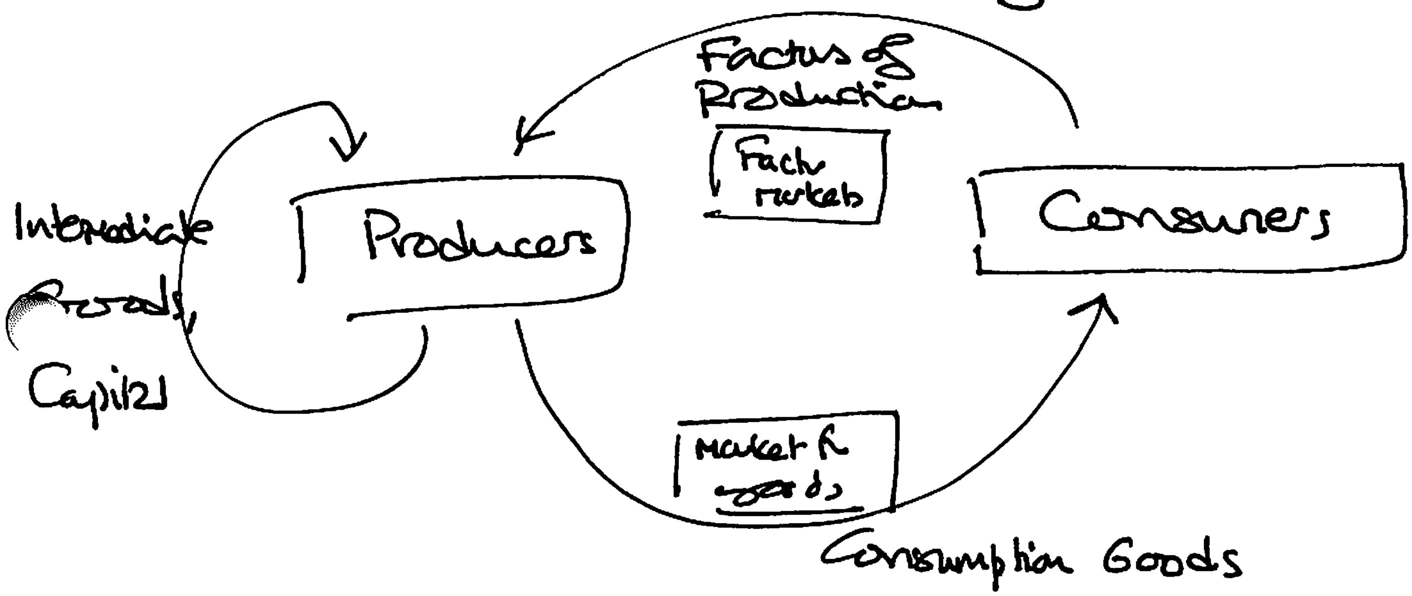
Then after redistribution by s_α, x_α will be the ~~Pareto~~ utility maximizing bundle (subject to budget) by agent α. i.e.

$$x_\alpha = \zeta_\alpha(pw_\alpha + s_\alpha). \qquad \square$$

Production

So far we have not considered at all where various goods arise from. We've just allowed our agents to exchange them among each other. Now we'll begin the analysis of production.

The standard model of the economy



— closed loop / "perpetual motion"

— flows per unit time constant at equilibrium (general Debreu model escapes this assumption..)

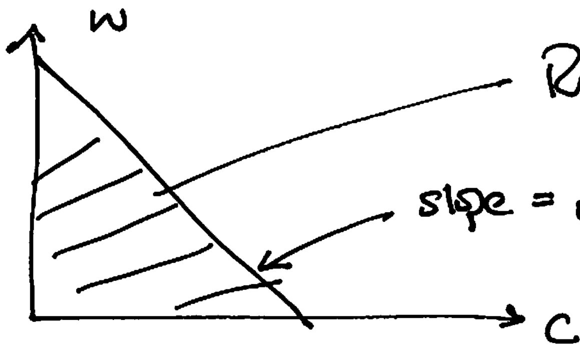
— Capital.... , time / space (Debreu).

