Cosmic-Ray Anisotropy

– Galactic Cosmic-Ray Puzzles (Forever Mystery?) –

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Phase-Space Distribution I

- cosmic-ray (CR) arrival directions described by phase-space distribution

$$f(t,\mathbf{x},\mathbf{p}) = \underbrace{\phi(t,\mathbf{x},p)/(4\pi)}_{\text{monopole}} + 3\,\widehat{\mathbf{p}}\,\underbrace{\boldsymbol{\Phi}(t,\mathbf{x},p)/(4\pi)}_{\text{dipole}} + \ldots$$

- local CR spectral density $[\mathrm{GeV}^{-1}\mathrm{cm}^{-3}]$

$$n_{\rm CR}(p) = \frac{1}{T_{\rm exp}}\int_0^{T_{\rm exp}} \mathrm{d}t\, p^2 \underbrace{\phi(t,p)}_{\propto\, p^{-(\Gamma_{\rm CR}+2)}} \propto p^{-\Gamma_{\rm CR}}$$

- in the absence of sources, follows Liouville's equation, $\dot{f} = 0$, or

$$\partial_t f + \dot{\mathbf{x}}\nabla_{\mathbf{x}} f + \dot{\mathbf{p}}\nabla_{\mathbf{p}} f = 0$$

- **from here on:** neglect energy losses and assume $p \gg m$
- rotation in regular ($\boldsymbol{\Omega}$) and turbulent ($\boldsymbol{\omega}(\mathbf{x})$) fields with operator $\mathbf{L} \equiv -\,\mathbf{p}\times\nabla_{\mathbf{p}}$:

$$\dot{\mathbf{p}} = -(\boldsymbol{\Omega}+\boldsymbol{\omega}(\mathbf{x}))\times\mathbf{p} \quad \text{and} \quad \dot{\mathbf{p}}\nabla_{\mathbf{p}} \rightarrow -i(\boldsymbol{\Omega}+\boldsymbol{\omega}(\mathbf{x}))\mathbf{L}$$

Phase-Space Distribution II

- splitting the phase space distribution into $f = \langle f \rangle + \delta f$ with the magnetic ensemble-average $\langle f \rangle$ (assuming 3D-isotropic turbulence)

$$\partial_t \langle f \rangle + \hat{\mathbf{p}} \nabla_{\mathbf{x}} \langle f \rangle - i \boldsymbol{\Omega} \mathbf{L} \langle f \rangle = \langle i \boldsymbol{\omega} \mathbf{L} \delta f \rangle$$

+ BGK approximation, [Bhatnagaer, Gross & Krook'54]

$$\langle i \boldsymbol{\omega} \mathbf{L} \delta f \rangle \to \underbrace{-\nu [\langle f \rangle - \phi/(4\pi)]}_{\text{isotropization}}$$

+ dipole approximation,

$$\partial_t \phi + \nabla_{\mathbf{x}} \boldsymbol{\Phi} \simeq 0 \quad \text{and} \quad \partial_t \boldsymbol{\Phi} + (1/3) \nabla_{\mathbf{x}} \phi + \boldsymbol{\Omega} \times \boldsymbol{\Phi} \simeq -\nu \boldsymbol{\Phi}$$

+ diffusion approximation, $\partial_t \boldsymbol{\Phi} \simeq 0$,

$$\underbrace{\partial_t \phi \simeq \nabla_{\mathbf{x}} (\mathbf{K} \nabla_{\mathbf{x}} \phi)}_{\text{diffusion equation}} \quad \text{and} \quad \underbrace{\boldsymbol{\Phi} \simeq -\mathbf{K} \nabla_{\mathbf{x}} \phi}_{\text{Fick's law}}$$

- diffusion tensor $\mathbf{K}$ (in BGK approximation)

$$K_{ij} = \underbrace{\frac{1}{3\nu} \hat{B}_i \hat{B}_j}_{\text{parallel}} + \underbrace{\frac{\nu}{3(\nu^2 + \Omega^2)} (\delta_{ij} - \hat{B}_i \hat{B}_j)}_{\text{perpendicular}} + \underbrace{\frac{\Omega}{3(\nu^2 + \Omega^2)} \epsilon_{ijk} \hat{B}_k}_{\text{drift}}$$

CR Dipole Anisotropy

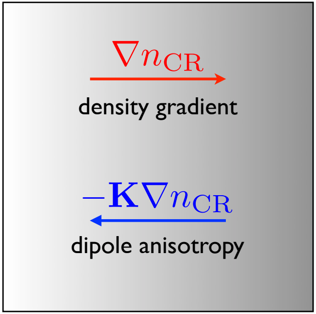
- diffusion tensor $\mathbf{K}$ (general):

$$K_{ij} = \frac{\hat{B}_i\hat{B}_j}{3\nu_{\|}} + \frac{\delta_{ij} - \hat{B}_i\hat{B}_j}{3\nu_{\perp}} + \frac{\epsilon_{ijk}\hat{B}_k}{3\nu_A}$$

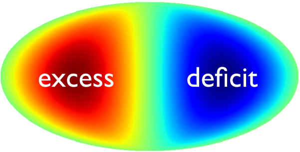
- expected **dipole anisotropy**:

$$\boldsymbol{\delta} \equiv \frac{f_{\max} - f_{\min}}{f_{\max} + f_{\min}} = 3\mathbf{K}\frac{\nabla n_{\rm CR}}{n_{\rm CR}}$$

- amplitude and phase depend on:
  - rigidity dependence of diffusion, $\mathbf{K} \propto \rho^{0.3-0.6}$
  - observational limitations! ➜ Dan's talk
  - (local) source distribution [Erlykin & Wolfendale'06] [Blasi & Amato'12; Sveshnikova *et al.*'13; Pohl & Eichler'13]
  - (local) ordered magnetic field $\mathbf{B}$ [*e.g.* Schwadron *et al.*'14; Mertsch & Funk'14]
  - relative velocity of the medium [Compton & Getting'35]



relative intensity



Compton-Getting Effect

- phase-space distribution is Lorentz-invariant, $f^\star(\mathbf{p}^\star) = f(\mathbf{p})$ [Forman'70]
- Lorentz boost (starred quantities in plasma rest-frame):

$$\mathbf{p}^\star = \mathbf{p} + \left(p + \frac{1}{2}\boldsymbol{\beta}\cdot\mathbf{p}\right)\boldsymbol{\beta} + \mathcal{O}(\beta^3)$$

