Who Controls the Agenda Controls the Polity

Nageeb Ali
Penn State

Doug Bernheim
Stanford

Alex Bloedel
UCLA

Silvia Console-Battilana
Auctionomics

Real-time Agenda Control + Manipulable Preferences

$$\Downarrow$$

Dictatorial Power

Collective Choice Problem

Policy space X, compact and metrizable (e.g., finite set or subset of Euclidean space).

Policy chosen by single agenda setter and n voters, where n is odd. Agenda setter doesn't vote.

Preferences:

- Agenda setter's (continuous) preference relation is $\succcurlyeq_A$.
- Voter i's (continuous) preference relation is $\succcurlyeq_i$.
- $\succ_M$ denotes strict (typically intransitive) majority relation:

$$x \succ_M y \iff x \succ_i y \text{ for at least } \frac{n+1}{2} \text{ voters}$$

- Complete Information: All preferences commonly known.

Extensive Form: Amendment Procedure

Legislature begins with initial (exogenous) default $x^0 \in X$.

In each round $t \in \{1, \ldots, T\}$, agenda setter proposes amendment a^t that is put to a vote against prevailing default.

Extensive Form: Amendment Procedure

Legislature begins with initial (exogenous) default $x^0 \in X$.

In each round $t \in \{1, \ldots, T\}$, agenda setter proposes amendment a^t that is put to a vote against prevailing default.

Round 1: Agenda setter proposes a^1.

- Voters vote between a^1 and x^0.
- Policy with majority support becomes new default x^1.

Extensive Form: Amendment Procedure

Legislature begins with initial (exogenous) default $x^0 \in X$.

In each round $t \in \{1, \ldots, T\}$, agenda setter proposes amendment a^t that is put to a vote against prevailing default.

Round 1: Agenda setter proposes a^1.

- Voters vote between a^1 and x^0.
- Policy with majority support becomes new default x^1.

Round 2: Just like Round 1, but with default x^1.

$\ldots$

Round T: Policy with majority support is implemented.

Extensive Form: Amendment Procedure

Legislature begins with initial (exogenous) default $x^0 \in X$.

In each round $t \in \{1, \ldots, T\}$, agenda setter proposes amendment a^t that is put to a vote against prevailing default.

Suppose default option at start of round t is x^{t-1}.

- If $t < T$: policy with majority support is default for $t + 1$.
- If $t = T$: policy with majority support is implemented.

Large $T < \infty$: Dynamic procedure for a static collective choice problem.

Interpretation

We view this as a dynamic procedure used to solve a static collective choice problem.

Negotiations concern a time-dated policy and cannot proceed past the (known) implementation date .

For example, the budget for 2024 must be decided by January 1, 2024.

$\Longrightarrow$ A finite number of rounds, T, at which offers can be made.

To interpret evolving defaults:

- "Provisional bills" arising during negotiations, prior to final passage of any actual bill.
- Distinct bills are passed (and supersede previously passed bills) prior to implementation date.

We also show results extend to other amendment procedures.

Equilibrium Concept

All players can condition actions on history of prior actions, and is sequentially rational.

Standard solution concept: Subgame Perfect equilibria with "as-if pivotal" voting

- Each voter compares continuation outcome if current amendment passes to that if it fails. If she has strict preferences between two outcomes, she votes accordingly.
- Outcome-equivalent to "roll call voting" with fixed sequential order in each round.

How We Depart from the Literature

Prior work: Agenda setter commits to a fixed slate of proposals $(a^1, \ldots, a^T)$.
Proposals cannot be tailored to prevailing default option.

How We Depart from the Literature

Prior work: Agenda setter commits to a fixed slate of proposals $(a^1, \ldots, a^T)$.
Proposals cannot be tailored to prevailing default option.

McKelvey ('76, '79): If voters are myopic, agenda setter achieves favorite policy.

Generically, $\succ_M$ is globally intransitive $\implies$ $\exists$ a majority chain from $x^0 \to x_A^*$.

How We Depart from the Literature

Prior work: Agenda setter commits to a fixed slate of proposals $(a^1, \ldots, a^T)$.
Proposals cannot be tailored to prevailing default option.

McKelvey ('76, '79): If voters are myopic, agenda setter achieves favorite policy.

Generically, $\succ_M$ is globally intransitive $\implies \exists$ a majority chain from $x^0 \to x_A^*$.

Shepsle-Weingast ('84): If voters are sophisticated, agenda setter is limited to 2-chains.

She obtains x where $x \succ_M x^0$ or $\exists y$ such that $x \succ_M y$ and $y \succ_M x^0$.

How We Depart from the Literature

Prior work: Agenda setter commits to a fixed slate of proposals $(a^1, \dots, a^T)$.
Proposals cannot be tailored to prevailing default option.

McKelvey ('76, '79): If voters are myopic, agenda setter achieves favorite policy.