(2) Limits on physical possibility constrain what can be achieved on the production side. Defn production set = set of possible productions.

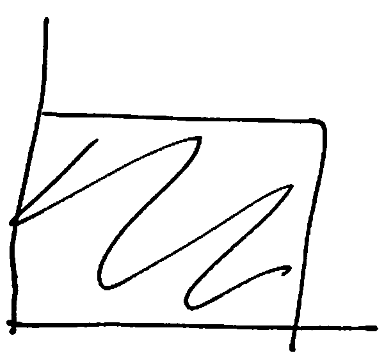
Examples (Ricardo)

National economy can produce only wine and cheese.

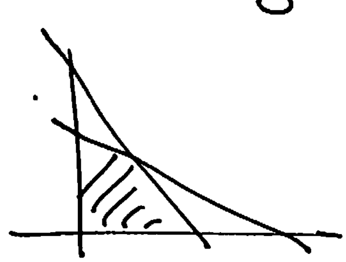
a) Fixed quantity of labor (only factor) freely mobile.



b) Immobile factors: winemakers & cheesemakers.



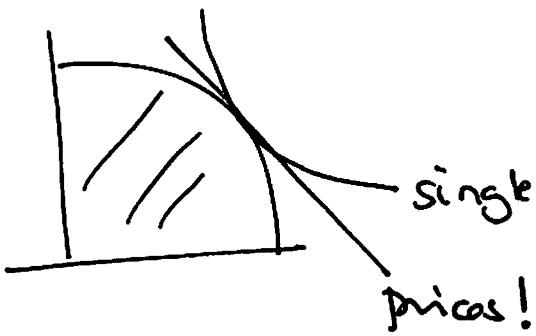
c) Two constraints (e.g. labor and land)



③

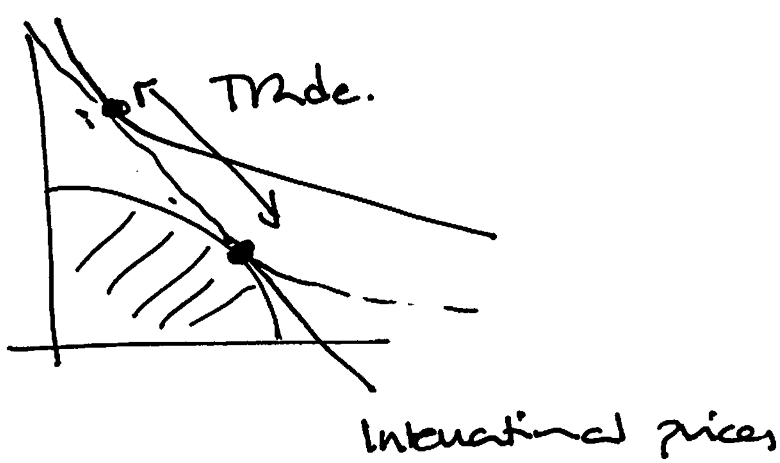
d) in general – convex regi.

What does the economy do..



[w. multiple fns, there will exist a price vector give. competitive allocation ...]

International trade ...



comparative advantage!

discuss 2 nations

Effectively –

(4)

Effectively we have replaced the production set by a simplex containing it (determined by the price vect.)
– cf CAPM, later.

