Penn State RET in Interdisciplinary Materials

Teacher's Preparatory Guide

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Structural Color Resulting from Total Internal Reflection

Purpose This lab is designed to help students understand the behavior of light as waves, including interference effects and angles of reflection. The lecture and associated lab will introduce students to the concept of total internal reflection, which is a result of Snell's law. The lecture will expand this topic to explain how total internal reflection causes interference effects of exiting light, yielding structural color.

Objectives Introduce students to Snell's law, total internal reflection, wave-particle duality, interference effects, structural color.

Time required ≈ 2 hours (30-minute prelab lecture, 30-minute lab, 45-minute post lab lecture, 15-minute demonstration)

Level High school

National Science Education Standards [Grades 11-12]

Content Standard A
- Abilities necessary to do scientific inquiry

Content Standard B
- Structure and properties of matter
- Chemical reactions

Teacher Background Basics of optics. This lecture and lab are most useful for a course/lesson in classical optics, particularly after introducing the critical angle that is calculated using Snell's law. The following sources may be useful:
https://pubs.rsc.org/en/content/articlelanding/2013/ra/c3ra41096j
https://en.wikipedia.org/wiki/Structural_coloration
https://medium.com/@rawwerks/what-is-structural-color-536ee6fe46d4
https://en.wikipedia.org/wiki/Snell%27s_law

Lecture Notes:

Pre-Lab Lecture

Refractive Index (Index of Refraction) is **a value calculated from the ratio of the speed of light in a vacuum to that in a second medium of greater density.**

Common refractive indices: (Note: The water and crown glass indices will be calculated during the lab)
air, 1.0003; water, 1.333; crown glass,1.517; dense flint glass, 1.655; and diamond, 2.417.

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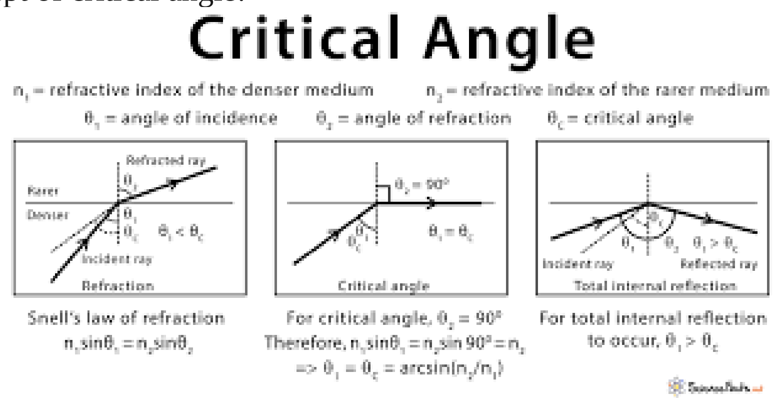
Introduce Snell's Law

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Equation may be rearranged to solve for unknown refractive index. (Part 1 of Lab)

$$n_2 = \frac{n_1 \sin \theta_1}{\sin \theta_2}$$

Introduce concept of critical angle:



The critical angle differentiates refraction from reflection. Refraction refers to the change in direction of light when propagating through a material with a different refractive index. Reflection refers to the light bouncing off the surface and reflecting with various trajectories.

Fun fact: When travelling between two points, even through multiple refractive indices, light will always take the path that leads to traversing from point A to point B in the least amount of time. This is called Fermat's Principle and is a classic example of the *spookiness* associated with quantum phenomena. Photons are weird!

Critical angle formula: (Part 2 of Lab)

$\sin \theta_1 = \frac{n_2 \sin \theta_2}{n_1}$ where $\sin \theta_2 = 90°$

$$\Rightarrow \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

LAB DESCRIBED BELOW (SEE LAB DESCRIPTION)

<u>Post-Lab Lecture</u>

Wave interference occurs when two or more waves make contact and combine to form a new wave. The resulting wave is the net combination of the previous waves.

Wave interference can be constructive or destructive.