Generically, $\succ_M$ is globally intransitive $\implies$ $\exists$ a majority chain from $x^0 \to x_A^*$.

Shepsle-Weingast ('84): If voters are sophisticated, agenda setter is limited to 2-chains.

She obtains x where $x \succ_M x^0$ or $\exists y$ such that $x \succ_M y$ and $y \succ_M x^0$.

This paper: Agenda setter proposes policies from X in real time.

- Flexibility: She can tailor her proposal to current default option.
- No commitment: Each proposal must be sequentially rational for her.

✓ Real-time Agenda Control + Manipulable Preferences

$\Downarrow$

Dictatorial Power

What is Manipulability?

Improvability and Manipulability

Definition

Policy x is improvable if $\exists\ y$ such that $y \succ_A x$ and $y \succ_M x$. Otherwise, x is unimprovable.

- Unimprovable policies are core of suitably defined cooperative game.
- Any policy in agenda setter's favorite set $X_A^* := \arg\max_{x \in X} u_A(x)$ is unimprovable.

Improvability and Manipulability

Definition

Policy x is improvable if $\exists\ y$ such that $y \succ_A x$ and $y \succ_M x$. Otherwise, x is unimprovable.

- Unimprovable policies are core of suitably defined cooperative game.
- Any policy in agenda setter's favorite set $X_A^* := \arg\max_{x \in X} u_A(x)$ is unimprovable.

Definition

A collective choice problem is Manipulable if every $x \notin X_A^*$ is improvable.

In other words, the only unimprovable policies are agenda setter's favorites.

Whenever the agenda setter can find an improvement for herself then she can also find an improvement that is better for herself and some majority of voters.

Definition

A collective choice problem is Manipulable if every $x \notin X_A^*$ is improvable.

Related to prevalence of intransitivities in majority relation $\succ_M$:

- If there is a Condorcet Winner, manipulability holds iff $X_A^* = \{CW\}$.
- More intransitivity expands scope for improvement.
- Distinct from McKelvey's Chaos: $\forall$ *x and y*, $\exists$ *sequence of majority improvements from x to y.*

Definition

A collective choice problem is Manipulable if every $x \notin X_A^*$ is improvable.

Satisfied generically in canonical settings:

- Distributive politics.
- Spatial politics with 3 or more dimensions.

Distributive Politics: Divide-the-Dollar is Manipulable

$X = \{x \in [0,1]^{n+1} : x_A + x_1 + \cdots + x_n \leq 1\}$, $u_i(x) = x_i$.

This problem is manipulable: if $x_A \neq 1$, then either:

- x is inefficient $\implies \exists y$ such that all are strictly better off.
- Some voter i has positive share $(x_i > 0)$
  $\implies$ AS can extract x_i and divide among herself and remaining $n-1$ voters.

Logic generalizes to a broad class of Distribution Problems.

Spatial Politics: Manipulability in 3 or more dimensions

Suppose $X = \mathbb{R}^d$ and player i's preferences are $u_i(x) = -\frac{1}{2}||x - x_i^*||^2$.

Theorem

If $d \geq 3$, problem is Manipulable for generic specifications of $(x_A^*, x_1^*, \ldots, x_n^*) \in \mathbb{R}^{d(n+1)}$.

Generic = Full-measure and open-dense set.

1. Model & Manipulability

2. Main Results

3. Distributive Politics

4. Spatial Politics

5. Commitments, Procedures, and Deadlines

6. Conclusion

Dictatorial Power

Main finding is (informally) that:

Manipulability $\Leftrightarrow$ AS obtains her favorite policy in every equilibrium (given sufficiently many rounds).

We establish this under different technical conditions:

- **Theorem 1**: Exact result if $|X| < \infty$ and preferences are strict.
- **Theorem 2**: Approximate result for continuous X and preferences, if discretized to finite grid.
- **Theorem 3**: Approximate result for continuous X and preferences, in class of equilibria.

Definition. A collective choice problem has Generic Finite Alternatives if X is finite and each player's preferences are antisymmetric.

Definition. A collective choice problem has Generic Finite Alternatives if X is finite and each player's preferences are antisymmetric.

Theorem 1

Suppose the collective choice problem satisfies Generic Finite Alternatives.

The collective choice problem is Manipulable.

$$\Updownarrow$$

Agenda setter obtains her favorite policy in every equilibrium for every initial default, if # of rounds exceeds $|X| - 1$.

Recall that Manipulability means that for every $x \notin X_A^*$, $\exists y$ such that $y \succ_A x$ and $y \succ_M x$.

Agenda Setter's Favorite Improvement

Proof uses the operator: $\phi(x) \equiv \arg\max_{y \succcurlyeq_M x} u_A(y)$.