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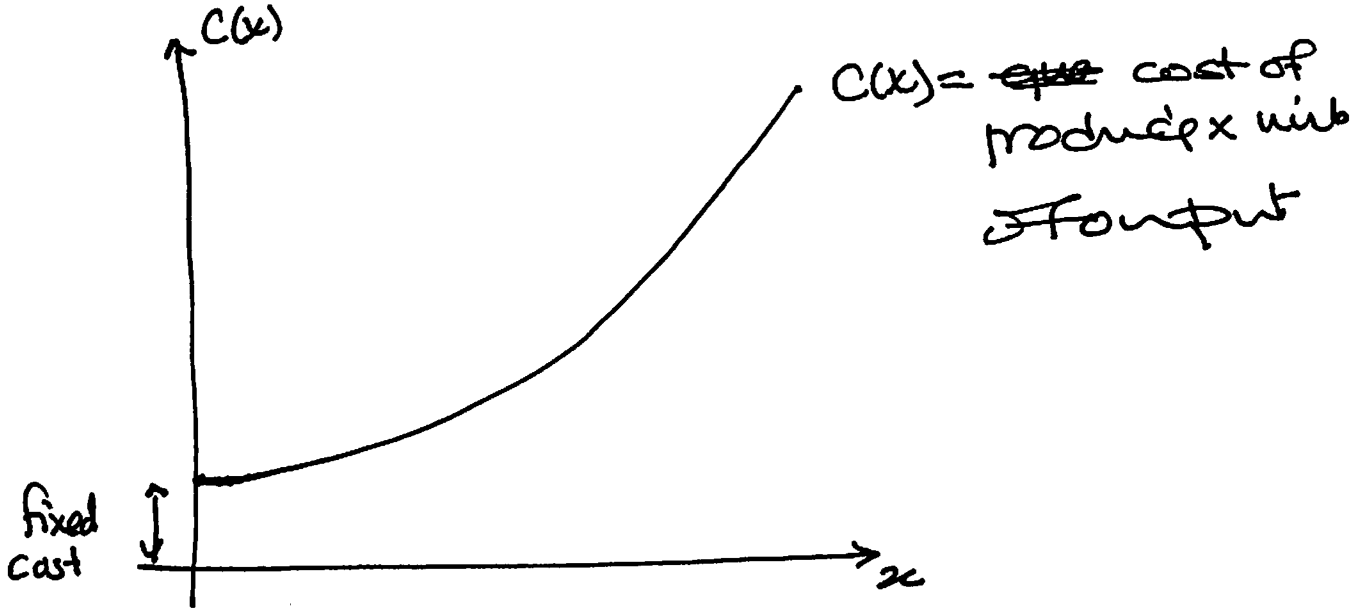
Let's consider a simplified model of what happens from the point of view of an individual producer (firm)



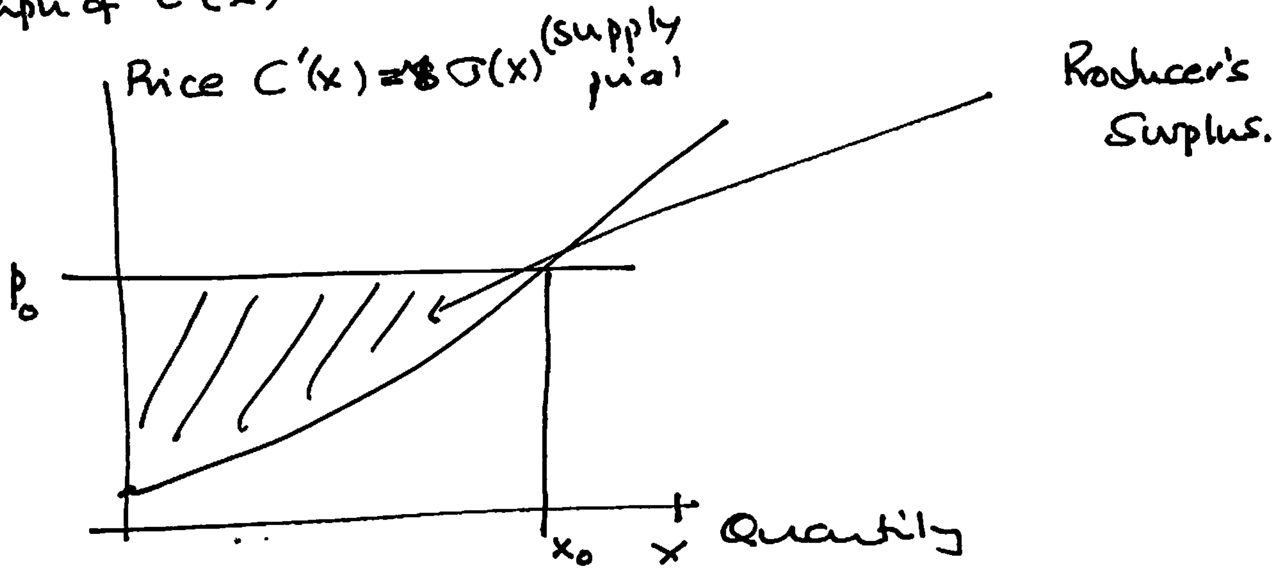
convex upwards = diminishing returns to scale.