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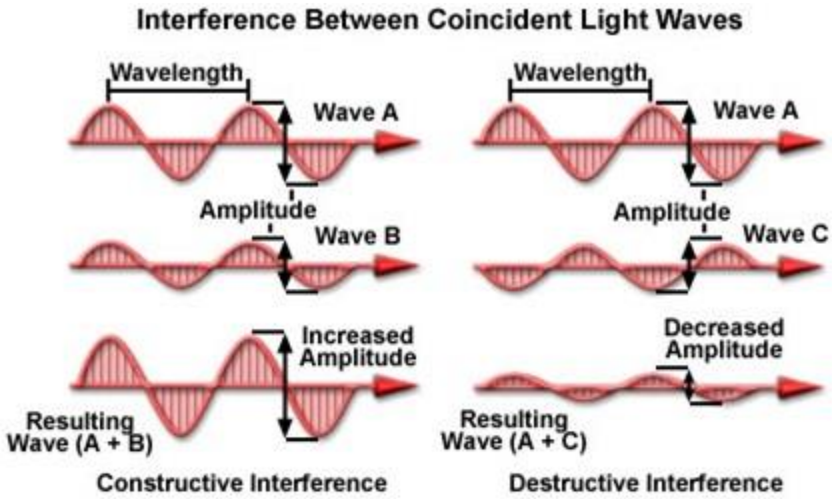
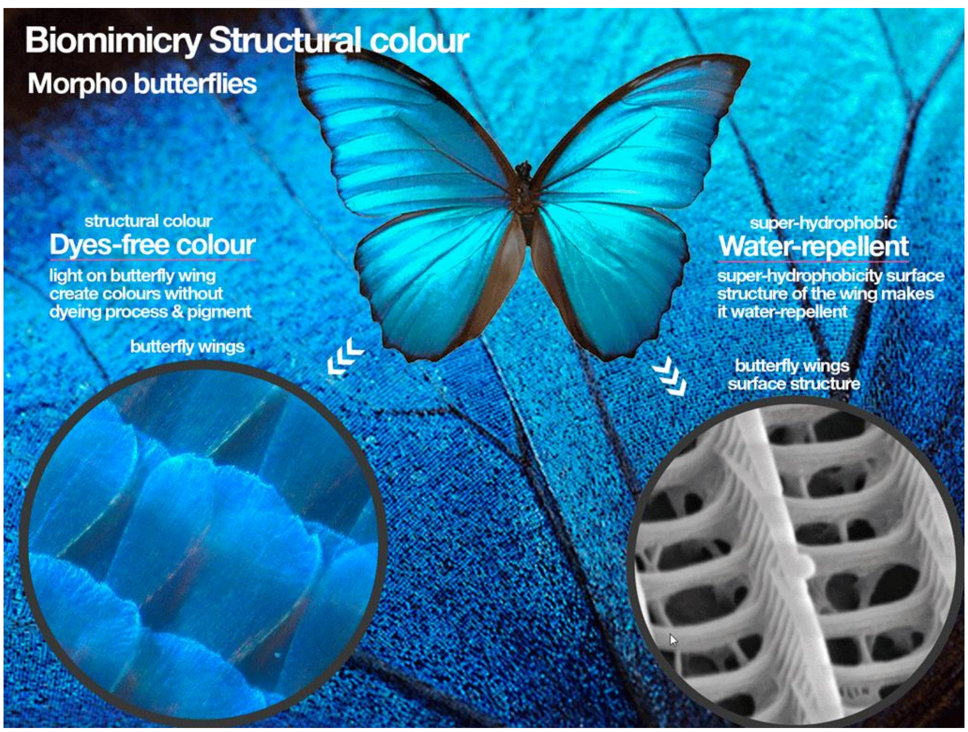


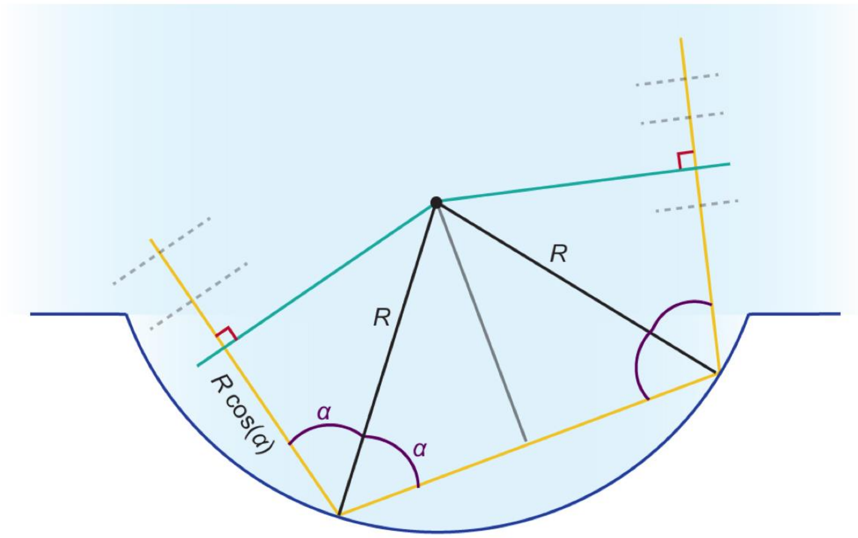
Interference

The composite waveform will have a resulting color because of the constructive interference from the initial waves. This color is what we call Structural Color, aka color resulting from interference caused by microscopic structural details.



Structural Color is found throughout nature. The most common mechanisms of structural colors are film interference, diffraction grating, scattering and photonic crystals.

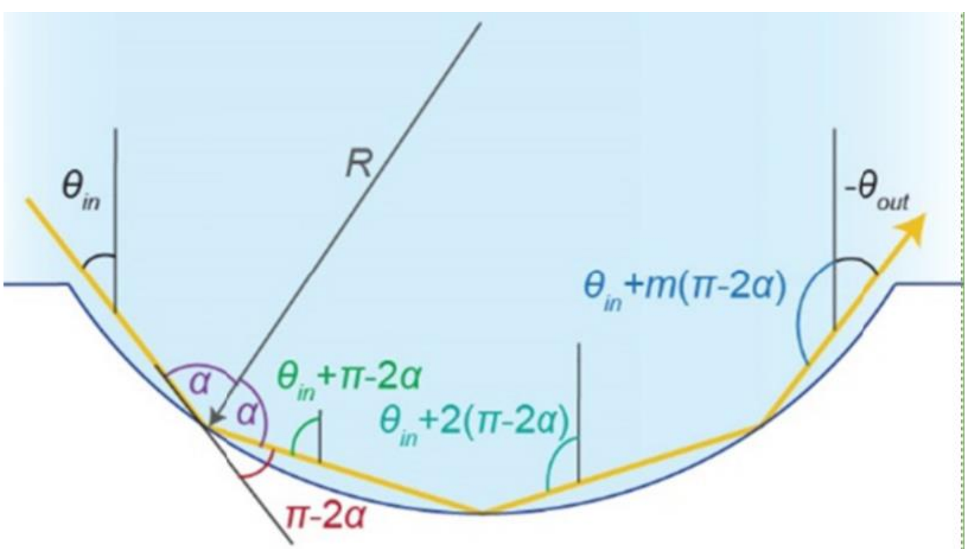
Behavior of light undergoing total internal reflection inside microscale cylinders.