- Taylor expansion

$$f(\mathbf{p}) \simeq f^\star(\mathbf{p}) + (\mathbf{p}^\star - \mathbf{p})\nabla_{\mathbf{p}^\star} f^\star(\mathbf{p}) + \mathcal{O}(\beta^2) \simeq f^\star(\mathbf{p}) + p\boldsymbol{\beta}\nabla_{\mathbf{p}^\star} f^\star(\mathbf{p}) + \mathcal{O}(\beta^2)$$

➜ splitting in ϕ and $\mathbf{\Phi}$ is not invariant: [Compton & Getting'35;Jones'90]

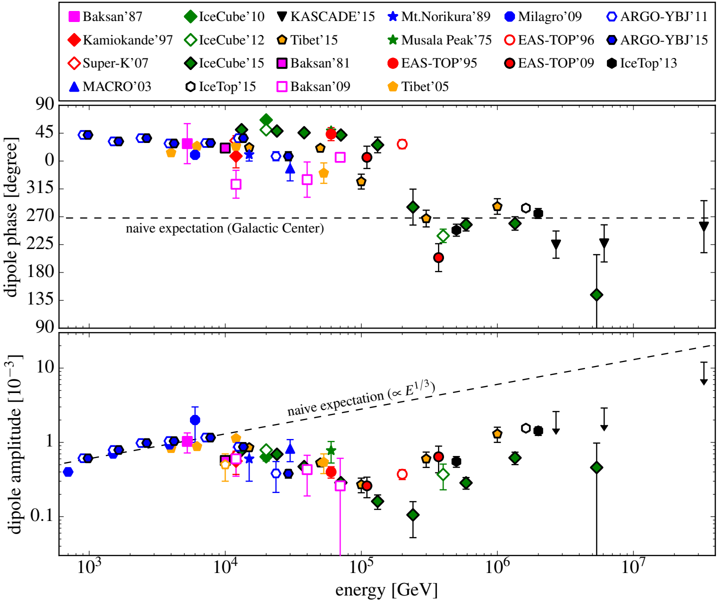
$$\phi = \phi^\star \qquad \mathbf{\Phi} = \mathbf{\Phi}^\star + \frac{1}{3}\boldsymbol{\beta}\frac{\partial\phi^\star}{\partial\ln p}$$

- remember: $\phi \sim p^{-2} n_{\rm CR} \propto p^{-(2+\Gamma_{\rm CR})}$

$$\boldsymbol{\delta} = \boldsymbol{\delta}^\star + \underbrace{(2+\Gamma_{\rm CR})\boldsymbol{\beta}}_{\text{Compton-Getting effect}}$$

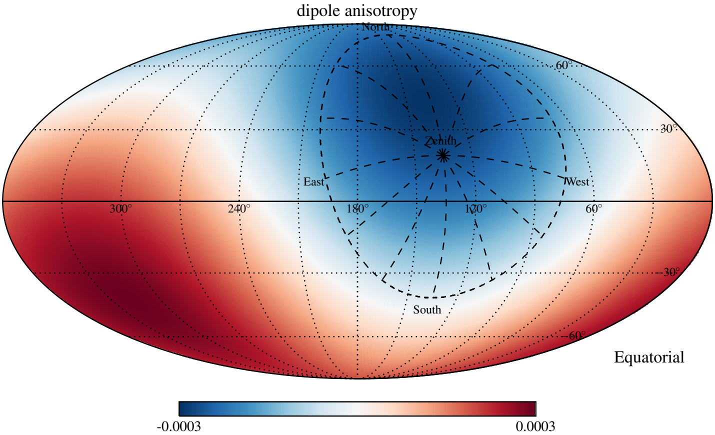
✘ However, what is the correct plasma rest-frame?

Observed Dipole Amplitude and Phase



[MA'16]

Observational Limitations



- ground-based detectors are calibrated by CR data ➜ **reduces anisotropies**
- true CR dipole defined by amplitude A_1, and orientation (RA,DEC) = (α_1,δ_1)

✘ observable only **projected dipole** with amplitude $A_1 \cos\delta_1$ and orientation $(\alpha_1,0)$

✘ further problems by limited field of view (cross-talk with small-scale structure)

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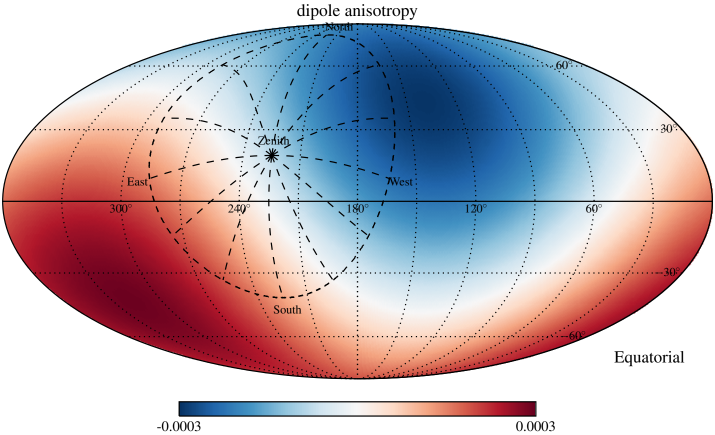