Suppose that the company can sell its product for price p_0. What quantity of goods will it produce? Each additional unit of production, δx, increases cost by $C'(x)\delta x$. If this is $< p_0$ then it is worth producing the extra unit; if it is $> p_0$ it would have been better to produce a unit less. Thus, the production level ~~with~~ x_0

(5)

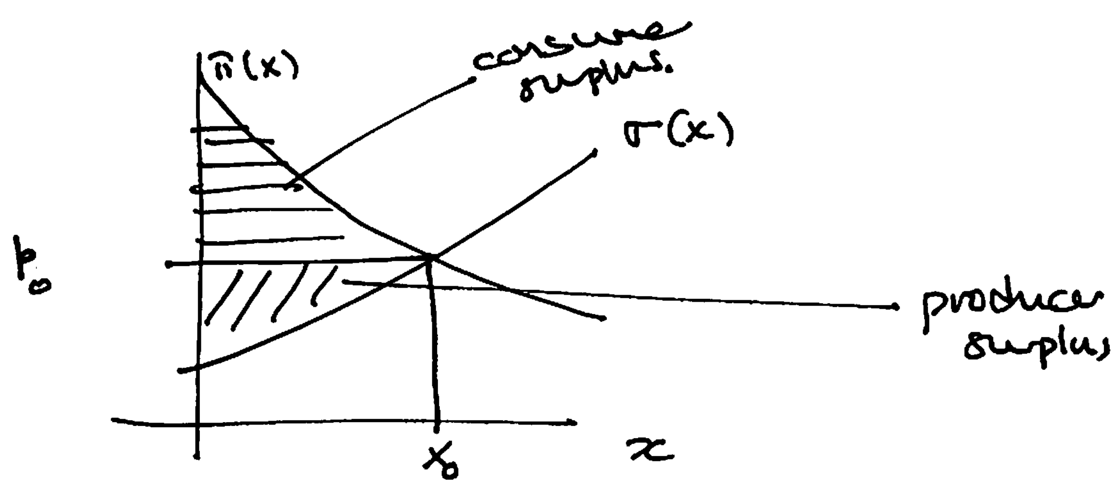
chosen will be s.th. $C'(x_0) = p_0$. (This is the short term analysis). — The price equals the marginal cost of the last unit produced.