From this image, we can derive that the path length taken by each ray of light is given by:

$$l = R\cos(\alpha) + (m-1)2R\cos(\alpha) + R\cos(\alpha) = 2mR\cos(\alpha)$$

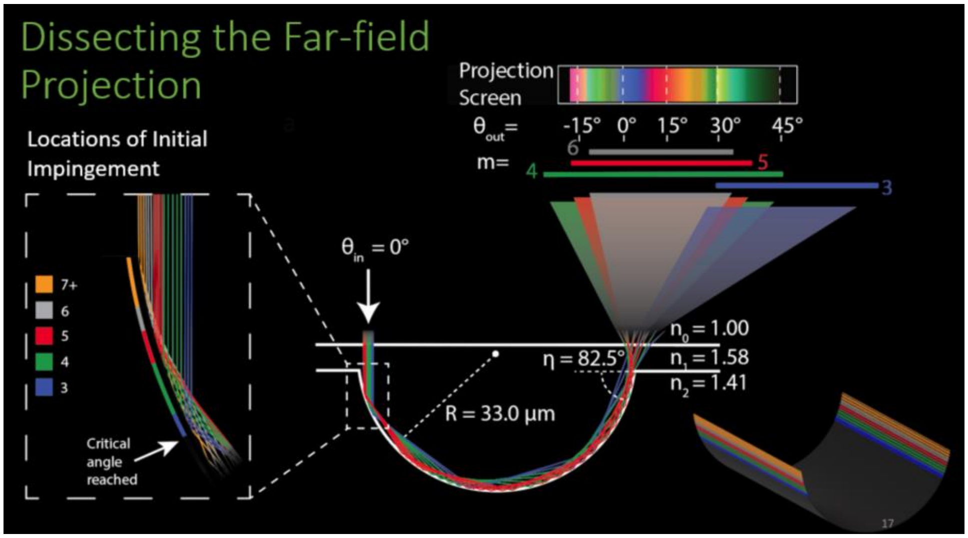
l= path length
m = number of bounces
R = Radius of Curvature
α = local angle of incidence



From this image, we can derive the relationship between the incident light angle and the outgoing light angle:

$$\pi + \theta_{out} = \theta_{in} + m(\pi - 2\alpha)$$

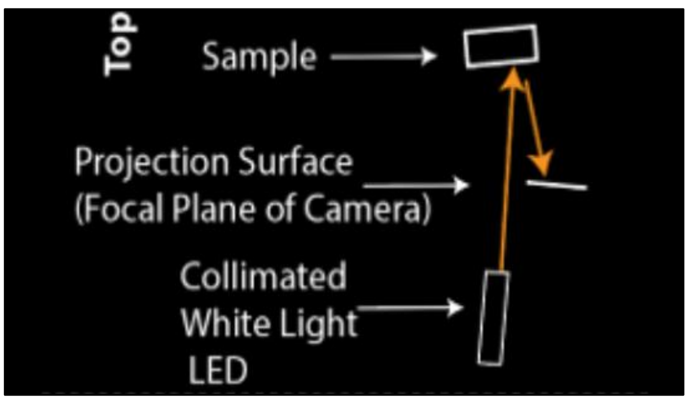
Therefore, we can deduce that the exit angle is directly dependent on the number of bounces the light undergoes inside the cylinder. This means that the light exiting the cylinder can be thought of as a function of optical path length. The exiting light will contact a projection screen. This description can be supplemented by the image below.

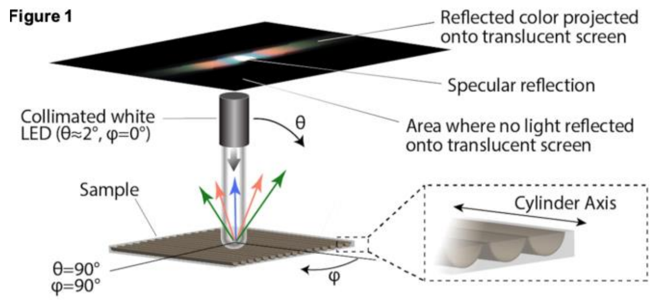


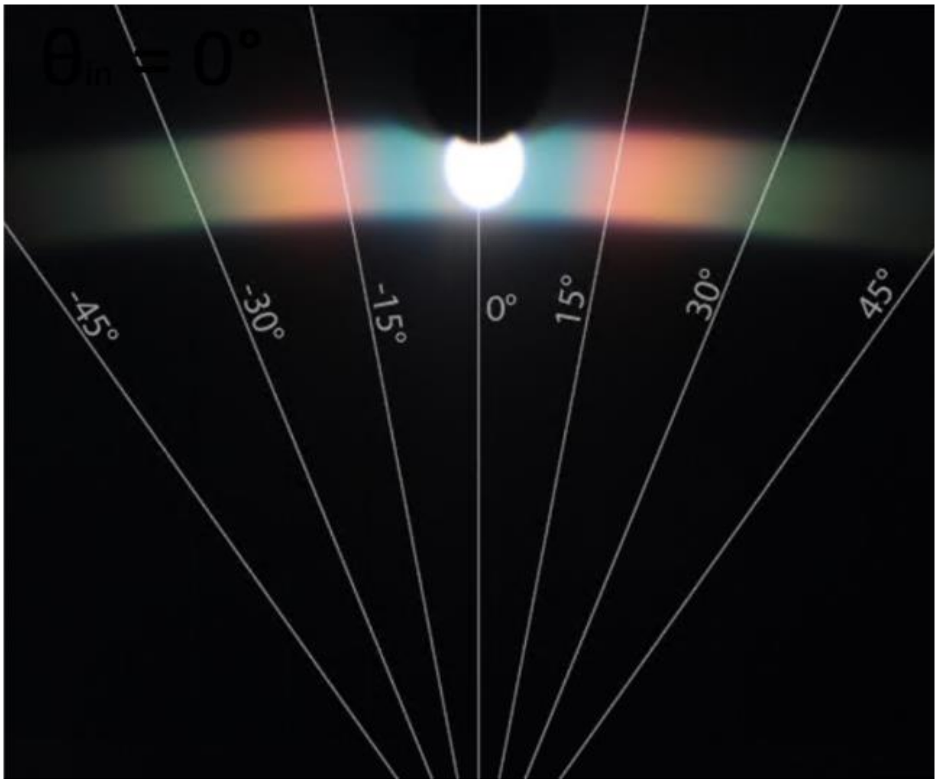
This image depicts the potential exit angles for each bounce trajectory. The interference effects of many combinations of light result in a complex spectrum of color output. Note: The colors given to the different numbers of bounce trajectories do not represent the color of that light, the color code is just used as a visual tool. The incident light for this output must be white.

