Superluminal Solitons: Don't Worry, It's Okay

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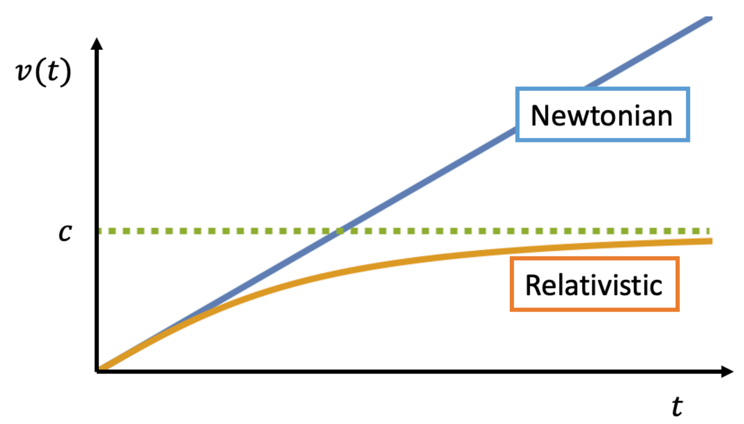
Abstract

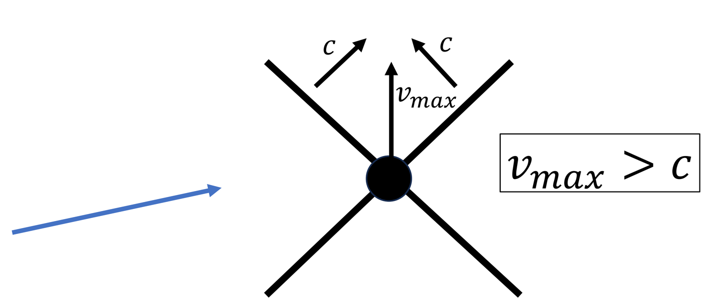
Theoretical investigations and numerical work have shown that kink solitons in 2-dimensional relativistic field theories can be accelerated to speeds greater than the speed of light. We explain how these configurations maintain consistency with the tenets of special relativity and exhibit a toy model that provides provides intuition for this phenomenon.

Introduction

- **"Nothing travels faster than the speed of light"** is inaccurate and not implied by special relativity. What is true is that energy can not be transported faster than the speed of light.
- Classic example: Point particle of mass m accelerated by a constant force F.



- More interesting examples in field theories like Electricity and Magnetism:
  - A laser swept across the surface the moon
  - X-wave formed by 2 plane waves crossing at angles



- For continuous systems, the **rate of energy transport** is measured by **energy flow velocity** – can prove this is less than the speed of light (The velocity of the max energy density is different and can be greater than the speed of light!)
- Field theories with **solitons** are a subtle and interesting case:
  - Solitons are lumps of energy held together by nonlinear self-interactions of the field. Examples include **kinks**, vortices, monopoles
  - They have both particle-like and field-like properties, so…

Key Question:

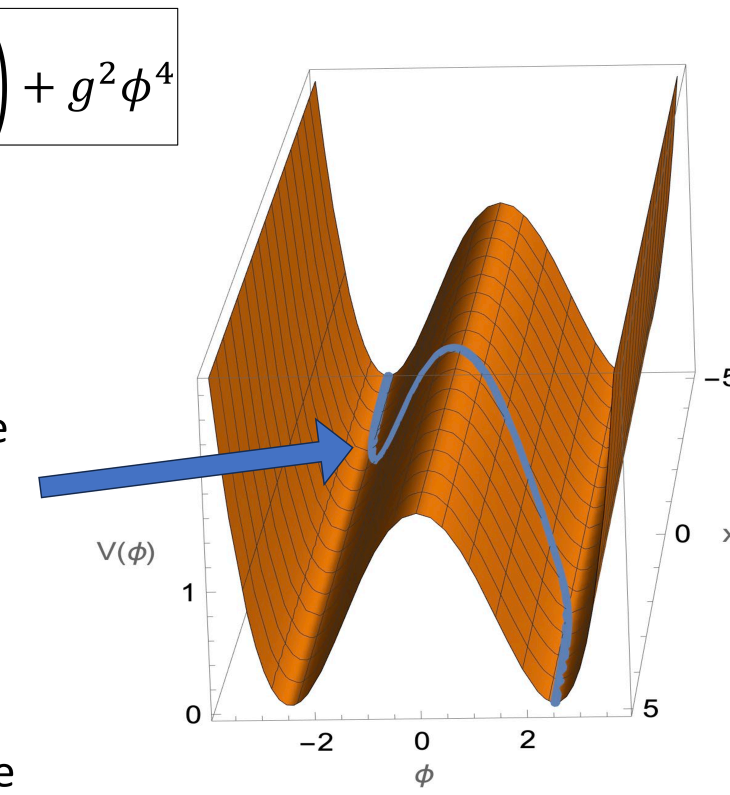
Can a soliton move faster than the speed of light, and if so, how do we reconcile this with special relativity?

