Q1: Does the APC-I model solve the identification problem?

A: The APC-I is fully identified in that it is estimable just as any identified general linear model and does not require additional constraints other than a regular coding scheme. However, this does not mean that it solves or is intended to solve the identification problem inherent in the traditional APC accounting framework. In my view, the identification problem is a blessing because it urges researchers to step back and think what they intend to estimate using the accounting model. Technically, the identification problem arises from the assumption about additive cohort effects that are independent of age and period effects. Substantively, this is equivalent to assuming that cohort differentiation can occur without social changes differentially affecting individuals of different ages. In contrast, the APC-I model explicitly considers cohort effects as age-period interactions based on Ryder's (1965) conceptualization of cohort effects that are implanted in age-period specifications.

Q2: Can the APC-I model be used to recover the linear trend in cohort effects?

A: If "the linear trend in cohort effects" refers to the linear component of the cohort effects in the APC accounting model, then the answer is no. In fact, this is precisely what the APC-I model is designed NOT to do.

I stress that the APC-I model is not intended to recover that kind of cohort effects—linear or nonlinear—defined by the traditional APC accounting model. That is, the cohort effects estimated in the APC-I model naturally differ from those in the accounting model because the two depart from each other on theoretical motivations: the traditional APC accounting model attempts to estimate the additive and independent cohort effects, whereas the APC-I approach operationalized cohort effects as the interactive and dependent effects between age and time periods. The latter better represents, I contend, the sociological and demographic concept of what cohort effects are and when such effects can be observed.

Q3: Some other APC methods focus on the nonlinear cohort effects. How is the APC-I model different from them?

A: A few methods have been proposed to focus on only the nonlinear cohort effects (Chauvel and Schröder 2015; Keyes et al. 2010; O'Brien, Hudson, and Stockard 2008). However, the APC-I model differs from such methods in two important ways: First, such methods were all developed under the APC accounting framework's additive-cohort-effects assumption—that is, cohort effects are assumed to occur independently of social changes and the aging process *and* without differential effects of social changes for individuals of different ages. That is, previous methods do not question the validity of the APC accounting framework's assumption of additive cohort effects and thus offer little theoretical motivation for their operationalization of cohort effects. In contrast, the APC-I approach challenges this questionable and arbitrary assumption. Luo and Hodges (2022) provide a detailed theoretical and conceptual justification for modeling cohort effects as a specific form of the age-by-period interaction. As a result, although under certain circumstances cohort effects based on nonlinear cohort models may be numerically similar to the average cohort deviations estimated in Step 2 of the APC-I model, these statistical quantities have different meanings and interpretations.

Second, the APC-I model allows a researcher to investigate life-course dynamics based on the multiple age-period interaction terms contained in the same cohort diagonal. Nonlinear cohort effect in the APC accounting framework cannot be used for this purpose.

Q4: Because Cohort = Period – Age, the effect of any one of the three variables can be expressed as the interaction between the other two. Why does the APC-I model consider cohort as the age-by-period interaction? Why not period effect as the age-by-cohort interaction?

A: Technically it is true that one could model age or period as the interaction of the other two effects, giving a model fit identical to the one given by the APC-I approach. The APC-I model makes an explicit and theoretically grounded choice to model cohorts using specific forms of the age-by-period interaction. However, the modeling choice ultimately depends on the relative theoretical importance of the three variables for specific research questions. For example, Harding and Jencks (2003) proposed an age-cohort model based on a parsimony criterion. This model may be a viable alternative to the APC-I strategy with the presence of substantial qualitative or cross-over interactions, a circumstance in which the trend in the effect of period has a different direction depending on age and thus poses a challenge for interpreting main effects. More broadly, substantial qualitative age-by-period interactions may imply that a more sensible study design would compare life course trajectories between cohorts because each cohort has a distinct age or period pattern so that there is no general age or period trend. As Abbott (2005:7) argued, "we cannot write a history of periods. We customarily write the history of a population in terms of periods."

Q5: The APC-I model uses model fit statistics to determine if a cohort effect exists. Other scholars also recommended using model fit statistics to determine APC model specification. What are the differences?

A: The idea of using these test statistics in APC analysis is not new. For example, Clayton and Schifflers (1987) recommended using deviance or likelihood-ratio tests to choose among an age-only model, an age-period model, an age-cohort model, or a full age-period-cohort model. Also, Yang (2008) suggested comparing these models, though arguing that certain test outcomes justify using a constrained approach like the intrinsic estimator. However, the purpose of the global deviance test proposed in the APC-I model is neither model selection nor verifying technical constraints on the unknown parameters. Rather, because we consider cohort effects as the interaction between age and period, the global deviance test of the age-by-period interaction in the APC-I model serves as an explicit measure of and necessary condition for cohort effects.

Q6: The numeric values of interactive terms differ across coding schemes. Because the APC-I model contains interaction terms, does it mean that the APC-I is not identified?