By definition,

- For every x, $\phi^{t+1}(x) \succcurlyeq_A \phi^t(x)$.
- The fixed points of ϕ are unimprovable.
- If $T \geq |X| - 1$, then policy $\phi^T(x)$ is unimprovable for every x.

(Recall: a policy x is unimprovable if $\nexists\, y$ such that $y \succ_A x$ and $y \succ_M x$.)

Equilibrium Characterization

For game with T rounds & initial default x^0, let

$$f_T(x^0) \equiv \bigcup_{\text{equilibria}} \{\text{policies chosen w.p. } > 0 \text{ in equilibrium}\}$$

Equilibrium Characterization

For game with T rounds & initial default x^0, let

$$f_T(x^0) \equiv \bigcup_{\text{equilibria}} \{\text{policies chosen w.p. } > 0 \text{ in equilibrium}\}$$

Lemma 1

Under Generic Finite Alternatives, for every horizon T and initial default x_0,

$$f_T(x^0) = \{\phi^T(x^0)\}.$$

- For every T and unimprovable x^0, $f_T(x^0) = \{x^0\}$.
- For every $T \geq |X| - 1$, $\bigcup_{x^0 \in X} f_T(x^0) = \{\text{Unimprovable Policies}\}$.

Equilibrium Characterization

For game with T rounds & initial default x^0, let

$$f_T(x^0) \equiv \bigcup_{\text{equilibria}} \{\text{policies chosen w.p. } > 0 \text{ in equilibrium}\}$$

Lemma 1

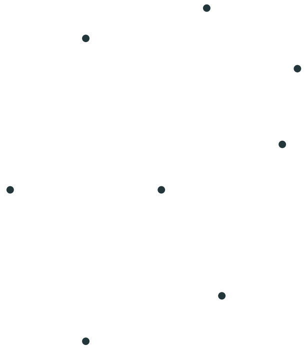
Under Generic Finite Alternatives, for every horizon T and initial default x_0,

$$f_T(x^0) = \{\phi^T(x^0)\}.$$

- For every T and unimprovable x^0, $f_T(x^0) = \{x^0\}$.
- For every $T \geq |X| - 1$, $\bigcup_{x^0 \in X} f_T(x^0) = \{\text{Unimprovable Policies}\}$.

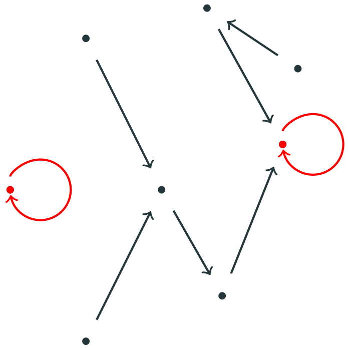
Lemma 1 $\Rightarrow$ Theorem 1 as Manipulability asserts Unimprovable Policies $=$ AS Favorite Policies.

$$\phi(x) \equiv \arg\max_{y \succcurlyeq_M x} u_A(y)$$



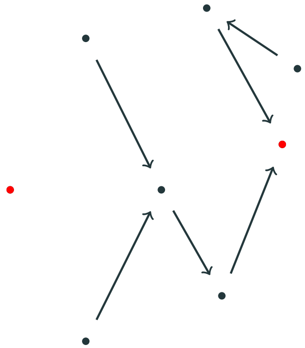
$$\phi(x) \equiv \arg\max_{y \succcurlyeq_M x} u_A(y)$$

$a \to b$ means that $b = \phi(a)$



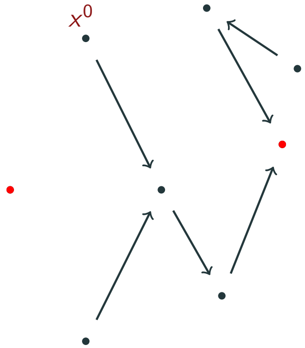
$$\phi(x) \equiv \arg\max_{y \succcurlyeq_M x} u_A(y)$$

$a \rightarrow b$ means that $b = \phi(a)$



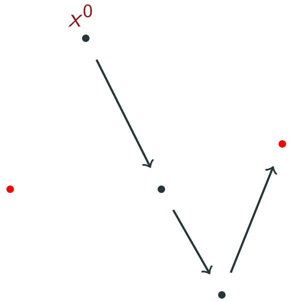
$$\phi(x) \equiv \arg\max_{y \succcurlyeq_M x} u_A(y)$$

Suppose x^0 is initial default option.



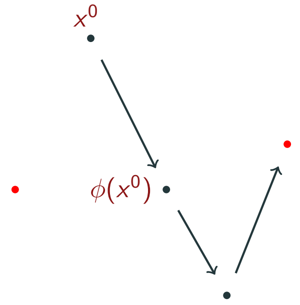
$$\phi(x) \equiv \arg\max_{y \succcurlyeq_M x} u_A(y)$$

Suppose x^0 is initial default option.