This is classically expressed as the supply curve = graph of $C'(x)$

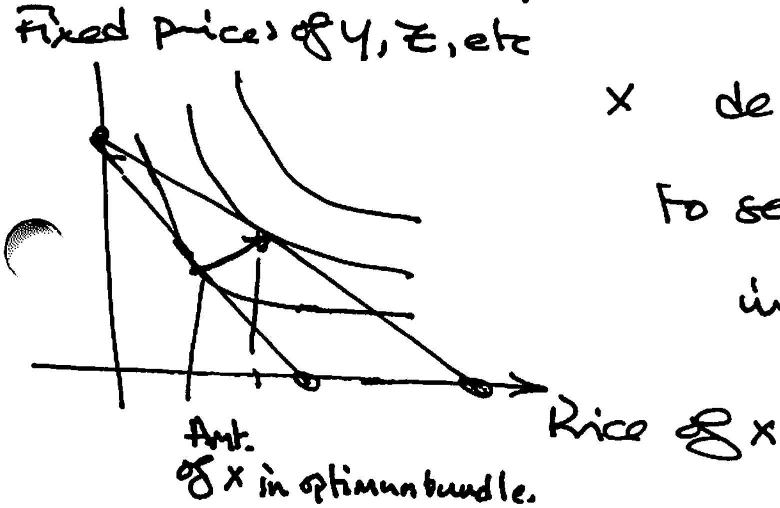


The area $p_0 x_0 - \int_0^{x_0} \sigma(x)\,dx$ is called the producers surplus; it is a measure of the profit in selling all his units at the fixed price p_0 rather than selling each one for its marginal cost. The firm is profitable (at price p_0) if the producers surplus is greater than its fixed cost $C(0)$; otherwise it is unprofitable.

⑥ We are taking the price as fixed because we assume that the scale of the firm is small relative to the scale of the whole industry – i.e. its production has no appreciable effect on prices. But we can also aggregate the supply curves for various producers to get a total supply curve for the whole industry. Now there is also a total demand curve (no longer a horizontal straight line) representing the quantity (other things being equal) that consumers will be willing to purchase at the specified price.



The ~~consumer surplus~~ demand price curve can be derived from consumer preferences. As the price of x decreases, we expect to see more of it demanded in aggregate.



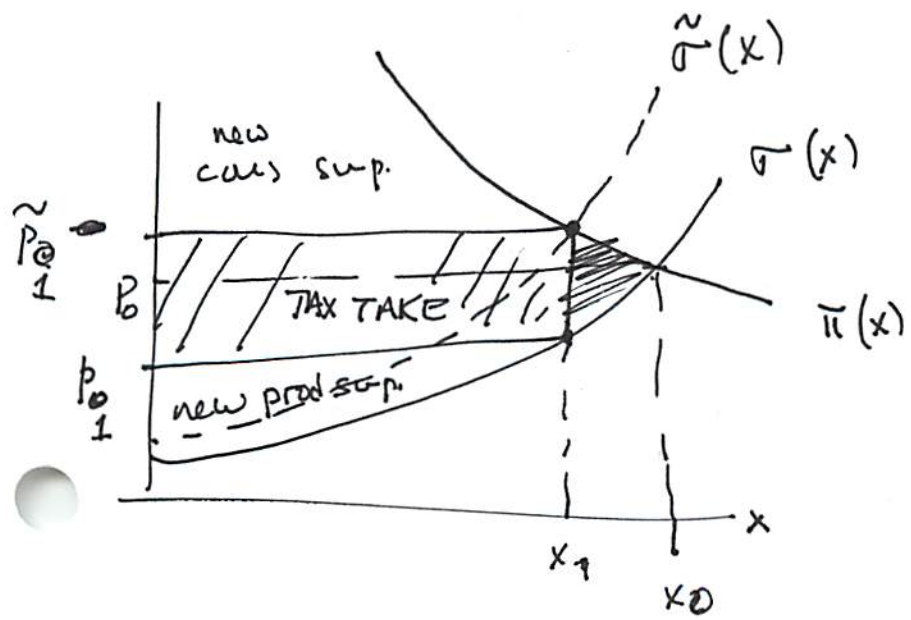
(7) 3 See our earlier comments about inferior goods etc.

Analogous to the "producer surplus", the "consumer surplus" is the notional amount by which consumers have benefited through being able to buy all their stuff at the price p_0 rather than the price they "would have been willing to pay".

— Constant prices are a social convention
— e.g. stock market valuations.

Classical analyses based on this picture

• Sales taxes Suppose that the government imposes a sales tax of 100θ %. The effect of this is that the curve $\sigma(x)$ is shifted up to $(1+\theta)\sigma(x) = \tilde{\sigma}(x)$



Per picture, the price p_1 that the producer receives is ~~exactly~~ different from the price $\tilde{p}_1$ that the consumer pays, with

(8)

$P_1 < P_0 < \tilde{P}_1$. The burden of the tax is divided between producer & consumer (& would have been divided in exactly the same way if a purchase tax of $\frac{\theta}{1+\theta}$ had been imposed on the consumer)...