A: Some estimators developed under the traditional APC framework (e.g., the intrinsic estimator) give estimates that are dependent on the choice of coding schemes. In other words, the estimates under different coding schemes are not equivalent (see te Grotenhuis et al. 2016; Luo et al. 2016, for a more detailed discussion). The APC-I model does not have a rank deficiency problem in the sense that it does not require more constraints than a usual generalized linear model with main effects and their interactions. For any identified model including the APC-I model, the estimates are equivalent; that is, the estimated cell means are the same for all coding schemes. Although this equivalence holds for both main effects and interaction estimates in the APC-I model, it is less obvious for interaction terms because the interpretation of the specific parameter estimates do change with coding schemes. To illustrate, consider an example of applying the APC-I model to health data with three age categories and three periods, shown in table 1 below. Under dummy coding—for example, the youngest age group 20-24 and the beginning survey period of 2000 are set to zero or omitted as the referent—the interaction for ages 25-29 and period 2005 in cell Y represents the difference in a health outcome between periods 2000 and 2005 for age 25-29 or equivalently, health difference between ages 25-29 and 20-24 for the period 2005. That is, interactions under dummy coding represents a directional difference from a particular reference group.

		Period		
		2000	2005	2010
Age	20-24			
	25-29		Y	
	30-34			

Table 1: Hypothetical data with three age categories and three periods illustrating a shift in meaning and interpretation of interaction terms under different coding schemes. The interaction terms under different types of coding in cell Y necessarily have different numerical values because of different reference groups. For example, the interaction term in cell Y under dummy coding represents a directional difference from a particular reference group (e.g., age 20-24 in year 2000). Under effect coding, the same interaction term in cell Y represents the deviation in the outcome from the age main effect plus period main effect for the group of individuals who were age 25-29 and surveyed in period 2005. Such different numeric values do not arise from an identification problem but rather from a shift in what these quantities represent.

By contrast, under effect coding (i.e., sum-to-zero or the usual ANOVA coding), the same interaction term in cell Y represents the deviation in the health outcome from the age main effect plus period main effect for the group of individuals who were age 25-29 and surveyed in period 2005. The estimated interaction terms under these two types of coding in cell Y thus necessarily have different numerical values. However, this difference does not arise from an identification problem but rather from a shift in what these quantities represent. That is, the two interaction terms can be transformed to be equivalent so that the means in Y after considering age and period main effects are the same under the two coding schemes. We recommend using effect or sum-to-zero coding for estimating the APC-I model for the following reasons: when characterizing cohort effects as a set of age-by-period interactions, we are less concerned about any direction of the interactions; that is, we are not particularly interested in the difference between two cohorts at a particular age or time period. Rather, we focus on particular structures of these interactions that may represent theoretically interesting patterns during a cohort's life span. Effect coding is helpful because they all have the same referent group — the next lower level in the hierarchy of main effects and interactions. That is, we choose effect coding for the purpose of easy interpretation. This is also consistent with the recommendation of coding schemes in the presence of interactions (see Aiken, West, and Reno 1991; Jaccard and Turrisi 2003).

References

Abbott, Andrew. 2005. "The Historicality of Individuals." *Social Science History* 29(1):1–13.

Aiken, Leona S., Stephen G. West, and Raymond R. Reno. 1991. *Multiple Regression: Testing and Interpreting Interactions*. SAGE.

Chauvel, Louis, and Martin Schröder. 2015. "The Impact of Cohort Membership on Disposable Incomes in West Germany, France, and the United States." *European Sociological Review* 31(3):298–311. doi: 10.1093/esr/jcu091.

Clayton, D., and E. Schifflers. 1987. "Models for Temporal Variation in Cancer Rates. II: Age-Period-Cohort Models." *Statistics in Medicine* 6(4):469–81.

te Grotenhuis, Manfred, Ben Pelzer, Liying Luo, and Alexander W. Schmidt-Catran. 2016. "The Intrinsic Estimator, Alternative Estimates, and Predictions of Mortality Trends: A Comment on Masters, Hummer, Powers, Beck, Lin, and Finch." *Demography* 53(4):1245–52. doi: 10.1007/s13524-016-0476-8.

Jaccard, James, and Robert Turrisi. 2003. *Interaction Effects in Multiple Regression*. SAGE.

Keyes, Katherine M., Rebecca L. Utz, Whitney Robinson, and Guohua Li. 2010. "What Is a Cohort Effect? Comparison of Three Statistical Methods for Modeling Cohort Effects in Obesity Prevalence in the United States, 1971–2006." *Social Science & Medicine* 70(7):1100–1108. doi: 10.1016/j.socscimed.2009.12.018.

Luo, Liying, James Hodges, Christopher Winship, and Daniel Powers. 2016. "The Sensitivity of the Intrinsic Estimator to Coding Schemes: Comment on Yang, Schulhofer-Wohl, Fu, and Land." *American Journal of Sociology* 122(3):930–61. doi: 10.1086/689830.

O'Brien, Robert M., Kenneth Hudson, and Jean Stockard. 2008. "A Mixed Model Estimation of Age, Period, and Cohort Effects." *Sociological Methods & Research* 36(3):402–28. doi: 10.1177/0049124106290392.

Ryder, Norman B. 1965. "The Cohort as a Concept in the Study of Social Change." *American Sociological Review* 30(6):843–61. doi: 10.2307/2090964.

Yang, Yang. 2008. "Social Inequalities in Happiness in the United States, 1972 to 2004: An Age-Period-Cohort Analysis." *American Sociological Review* 73(2):204–26.