Presentation of Poster (all necessary information will be found on poster)
https://drive.google.com/file/d/1f27ncn6A7z6qNm-ehSF0PVVVGTrQLxzu/view?usp=sharing

Demonstration of structural color resulting from method described above:
Place sample vertically with black background. Shine collimated white light onto sample a few degrees from the normal. Place a white screen a few inches away, directly next to the light source. Structural color reflection will be visible on the projection screen. The structural details of the sample are described in the image above. The image below shows the set up for the far-field projection.







Expected result for Far-field projection.

LAB DESCRIPTION

Materials

- Hemi-cylindrical dish filled with Vegetable oil (Refractive index ≈ 1.47)
- Hemi-cylindrical crown glass block (Refractive index ≈ 1.52)
- Protractor or polar graph paper with listed angles.
- Monochromatic red laser
- Hemi-cylindrical dish filled with water (Refractive index ≈ 1.33)
- Calculator
- Lab Notebook
- Pen/pencil
- Safety goggles

Advance Preparation Each group of students should have a setup of 2 hemi-cylindrical glass dishes, and 1 hemi-cylindrical glass block. One of the dishes should be filled with vegetable oil, and the other should be filled with water. Each group must have a monochromatic red laser and a protractor/polar graph paper.

Safety Information
Laser should not be shined toward people's faces. Students should wear safety goggles throughout the lab.

Teaching Strategies

Students should perform the lab in groups of 3-4 students. Lab should be performed after the lecture described above introducing necessary concepts. Lab should be placed in a unit on Classical Optics, after the introduction of Snell's Law.

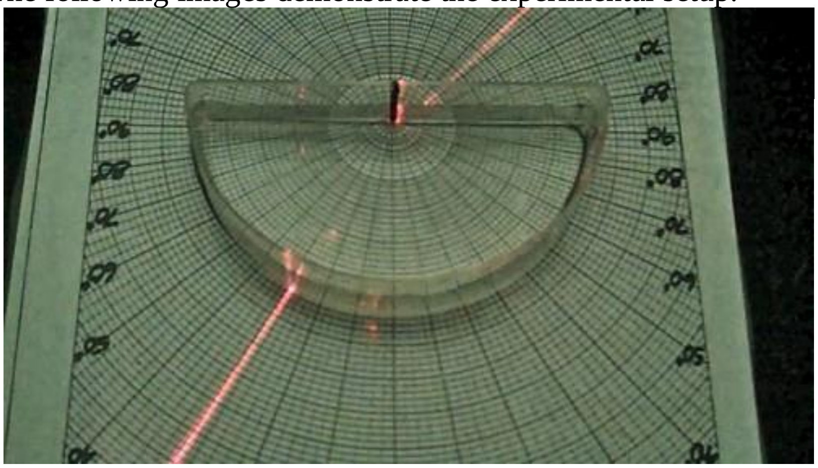
Resources: You may wish to use these resources either as background or as a resource for students to use in their inquiry-based design.
https://faculty.etsu.edu/lutter/courses/phys2021/Snells_Laws_Online_Lab.pdf
https://www.lehman.edu/faculty/dgaranin/Introductory_Physics/Online%20labs/Experiment_17-Refraction-online.pdf

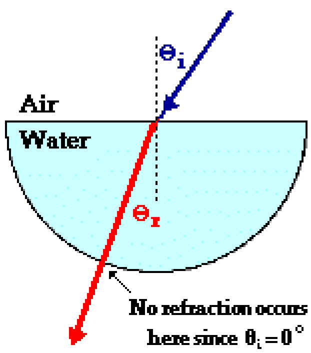
(Online labs that demonstrate the same concepts)
The sources listed under 'Teacher Resources' may also be useful.

Procedure
Part 1:
Note: Begin with in-class demonstration of experiment.
Students will calculate the refractive index of 3 unknown substances. (Water, vegetable oil, crown glass)

- Students will shine incident light directly along the normal into the flat side of the hemi-cylindrical bowls/blocks. Students will take measurements at 15° , 30° , and 45° from the normal.
- Students will calculate the refractive index for each measurement using $n_2 = \frac{n_1 \sin\theta_1}{\sin\theta_2}$
- The students will average results for each of the mystery indices and compare them to the true refractive indices for each substance.