Accelerating Kinks in Phi-four Theory

- Consider a single scalar field $\phi(t,x)$ in one spatial dimension whose energy density is:

$$\mathcal{E}(t,x) = \frac{1}{2}\left(\frac{1}{c^2}\left(\frac{\partial\phi}{\partial t}\right)^2 + \left(\frac{\partial\phi}{\partial x}\right)^2 - m^2\phi^2\right) + g^2\phi^4$$

- ϕ represents infinite string free to wiggle at the bottom of a frictionless infinite double-well trough:



- Static kink is a solution corresponding to the string passing over the central hump:

$$\phi(t,x) = \phi_0(x) = \frac{m}{2g}\tanh\left(\frac{mx}{\sqrt{2}}\right)$$

there are also solutions for a moving kink.

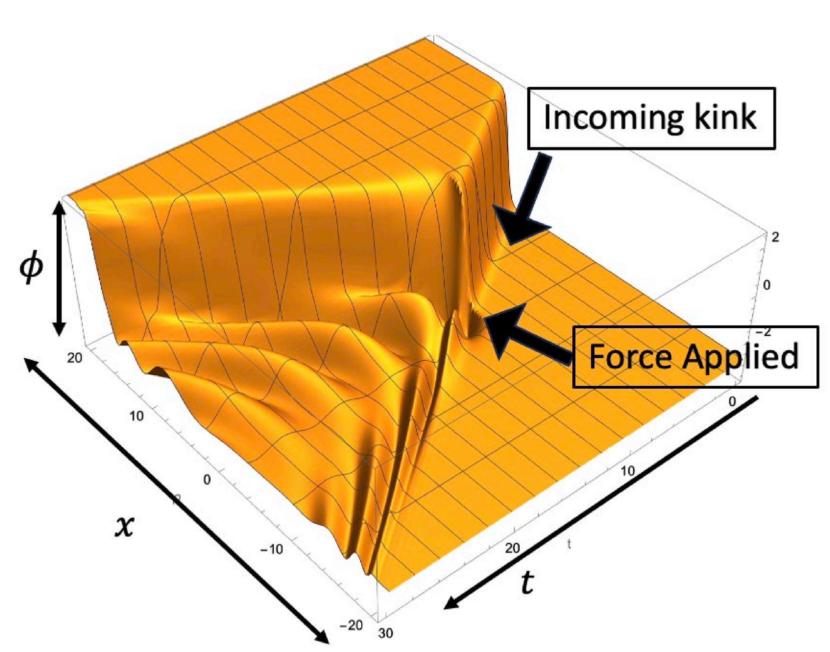
- Now suppose we apply a force $F(t,x)$ to the kink as it's moving. What happens? Must solve:

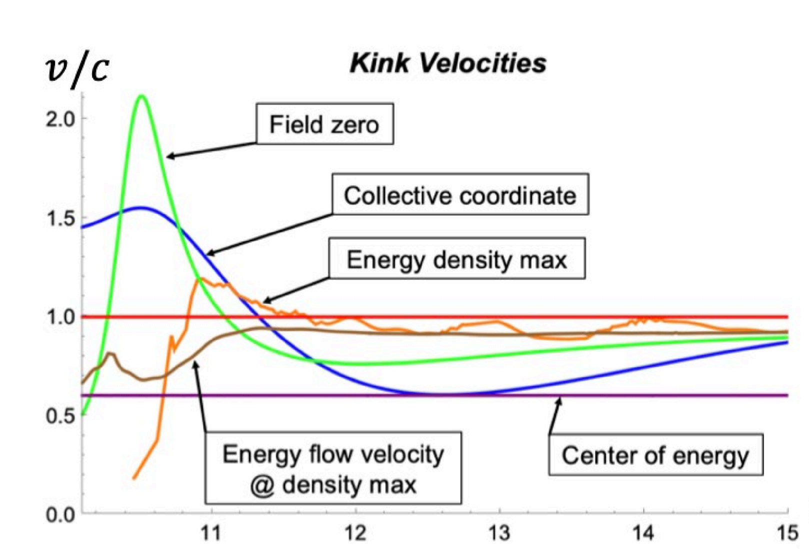
$$\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial x^2} - m^2\phi + 4g^2\phi^3 = F(t,x)$$

- To extract the kink's position $X(t)$ we can use the standard definition [1,2] of the **collective coordinate, X**:

$$\int \phi(t,x)\phi_0{}'(x - X(t))\, dx = 0$$

- One can also consider other definitions of the kink position:
  - the maximum of the energy density, and
  - the location where string passes over the central hump of the trough.