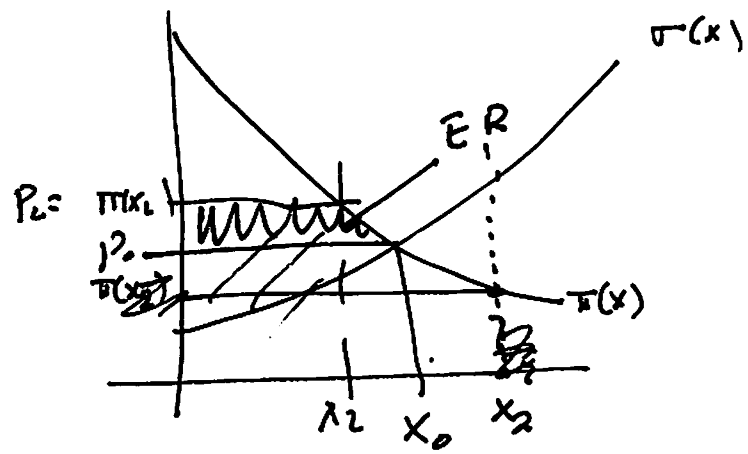
The area $\int_{x_1}^{x_0} (\pi(x) - \sigma(x))\,dx$ represents "net social loss" thought to arise from the tax scheme. (?)

• Competitive industries (classical theory).

Consider the long term future of a single firm producing at $p_0 = C'(x_0)$. If the firm is unprofitable ($x_0 p_0 < C(x_0)$) it will eventually go bust. In the long term this may be expected to reduce supply and cause price to rise. Conversely if the firm is profitable ($x_0 p_0 > C(x_0)$), I can make a profit too by making a clone of your firm, and this will cause prices to fall. (Note unlimited resource assumptions here.) In the long term, then, one expects the industry as a whole to adjust so that profit is zero (the producers surplus equals the fixed costs). The "invisible hand" will allocate all the benefit to the consumer.

Comments....
- cf. hyperbolic dynamics
- failure rates – 25% fail after 1 yr
  - 50% —— 4
  - 70%+ —— 10

What about the opposite — monopoly.



now σ(x) is the supply price for the monopolist only.

Where will the price be set?

Want to maximize $x_2 \pi(x_2) - \int_0^{x_2} \sigma(x)\,dx$

Differentiate & equate to 0: $x_2 \pi'(x_2) + \pi(x_2) - \sigma(x_2) = 0$

i.e. $p_2 = \sigma(x_2) - x_2 \pi'(x_2) > \pi(x_2)$

The economic rent of the producer is the area shaded ER above, i.e. ~~$x_2(p_2 - p_0) + \int_{x_2}$~~ $x_2 p_2 - x_0 p_0 + \int_{x_2}^{x_0} \pi(x)\,dx$

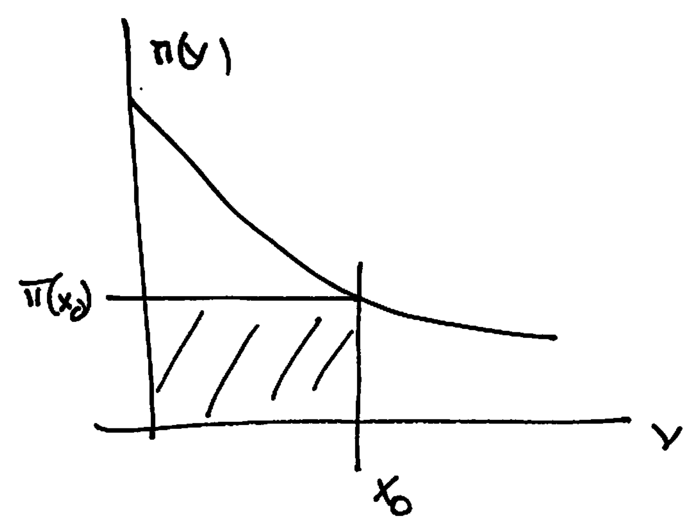
An economic rent is by definition the difference btw. what a producer receives b/c of exerting market power and what it wd. need to receive to remain in "its current occupⁿ", i.e. "its difference over the 'next best option'". Perfect competition should reduce such rents to zero. They present opportunities to tax w/o distorting the market (as, by hypothesis, reducing rent does not affect production decisions)