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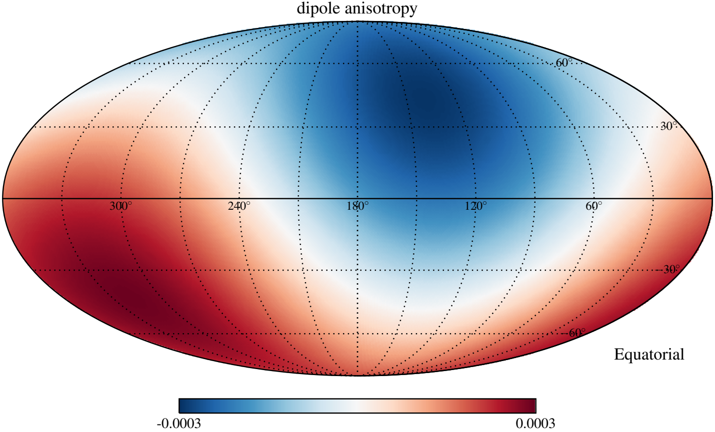

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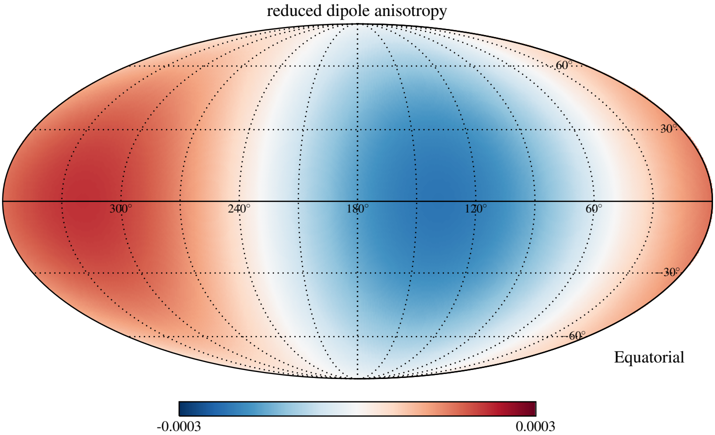

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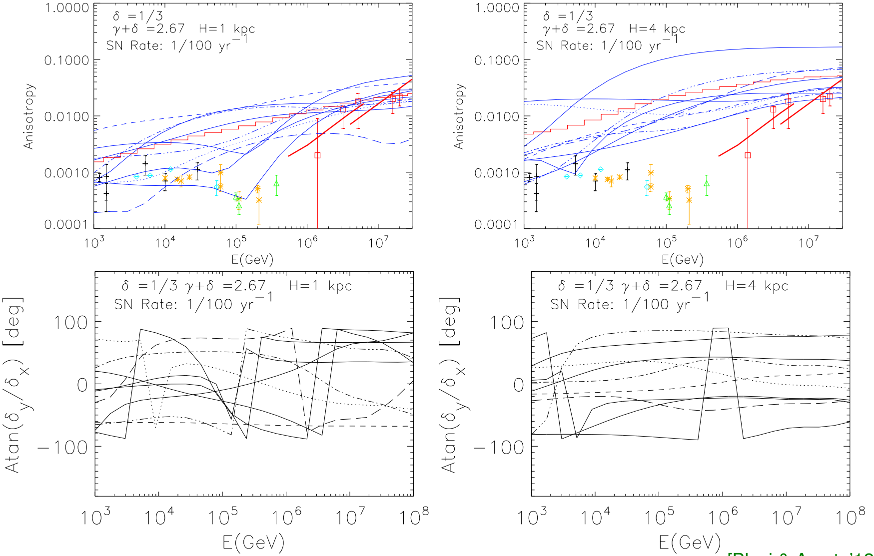
Local Sources



[Blasi & Amato'12]

[Erlykin & Wolfendale'06; Sveshnikova *et al.*'13; Pohl & Eichler'13]

Local Magnetic Field

- strong ordered magnetic fields in the local environment

➜ diffusion tensor reduces to projector:

$$K_{ij} \to \frac{\hat{B}_i \hat{B}_j}{3\nu_\parallel}$$

- TeV–PeV dipole data consistent with magnetic field direction inferred by IBEX data [McComas *et al.*'09]

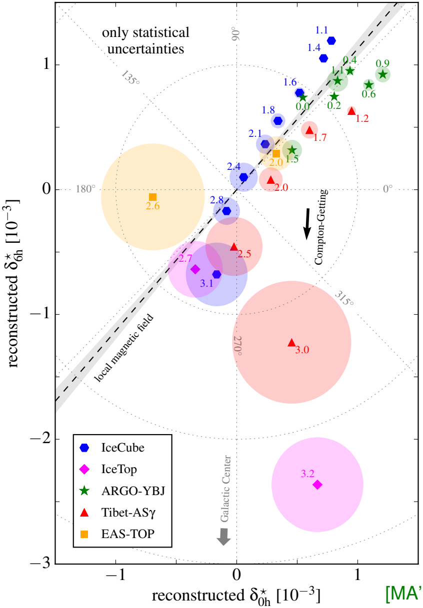
- 1–100 TeV phase indicates a local gradient within longitudes:

$$120^\circ \lesssim l \lesssim 300^\circ$$

- phase flip induced by Vela SNR? [MA'16]

- or a luminous 2Myr old SNR?

[Savchenko, Kachelrieß & Semikoz'15]



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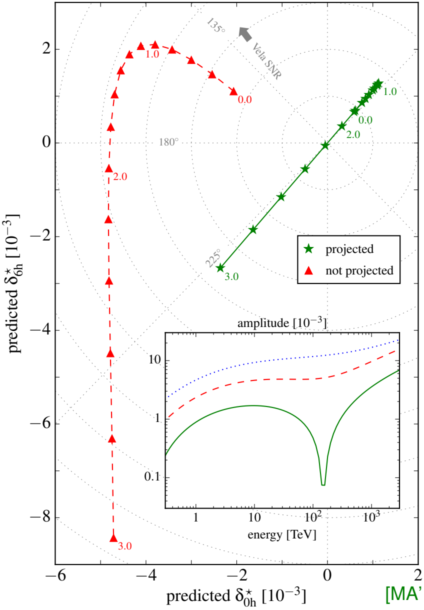
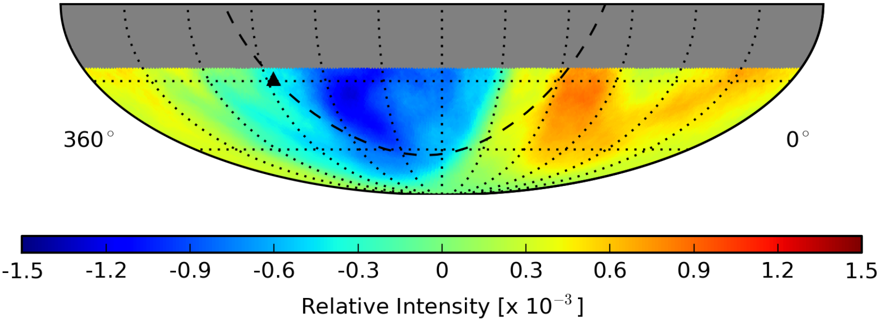