The following images demonstrate the experimental setup.





Center for Nanoscale Science www.mrsec.psu.edu

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Part 2:

Students will find the critical angle associated with each of the materials.

- Students will first calculate the critical angles for each material using
$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$
- The refractive indices found in part a will be used for n_1.
- Students will then experimentally find the critical angle for each material.

Example Student Worksheet or Guide

Finding Unknown Refractive Indices and Relative Critical Angles to Produce Total Internal Reflection (TIR) (with Answers in Red)

Introduction

This lab will demonstrate the trajectories taken by light through materials of different refractive indices, showing the conditions necessary to produce refraction and reflection. This will serve as a transition from standard textbook lessons about optics to current research into structural color and its potential applications. Furthermore, the behavior seen in today's lab is proof of some of the *spooky* behavior notoriously associated with quantum physics, notably Fermat's Principle (light travelling between two points will always take the path that requires the shortest amount of time to traverse). How does light know which path to take to accomplish this!?

Materials

- Hemi-cylindrical dish filled with Vegetable oil
- Hemi-cylindrical crown glass block
- Protractor or polar graph paper with listed angles.
- Monochromatic red laser
- Hemi-cylindrical dish filled with water
- Calculator
- Lab Notebook
- Pen/pencil
- Safety goggles

Procedure

Part 1: Students will calculate the refractive index of 3 substances by taking measurements of the refractive angles from 3 different incident light angles.

1. Place hemi-cylindrical glass block at center of polar graph paper. Shine red laser at 15° , 30,° and 45° from the normal (as demonstrated by your teacher).

2. Record your refractive angle (angle between exit light and normal) in TABLE 1 below.

3. Use $n_2 = \frac{n_1 \sin\theta_1}{\sin\theta_2}$ to calculate the refractive index of each material at each angle. Note: Air has a refractive index of 1.00.

4. Average the calculated refractive indices to determine your measured refractive index associated with the glass.

5. Repeat for the bowl filled with oil and the bowl filled with water (Tables 2 & 3).

6. Compare results with refractive indices given by your teacher at the end of the lab.

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Record Your Observations

TABLE 1: HEMI-CYLINDRICAL GLASS BLOCK

Angle of Incidence	Angle of Refraction	Calculated Refractive Index
15°	9.8°	1.52
30°	19.2°	1.52
45°	27.7°	1.52

Average Calculated Refractive Index: ______1.52________

TABLE 2: HEMI-CYLINDRICAL DISH FILLED WITH OIL

Angle of Incidence	Angle of Refraction	Calculated Refractive Index
15°	10.1°	1.47
30°	19.9°	1.47
45°	28.8°	1.47

Average Calculated Refractive Index: _______1.47_______

TABLE 3: HEMI-CYLINDRICAL DISH FILLED WITH WATER

Angle of Incidence	Angle of Refraction	Calculated Refractive Index
15°	11.2°	1.33
30°	22.1°	1.33
45°	32.1°	1.33

Average Calculated Refractive Index: ______1.33_______

Compare your results with the true refractive indices for each material.

TABLE 4: Compare Your Results

MATERIAL	TRUE REFRACTIVE INDEX	MEASURED REFRACTIVE INDEX
Glass	1.52	1.52
Oil	1.47	1.47
Water	1.33	1.33

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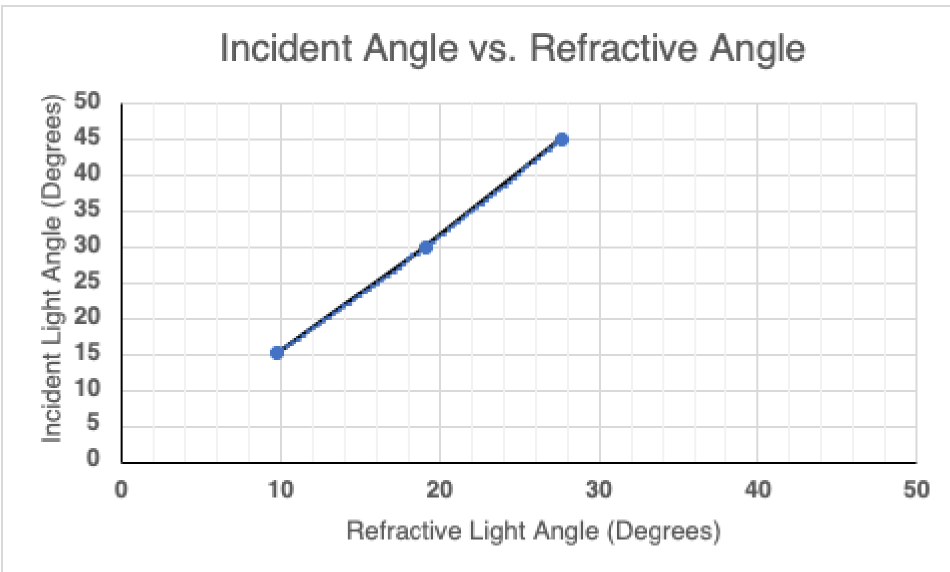
Reflection Questions:

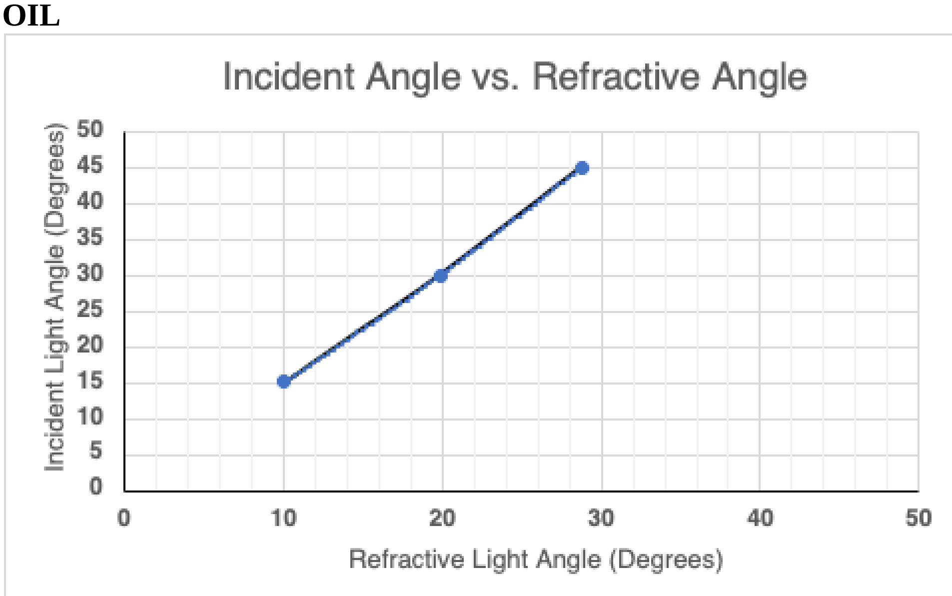
1. How did your results compare?

2. Graph the relationship between incidence and refraction angles for each material.

GLASS



OIL



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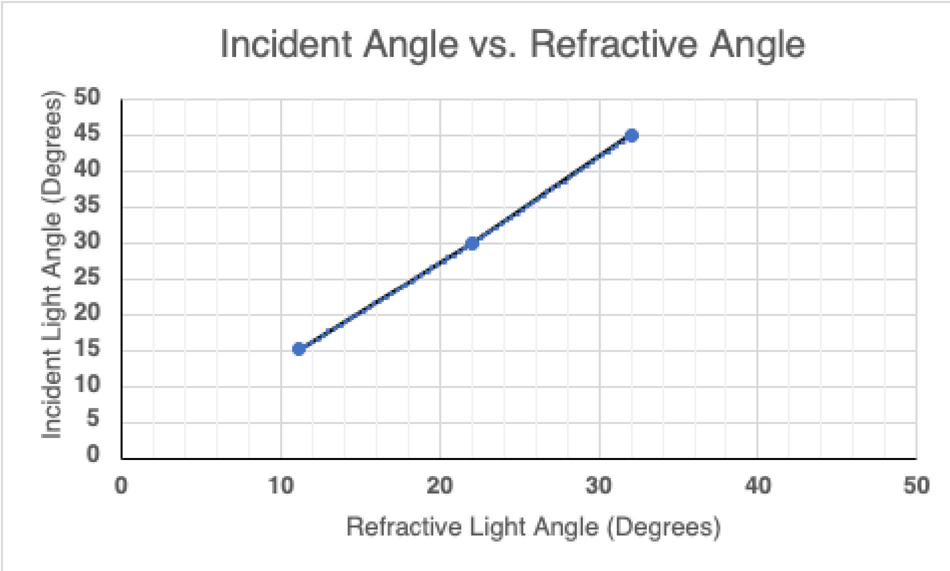
WATER



3. **What is the relationship between the angle of incidence and the angle of refraction? Does this relationship change with respect to the refractive index of the material?**

Refractive angle is proportional to incident light angle. A higher refractive index means a smaller refractive angle with respect to the incident light angle.

4. **Given what we know about the critical angle, hypothesize which of these three materials would have the lowest critical angle. Explain your reasoning.**

Glass would have the lowest critical angle since we will consider our material to be n1 and the air to be n2.

Part 2:
Procedure

1. Calculate the critical angle necessary for Total Internal Reflection for each material using

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Note: The light is reflecting from the material to the air. Therefore n_2 = air (1.00) and n_1 = material.

2. Measure the critical angle for each material and compare theoretical calculations to experimental data.

TABLE 5: Critical Angle to Achieve TIR

MATERIAL	CALCULATED CRITICAL ANGLE	MEASURED CRITICAL ANGLE
GLASS	41.1	41.1
OIL	42.9	42.9
WATER	48.6	48.6

Reflection Questions continued:

5. **How does the refractive index relate to the critical angle?**

The critical angle and refractive index are inversely proportional.

6. **Was your hypothesis from question #4 correct? If not, explain the relationship shown above.**

7. **Overall, were any of your calculations/measurements inaccurate? Explain potential causes for the discrepancies.**

Going Further: (This section is only appropriate for students that have completed at least one full course in calculus)

Fermat's Principle states that the path taken by a ray between two given points is the path that can be traveled in the least time. Snell's law can be rewritten as follows:

$$\frac{sin\theta_1}{sin\theta_2} = \frac{n_1}{n_2} = \frac{v_1}{v_2}$$

Here v_1 and v_2 refer to the phase velocity of the speed of light in each respective medium.