Duopoly

Let's make a simple analysis of the bottled water industry. US bottled water ~~[illegible]~~ consumption: 8.6 billion gallons per year for 29% of beverage market

Aquafina (Pepsi) and Dasani (Coke) originate from municipal water systems. Approx 50 billion PET bottles per annum in US. 75% end up in landfill or the ocean. Avg. wt. 13.8 gr. Approx 15M. barrels of oil per year ⇒ bottles. Cost ≈ 10^{3-4} times greater than tapwater, pref. in taste tests.

Assume demand price curve for bottled water is $\pi(x)$. ~~Price~~ cost of production = 0.



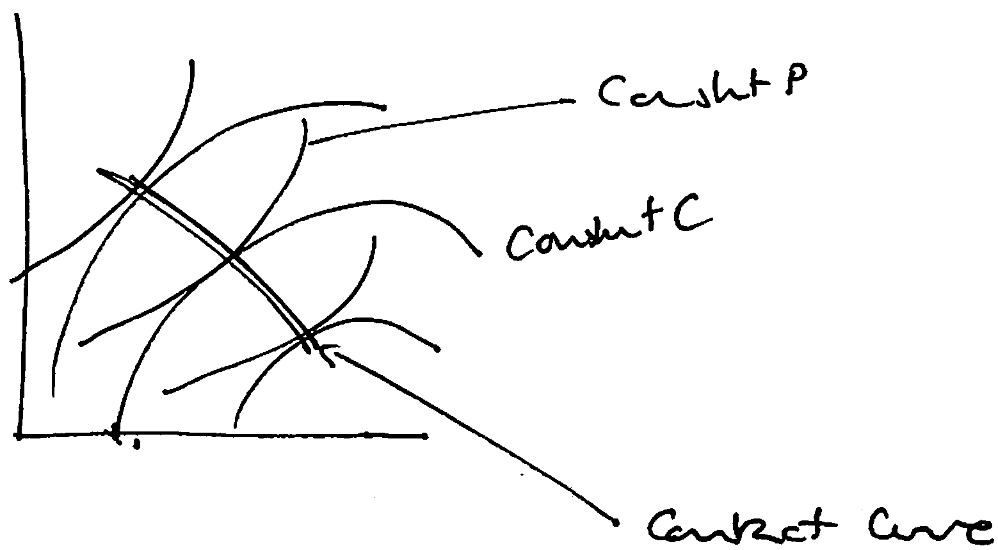
(11) Monopoly price x_0 will maximize $x_0\pi(x_0)$ so that

$$x_0\pi'(x_0) + \pi(x_0) = 0 \quad \ldots \quad (1)$$

Now consider duopoly: 2 producers (Coke & Pepsi) produce x and y respectively. The profit functions are

$$C(x,y) = x\,\pi(x+y)$$
$$P(x,y) = y\,\pi(x+y)$$

What happens? We can make an analysis like the Edgeworth box.



Eq. of cc is $\nabla C \times \nabla P = 0$

: (2)

$$\nabla C = (\pi + x\pi', x\pi')$$

$$\nabla P = (y\pi', \pi + y\pi')$$

Eq. is

$$xy\,\pi'^2 = (\pi + x\pi')(\pi + y\pi')$$

i.e. $\pi^2 + (x+y)\pi\pi' = 0$

or (fact π) $\pi(x+y) + (x+y)\pi'(x+y) = 0.$

Thus the total production gravitates to the monopoly level and consumers obtain no advantage over a monopoly situation. (This is a consequence of the zero cost assumption).