[MA'16]

Map of CR Arrival Directions

➜ Cosmic ray anisotropies up to the level of one-per-mille have been observed at various energies [Super-Kamiokande'07; Milagro'08; ARGO-YBJ'09,'13;EAS-TOP'09] [Tibet AS-γ'05,'06,'15;IceCube'10,'11,'16; HAWC'13,'14]



$E_{\rm CR} \simeq 10$ TeV, $N_{\rm CR} \sim 3.2 \times 10^{11}$ [IceCube (IC59-IC86-IV)'16]

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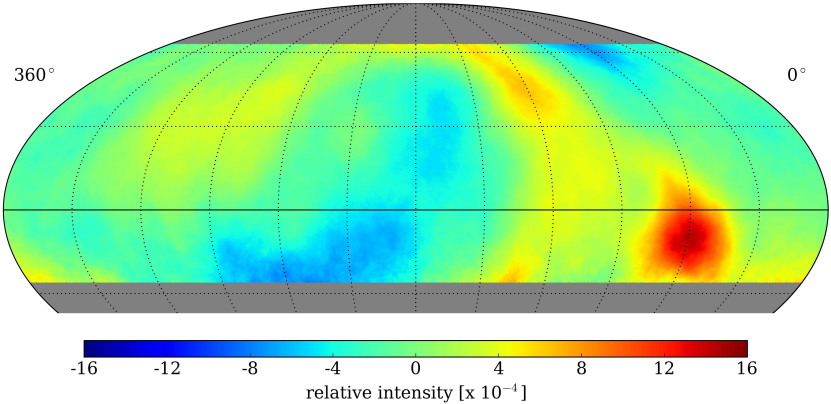


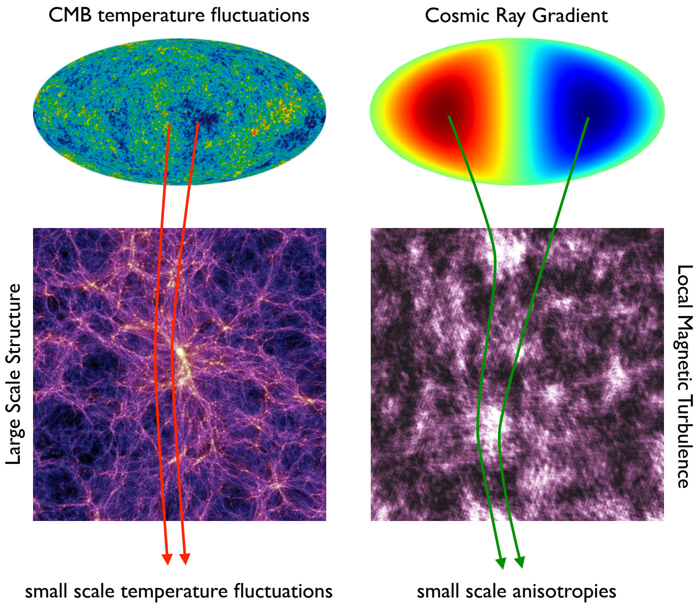
$E_{\rm CR} \simeq 1$ TeV, $N_{\rm CR} \sim 4.9 \times 10^{10}$ [HAWC'14]

Suggested Origin of Small-Scale Anisotropy

- local magnetic field structure with energy-dependent magnetic mirror leakage [Drury & Aharonian'08]
- preferred CR transport directions [Malkov, Diamond, Drury & Sagdeev'10]
- magnetic reconnections in the heliotail [Lazarian & Desiati'10]
- non-isotropic particle transport in the heliosheath [Desiati & Lazarian'11]
- heliospheric electric field structure [Drury'13]
- magnetized outflow from old SNRs [Biermann, Becker, Seo & Mandelartz'12]
- strangelet production in molecular clouds or neutron stars [Kotera, Perez-Garcia & Silk '13]

➜ small-scale anisotropies from local magnetic field mapping of a global dipole [Giacinti & Sigl'12; MA'14; MA & Mertsch'15]

Analogy to Gravitational Lensing



Gedanken Experiment

- **Idea:** local realization of magnetic turbulence introduces small-scale structure
  [Giacinti & Sigl'11]

- Particle transport in (static) magnetic fields is governed by Liouville's equation of the CR's phase-space distribution f:

$$\dot{f}(t, \mathbf{x}, \mathbf{p}) = 0$$

- "trivial" solution:

$$f(0, \mathbf{0}, \mathbf{p}) = f(-T, \mathbf{x}(-T), \mathbf{p}(-T))$$

- *Gedanken Experiment*:
  Assume that at look-back time $-T$ initial condition is **homogenous, but not isotropic**:

$$f(0, \mathbf{0}, \mathbf{p}) = \widetilde{f}(\mathbf{p}(-T))$$

Angular Power Spectrum

- Every smooth function $g(\theta,\phi)$ on a sphere can be decomposed in terms of spherical harmonics Y_m^ℓ:

$$g(\theta,\phi) = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} a_{\ell m} Y_\ell^m(\theta,\phi) \qquad \leftrightarrow \qquad a_{\ell m} = \int \mathrm{d}\Omega (Y_\ell^m)^*(\theta,\phi) g(\theta,\phi)$$