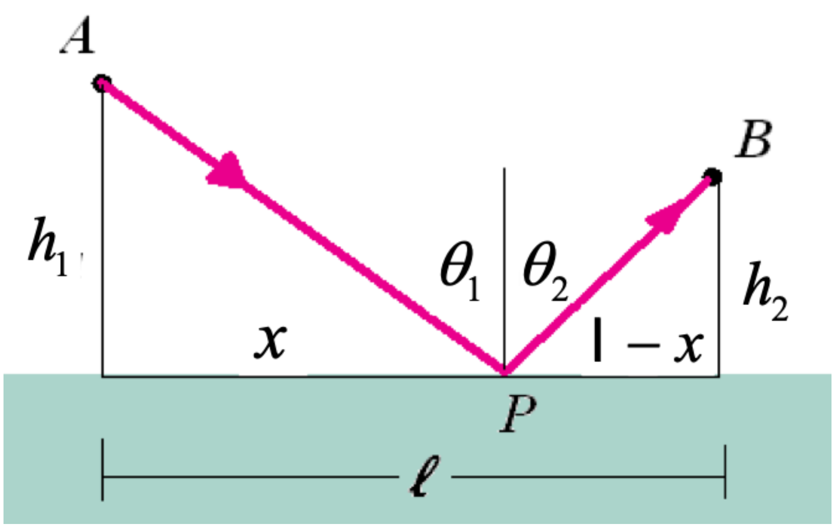
These phase velocities can also be written as:

$$v_1 = \frac{c}{n_1} \quad \text{and} \quad v_2 = \frac{c}{n_2}$$

1. **Show that Fermat's Principle, when applied to reflection, proves that the incident angle and reflective angle must be equal. ($\theta_1 = \theta_2$)**

We are looking for the shortest amount of time between points A and B in the image below.

Note: The shortest amount of time would be a straight line between points A and B, however in this scenario we are interested in the shortest amount of time on a trajectory that includes a reflection.



Hint: Begin with t=d/v (time is equal to distance over velocity). Use the Pythagorean theorem to express the distance traveled before the reflection and after the reflection. A good starting point is:

$$t = \frac{\sqrt{(h_1^2 + x^2)}}{v} + \frac{\sqrt{(h_2^2 + (1 - x)^2)}}{v}$$

Solution:
Minimize derivative of time with respect to x by setting it equal to 0.

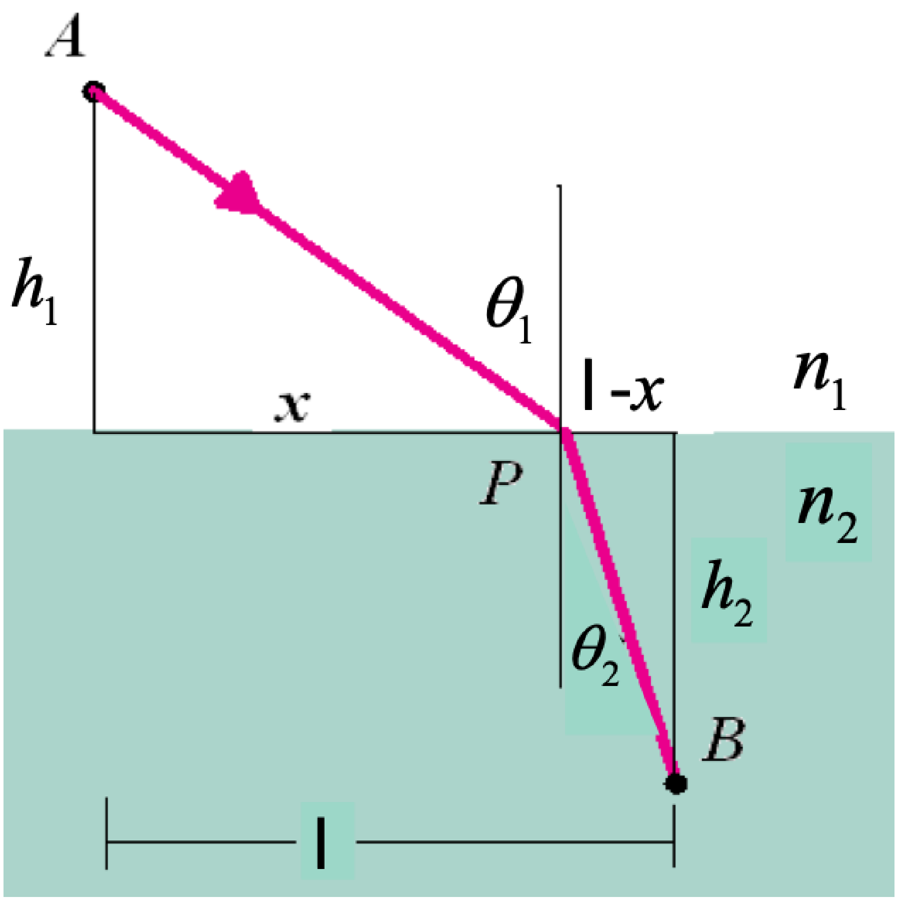
$$\frac{dt}{dx} = \frac{x}{v\sqrt{(h_1^2 + x^2)}} + \frac{-(1-x)}{v\sqrt{\left(h_2^2 + ((1-x)^2)\right)}} = 0$$

$$\Rightarrow \frac{x}{\sqrt{(h_1^2+x^2)}} = \frac{(1-x)}{\sqrt{\left(h_2^2+((1-x)^2)\right)}} = sin\theta_1 = sin\theta_2 = \theta_1 = \theta_2$$

(because sin=opp/hyp)

2. **Apply Fermat's Principle to Refraction to derive Snell's Law.**

In this scenario, a ray of light refracts into the second material. Consider the case when the ray of light travels through two different refractive indices between points A and B.