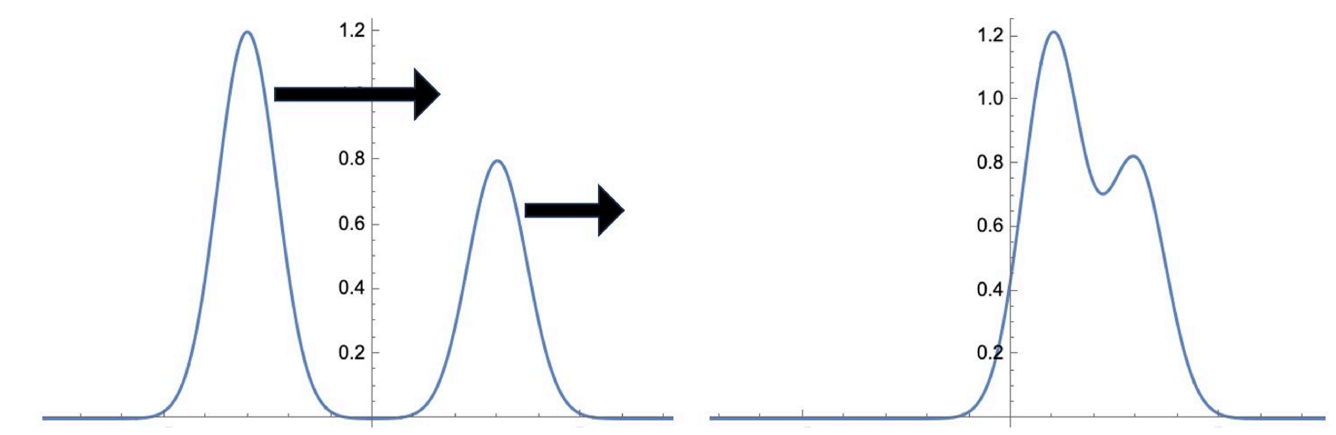


- Using numerics, Ref [3] evaluated the velocity of these quantities as functions of time on the solution shown above and compared them to the maximum value of the **energy flow velocity**. As one can see from the graph on the right all notions of "kink velocity" exceed the speed of light immediately after the impact, while the energy flow velocity remains subluminal.

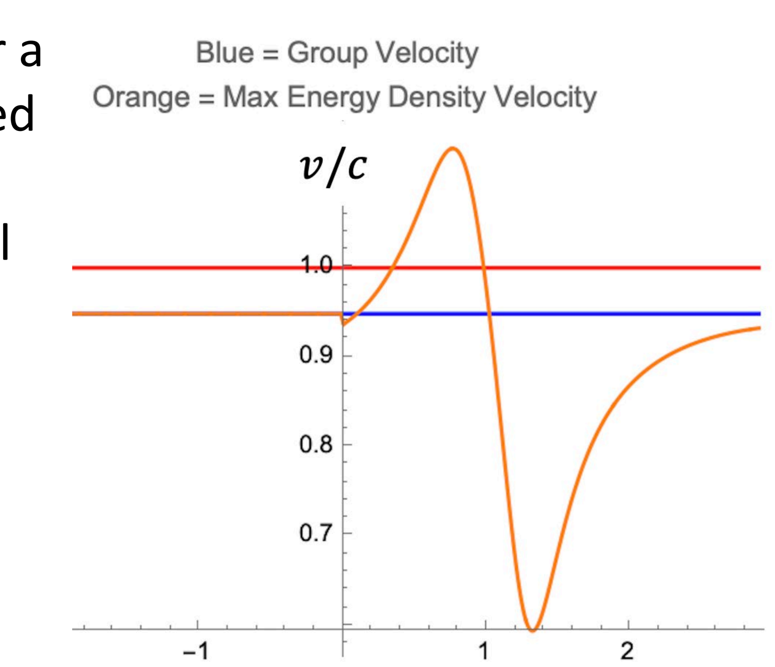
Explanation of Effect via Simple Toy Model

Basic Idea:

- The kink is a moving lump of energy. The applied force creates a second lump of energy. These two lumps generally travel at different speeds.
- As one passes over the other, and we follow the position of the maximum energy density, we see it can have greater speed than the speeds of either lump.



- To construct detailed toy model, we need a single theory which admits lumps as wave packets that can travel at different speeds, can be superimposed, and is simpler than ϕ^4 theory.
- **Klein Gordon Theory:**
  - This is a relativistic theory, related to ϕ^4 theory, but without solitons.
  - We use a wave packet as substitute and track the position of the max energy density after a force is applied.
  - Results show that for a strong enough applied force, the maximum of energy density will briefly exceed the speed of light with qualitatively similar features:



Conclusions

The soliton's collective coordinate is a nonlocal degree of freedom. A good substitute for it is the position of the max energy density in the field. In a theory with a nontrivial dispersion relation (like Klein Gordon), it is possible to construct solutions where the max energy density moves faster than the speed of light while, nonetheless, the energy flow speed is always and everywhere less then the speed of light.

Contacts

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References

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