- **angular power spectrum**:

$$C_\ell = \frac{1}{2\ell+1}\sum_{m=-\ell}^{\ell} |a_{\ell m}|^2$$

- related to the **two-point auto-correlation function**: ($\mathbf{n}_{1/2}$: unit vectors, $\mathbf{n}_1 \cdot \mathbf{n}_2 = \cos\eta$)

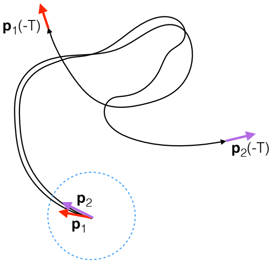
$$\xi(\eta) = \frac{1}{8\pi^2}\int \mathrm{d}\mathbf{n}_1 \int \mathrm{d}\mathbf{n}_2 \delta(\mathbf{n}_1\mathbf{n}_2 - \cos\eta) g(\mathbf{n}_1) g(\mathbf{n}_2) = \frac{1}{4\pi}\sum_\ell (2\ell+1) C_\ell P_\ell(\cos\eta)$$

➜ Note that individual C_ℓ's are **independent** of coordinate system (assuming full sky coverage).

Gedanken Experiment (continued)

- Initial configuration has power spectrum $\widetilde{C}_\ell$.
- For small correlation angles η flow remains correlated even beyond scattering sphere.
- Correlation function for $\eta = 0$:

$$\xi(0) = \frac{1}{4\pi}\int \mathrm{d}\hat{\mathbf{p}}_1 \widetilde{f}^{\,2}(\mathbf{p}_1(-T))$$



- On **average**, the rotation in an *isotropic* random rotation in the turbulent magnetic field leaves an isotropic distribution on a sphere **invariant**:

$$\langle \xi(0)\rangle = \frac{1}{4\pi}\int \mathrm{d}\hat{\mathbf{p}}_1 \widetilde{f}^{\,2}(\mathbf{p}_1)$$

➜ The weighted sum of $\langle C_\ell\rangle$'s remains constant:

$$\frac{1}{4\pi}\sum_{\ell\geq 0}(2\ell+1)\widetilde{C}_\ell = \frac{1}{4\pi}\sum_{\ell\geq 0}(2\ell+1)\langle C_\ell(T)\rangle$$

Evolution Model

- Diffusion theory motivates that each $\langle C_\ell \rangle$ decays exponentially with an effective **relaxation rate** [Yosida'49]

$$\nu_\ell \propto \mathbf{L}^2 \propto \ell(\ell+1)$$

- A **linear** $\langle C_\ell \rangle$ evolution equation with generation rates $\nu_{\ell\to\ell'}$ requires:

$$\partial_t \langle C_\ell \rangle = -\nu_\ell \langle C_\ell \rangle + \sum_{\ell' \geq 0} \nu_{\ell'\to\ell} \frac{2\ell'+1}{2\ell+1} \langle C_{\ell'} \rangle \quad \text{with} \quad \nu_\ell = \sum_{\ell'\geq 0} \nu_{\ell\to\ell'}$$

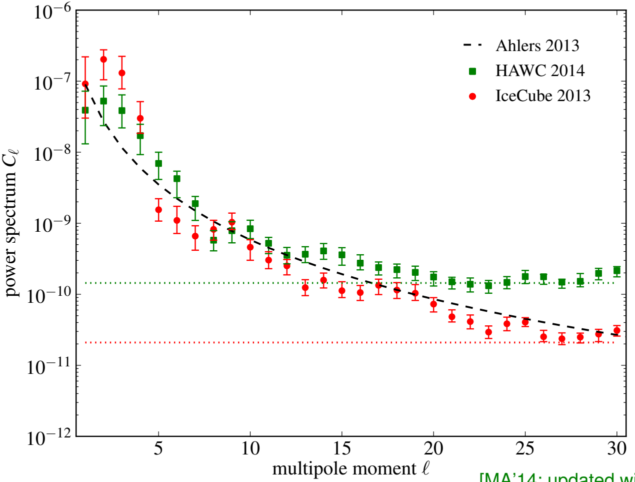
- For $\nu_\ell \simeq \nu_{\ell\to\ell+1}$ and $\widetilde{C}_\ell = 0$ for $l \geq 2$ this has the analytic solution:

$$\langle C_\ell \rangle(T) \simeq \frac{3\widetilde{C}_1}{2\ell+1} \prod_{m=1}^{\ell-1} \nu_m \sum_n \prod_{p=1(\neq n)}^{\ell} \frac{e^{-T\nu_n}}{\nu_p - \nu_n}$$

- For $\nu_\ell \simeq \ell(\ell+1)\nu$ we arrive at a finite asymptotic ratio:

$$\lim_{T\to\infty} \frac{\langle C_\ell \rangle(T)}{\langle C_1 \rangle(T)} \simeq \frac{18}{(2\ell+1)(\ell+2)(\ell+1)}$$

Comparison with CR Data



[MA'14; updated with HAWC data]

$$\lim_{T\to\infty}\frac{\langle C_\ell\rangle(T)}{\langle C_1\rangle(T)} \simeq \frac{18}{(2\ell+1)(\ell+2)(\ell+1)}$$

Multipole Cross-Talk

- relative intensity

$$I(\alpha,\delta) = 1 + \sum_{\ell\geq 1}\sum_{m\neq 0} a_{\ell m} Y_{\ell m}(\alpha, \pi/2-\delta)$$

- dipole: $a_{1-1} = (\delta_{0\mathrm{h}} + i\delta_{6\mathrm{h}})\sqrt{2\pi/3}$ and $a_{11} = -a^*_{1-1}$
- **traditional dipole analyses** extract amplitude "A_1" and phase "α_1" from data projected into right ascension ($s_{1/2} \equiv \sin\delta_{1/2}$)