Hint: Begin with t=d/v (time is equal to velocity over time), however in this scenario, v varies with respect to the refractive index of the material being traversed (see equation above). Use the Pythagorean theorem to express the distance traveled before the reflection and after the reflection.
A good starting point is:

$$t_= \frac{\sqrt{(h_1^2+x^2)}}{v_1} + \frac{\sqrt{(h_2^2+(1-x)^2)}}{v_2} \text{ where } v_1 = \frac{c}{n_1} \text{and } v_2 = \frac{c}{n_2}$$

Solution:
Minimize derivative of time with respect to x by setting it equal to 0.

$$\frac{dt}{dx} = \frac{1}{v_1}\frac{x}{\sqrt{(h_1^2 + x^2)}} + \frac{1}{v_2}\frac{-(1-x)}{\sqrt{\left(h_2^2 + ((1-x)^2)\right)}} = 0$$

$$\Rightarrow \frac{1}{v_1} sin\theta_1 = \frac{1}{v_2} sin\theta_2 = \frac{n_1}{c} sin\theta_1 = \frac{n_2}{c} sin\theta_2$$

Therefore,

$$n_1 \sin\theta_1 = n_2 sin\theta_2$$

Assessment:

Packets will be collected and graded for completeness. Each question must be answered thoroughly for full credit.

Structural Color Resulting from Total Internal Reflection Worksheet

$$l = 2mR\cos(\alpha)$$

$$\theta_{out} = \theta_{in} + 180°(m-1) - 2m\alpha$$

Use the equations above and notes from today's lecture to complete the worksheet. Consider a hypothetical structure with surface cylinders of radius = 32.5 microns. This hypothetical structure supports a 3-6 bounce trajectory.

1. If 0^{o} incident light is shined onto our structure, and bounces 3 times with a local incidence angle of $64°$, calculate the exit angle and path length associated with this trajectory?

Plugging into equations at the top, we find an exit angle of -24 degrees and a path length of 85.5 microns.

2. Calculate the local incidence angle of $0°$ incident light undergoing TIR and exiting after 4 bounces at an angle of $12°$.

Rearrange the second equation to isolate alpha.

$$\frac{\theta_{out} - \theta_{in} - 180°(m-1)}{-2m} = \alpha$$

Alpha = 66 degrees.

3. Given the # of bounces and the local incident angle range (α), calculate exit angle range (θ_{out}) and path length for each bounce trajectory.

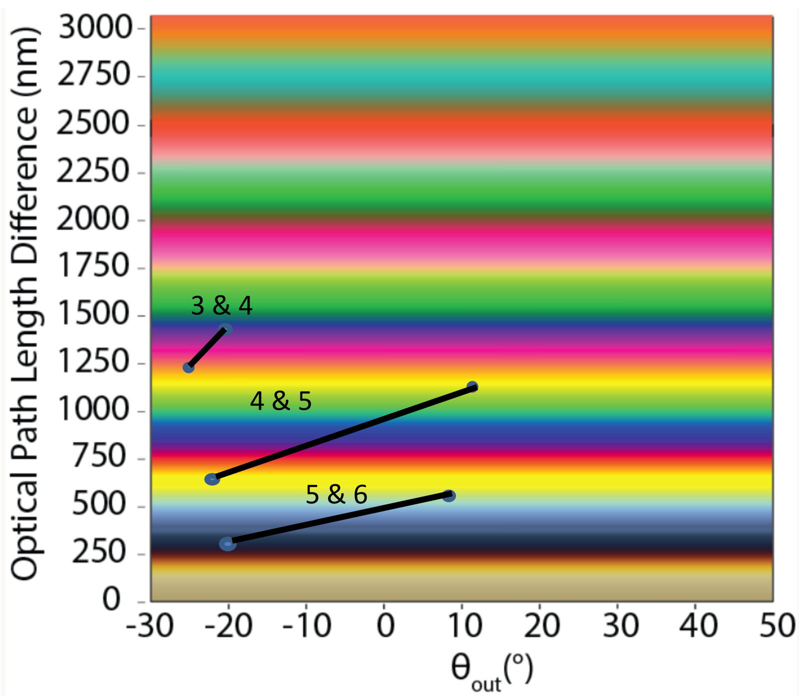
# of bounces	α range	θ_{out} Range	Path Length (μm)
3	63° - 65.4°	-18.0 to -32.4	81.2 - 88.5
4	65.4° - 70.8°	16.8 to -26.4	85.5-108.2
5	70.8° - 74.3°	12.0 to –23.0	87.9-106.9
6	74.3° - 76.7°	8.4 to -20.4	89.7-128.3

4. Using the table above, calculate the maximum and minimum overlapping exit angles for pairs of bounce trajectories. Calculate the minimum and maximum path length differences for each of these trajectories.

$$l = 2mR\cos\left(\frac{\theta_{out} - 180(m-1)}{-2m}\right)$$

Bounce Trajectory Pairs	Overlapping θ_{out} Range	Optical path Length Difference (Lower Exit Angle)	Optical path Length Difference (Larger Exit Angle)
3&4	-26.4 to –18.0	85.51-84.26 =1.25	89.99-88.53=1.46
4&5	-23.0 to 12.0	87.95-87.32=0.63	106.88-105.75=1.13
5&6	-20.4 to 8.4	89.72-89.36=0.36	105.53-104.95=0.58

5. Use the graph below to sketch the interference created by the bounce trajectory pairs in the table above.



6. What color would be seen as a result of interference between the 4 & 5 bounce trajectories at an exit angle of -22° ?

Yellow

7. Using $m = \frac{y_2 - y_1}{x_2 - x_1}$, calculate the slope of each of the path length difference for each bounce trajectory pair.

3&4 trajectory: 25 nm per degree
4&5 trajectory: 14.3 nm per degree
5&6 trajectory: 7.6 nm per degree

Assessment:

Packets will be collected and graded for completeness. Each question must be answered thoroughly for full credit.