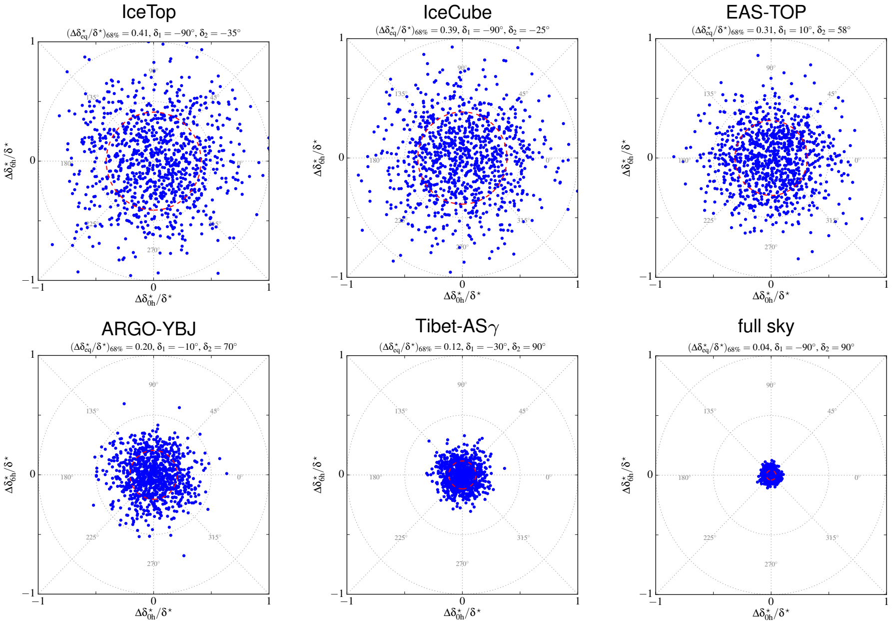
$$A_1 e^{i\alpha_1} = \frac{1}{\pi}\int_0^{2\pi} \mathrm{d}\alpha e^{i\alpha} \underbrace{\frac{1}{s_2-s_1}\int_{s_1}^{s_2} \mathrm{d}\sin\delta\, I(\alpha,\delta)}_{\text{projection}}$$

- the presence of high-ℓ multipole moments introduces **cross-talk**:

$$A_1 e^{i\alpha_1} = \sum_\ell \frac{1}{s_1-s_2}\sqrt{\frac{(2\ell+1)}{\pi\ell(\ell+1)}}\int_{s_1}^{s_2} \mathrm{d}s\, P^1_\ell(s) a_{\ell-1}$$

➜ Can now estimate the **systematic uncertainties** of dipole measures from dipole-induced small-scale power spectrum.

Systematic Uncertainty of CR Dipole



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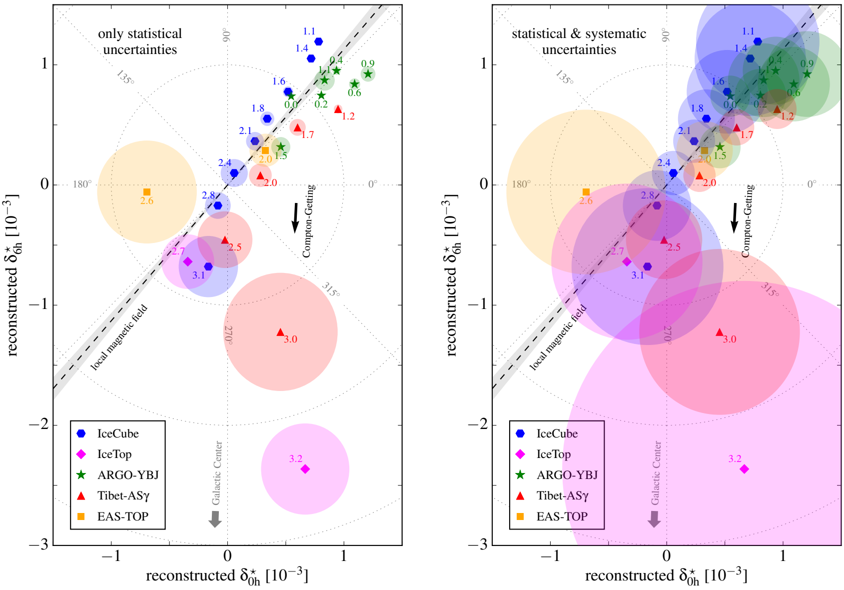


Simulation via Backtracking

- Consider a local (quasi-)stationary solution of the **diffusion approximation**:

$$4\pi\langle f\rangle \simeq \phi + \underbrace{\mathbf{r}\nabla\phi - 3\,\widehat{\mathbf{p}}\,\mathbf{K}\nabla\phi}_{\text{1st linear correction}}$$

- backtracking over long time-scales T and evaluating in mean field

$$4\pi f_i \simeq 4\pi\,\underbrace{\delta f(-T,\mathbf{r}_i(-T),\mathbf{p}_i(-T))}_{\text{deviation from }\langle f\rangle} + \phi + [\mathbf{r}_i(-T) - 3\hat{\mathbf{p}}_i(-T)\mathbf{K}]\nabla\phi$$

- Ensemble-averaged C_ℓ's ($\ell \geq 1$): [MA & Mertsch'15]

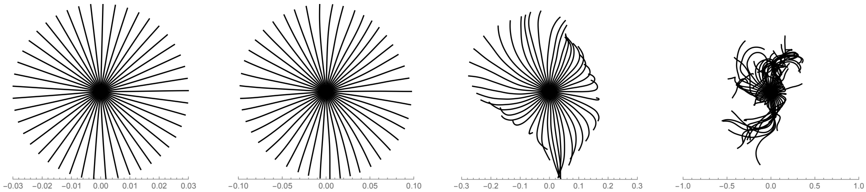
$$\frac{\langle C_\ell\rangle}{4\pi} \simeq \int\frac{\mathrm{d}\hat{\mathbf{p}}_1}{4\pi}\int\frac{\mathrm{d}\hat{\mathbf{p}}_2}{4\pi}P_\ell(\hat{\mathbf{p}}_1\hat{\mathbf{p}}_2)\lim_{T\to\infty}\underbrace{\langle\mathbf{r}_{1i}(-T)\mathbf{r}_{2j}(-T)\rangle}_{\textit{relative}\text{ diffusion}}\frac{\partial_i n_{\rm CR}\partial_j n_{\rm CR}}{n_{\rm CR}^2}$$

- Sum of $\langle C_\ell\rangle$ **related to diffusion tensor**:

$$\frac{1}{4\pi}\sum_{\ell\geq 0}(2\ell+1)\langle C_\ell\rangle \simeq 2TK^s_{ij}\frac{\partial_i n_{\rm CR}\partial_j n_{\rm CR}}{n_{\rm CR}^2}$$

Simulation via Backtracking

- CR arrival direction determined by backtracking of CRs towards a **homogeneous initial dipole anisotropy** (ballistic ➜ laminar ➜ turbulent)



- Kolmogorov turbulence with energy density comparable to $\mathbf{B}_0$
- two cases: dipole vector aligned with or perpendicular to $\mathbf{B}_0$
- asymptotically **limited by simulation noise**:

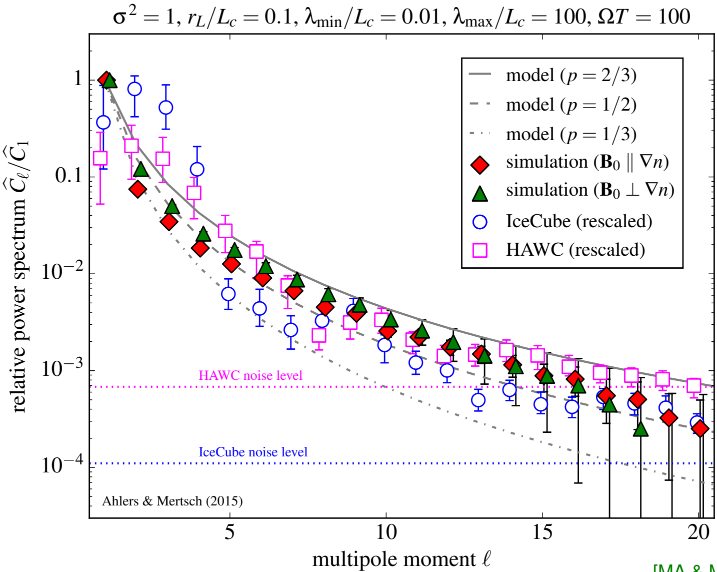
$$\mathcal{N} \simeq \frac{4\pi}{N_{\rm pix}} 2TK^s_{ij} \frac{\partial_i n_{\rm CR} \partial_j n_{\rm CR}}{n^2_{\rm CR}}$$

Movie 1 (link): Sample map of $z(-T)$ with $\mathbf{B} = B_0\mathbf{e}_z$ (monopole & dipole removed)

Movie 2 (link): Power spectrum convergence for $\nabla n_{\rm CR} \parallel \mathbf{B}$.

Movie 3 (link): Power spectrum convergence for $\nabla n_{\rm CR} \perp \mathbf{B}$.

Comparison to CR Data



[MA & Mertsch'15]

Summary

- **Large scale anisotropy** can be understood in the context of standard diffusion theory:
  - Dipole phase aligns with local ordered magnetic field.
  - Amplitude variations as a result of local sources (**Vela?**).
  - ➜ Need better data (reconstruction & analysis methods).

- **Small-scale anisotropy** can be a result of local magnetic turbulence:
  - Effect analogous to induced high-ℓ multipoles in CMB temperature power spectrum from **gravitational lensing** in small-scale structure.
  - Ensemble-averaged C_ℓ's ($\ell \geq 1$) related to **relative diffusion**:

$$\frac{\langle C_\ell \rangle}{4\pi} \simeq \int \frac{d\hat{\mathbf{p}}_1}{4\pi} \int \frac{d\hat{\mathbf{p}}_2}{4\pi} P_\ell(\hat{\mathbf{p}}_1\hat{\mathbf{p}}_2) \lim_{T\to\infty} \underbrace{\langle \mathbf{r}_{1i}(-T)\mathbf{r}_{2j}(-T)\rangle}_{\textit{relative}\text{ diffusion}} \frac{\partial_i n_{\rm CR} \partial_j n_{\rm CR}}{n_{\rm CR}^2}$$

  - **issues:** damping effects for CR rigidity distributions, partial sky coverage of experiments, angular resolution, noise. . .

Thank you for your attention!

Appendix

Local Description: Relative Scattering

- evolution of C_ℓ's: [MA & Mertsch'15]

$$\partial_t \langle C_\ell \rangle = -\frac{1}{2\pi} \int \mathrm{d}\hat{\mathbf{p}}_1 \int \mathrm{d}\hat{\mathbf{p}}_2 P_\ell(\hat{\mathbf{p}}_1 \hat{\mathbf{p}}_2) \langle (\mathbf{p}_1 \nabla f_1 + i\boldsymbol{\omega} \mathbf{L} f_1) f_2 \rangle$$

- large-scale dipole anisotropy gives an effective "source term":

$$-\frac{1}{2\pi} \int \mathrm{d}\hat{\mathbf{p}}_1 \int \mathrm{d}\hat{\mathbf{p}}_2 P_\ell(\hat{\mathbf{p}}_1 \hat{\mathbf{p}}_2) \langle (\mathbf{p}_1 \nabla f_1) f_2 \rangle \to Q_1 \delta_{\ell 1}$$

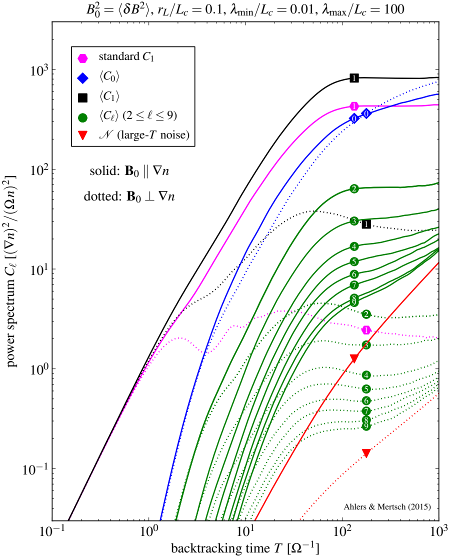
- BGK-like Ansatz for scattering term ($\langle i\boldsymbol{\omega} \mathbf{L} f \rangle \to -\frac{\nu}{2} \mathbf{L}^2 \langle f \rangle$) [Bhatnagaer, Gross & Krook'54]

$$-\frac{1}{2\pi} \int \mathrm{d}\hat{\mathbf{p}}_1 \int \mathrm{d}\hat{\mathbf{p}}_2 P_\ell(\hat{\mathbf{p}}_1 \hat{\mathbf{p}}_2) \langle (i\boldsymbol{\omega} \mathbf{L} f_1) f_2 \rangle \to \frac{1}{2\pi} \int \mathrm{d}\hat{\mathbf{p}}_1 \int \mathrm{d}\hat{\mathbf{p}}_2 P_\ell(\hat{\mathbf{p}}_1 \hat{\mathbf{p}}_2) \tilde{\nu}(\hat{\mathbf{p}}_1 \hat{\mathbf{p}}_2) \mathbf{L}^2 \langle f_1 f_2 \rangle$$

- Note that $\tilde{\nu}(1) = 0$ for vanishing regular magnetic field.

$$\tilde{\nu}(x) \simeq \nu_0 (1 - x)^p$$

Convergence of Power Spectrum



[MA & Mertsch'15]

Simulated Turbulence

- 3D-isotropic turbulence: [Giacalone & Jokipii'99]

$$\delta\mathbf{B}(\mathbf{x}) = \sum_{n=1}^{N} A(k_n)(\mathbf{a}_n \cos\alpha_n + \mathbf{b}_n \sin\alpha_n)\cos(\mathbf{k}_n\mathbf{x} + \beta_n)$$

- α_n and β_n are random phases in $[0, 2\pi)$, unit vectors $\mathbf{a}_n \propto \mathbf{k}_n \times \mathbf{e}_z$ and $\mathbf{b}_n \propto \mathbf{k}_n \times \mathbf{a}_n$
- with amplitude

$$A^2(k_n) = \frac{2\sigma^2 B_0^2 G(k_n)}{\sum_{n=1}^{N} G(k_n)} \qquad \text{with} \qquad G(k_n) = 4\pi k_n^2 \frac{k_n \Delta \ln k}{1 + (k_n L_c)^\gamma}$$

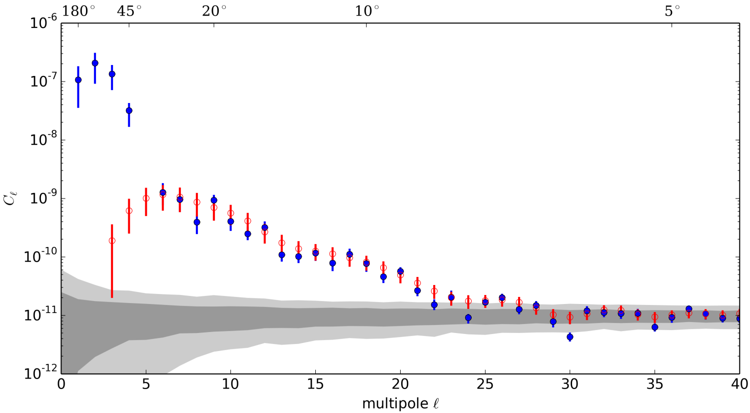
- Kolmogorov-type turbulence: $\gamma = 11/3$
- $N = 160$ wavevectors $\mathbf{k}_n$ with $|\mathbf{k}_n| = k_{\min} e^{(n-1)\Delta \ln k}$ and $\Delta \ln k = \ln(k_{\max}/k_{\min})/N$
- $\lambda_{\min} = 0.01 L_c$ and $\lambda_{\max} = 100 L_c$ [Fraschetti & Giacalone'12]
- rigidity: $r_L = 0.1 L_c$
- turbulence level: $\sigma^2 = \mathbf{B}_0^2/\langle\delta\mathbf{B}^2\rangle = 1$

Powerspectrum of CR Arrival Directions

➜ Cosmic ray anisotropies up to the level of one-per-mille have been observed at various energies

[Tibet AS-γ'05,'06; Super-Kamiokande'07; Milagro'08; ARGO-YBJ'09,'13;EAS-TOP'09]

[IceCube'10,'11; HAWC'13,'14]



[IceCube [arXiv:1603.01227]]

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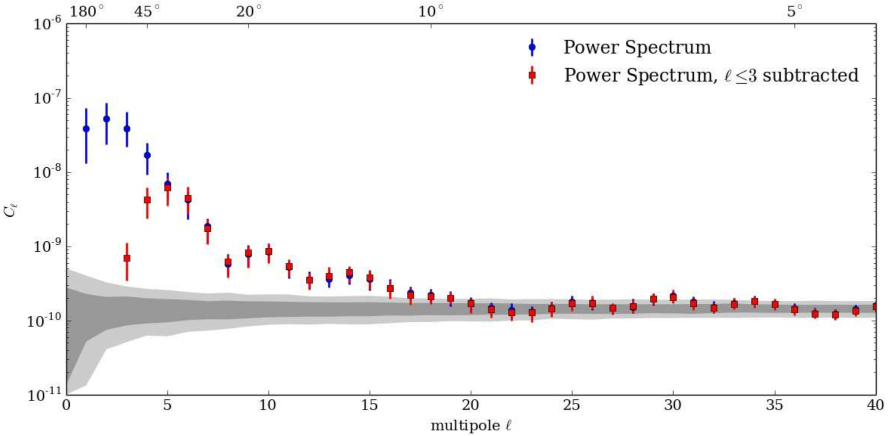

[HAWC [arXiv:1408.4805]; note: low$-\ell$ power under-estimated]