$\phi(x) \equiv \arg\max_{y \succcurlyeq_M x} u_A(y)$

Suppose x^0 is initial default option.

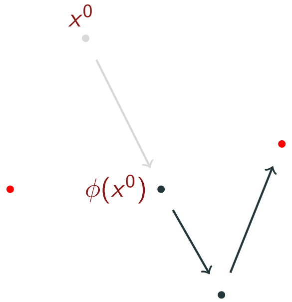


One-round Game

$\phi(x) \equiv \arg\max_{y \succcurlyeq_M x} u_A(y)$

Suppose x^0 *is initial default option.*

AS moves policy to $\phi(x^0)$*.*

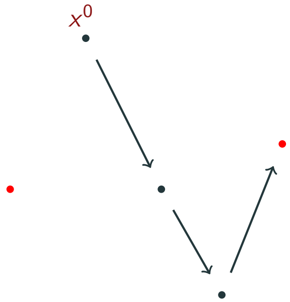


One-round Game

$\phi(x) \equiv \arg\max_{y \succcurlyeq_M x} u_A(y)$

Suppose x^0 is initial default option.

Rejecting first proposal leads to $\phi(x^0)$ in any eqm.



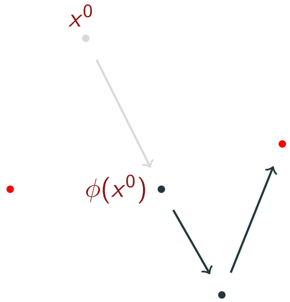
Two-round Game

$$\phi(x) \equiv \arg\max_{y \succcurlyeq_M x} u_A(y)$$

Suppose x^0 is initial default option.

Rejecting first proposal leads to $\phi(x^0)$ in any eqm.

Accepting first proposal y leads to $\phi(y)$ in any eqm.



Two-round Game

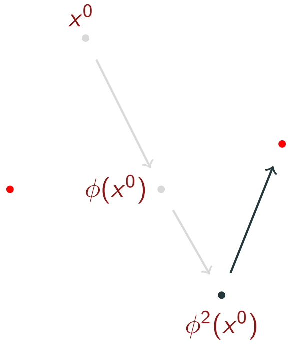
$$\phi(x) \equiv \arg\max_{y \succcurlyeq_M x} u_A(y)$$

Suppose x^0 is initial default option.

Rejecting first proposal leads to $\phi(x^0)$ in any eqm.

Accepting first proposal $\phi(x^0)$ leads to $\phi^2(x^0)$ in any eqm.

Agenda setter achieves $\phi^2(x^0)$.

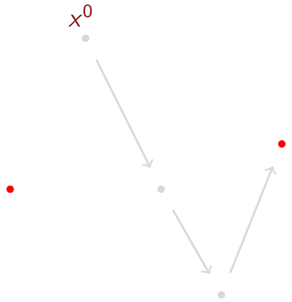


Two-round Game

$$\phi(x) \equiv \arg\max_{y \succcurlyeq_M x} u_A(y)$$

Suppose x^0 is initial default option.

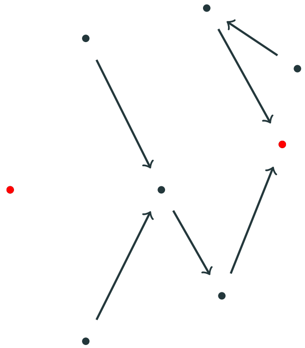
In a three-round game, unimprovable policy is reached.



Three-round Game

$$\phi(x) \equiv \arg\max_{y \succcurlyeq_M x} u_A(y)$$

$$a \rightarrow b \text{ means that } b = \phi(a)$$

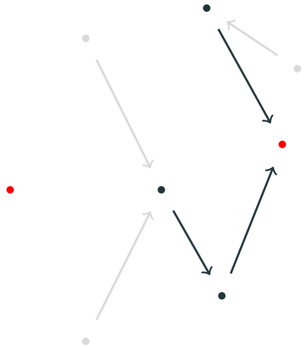


$$\phi(x) \equiv \arg\max_{y \succcurlyeq_M x} u_A(y)$$

$$a \to b \text{ means that } b = \phi(a)$$

In a one-round game, "effective policy space" is $\phi(X)$.

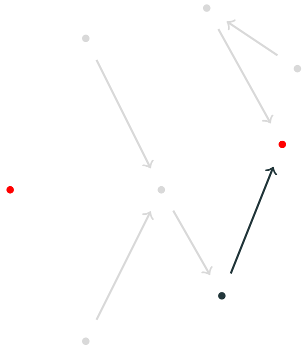
Players identify policy x with its continuation outcome $\phi(x)$.



$\phi(x) \equiv \arg\max_{y \succcurlyeq_M x} u_A(y)$

$a \rightarrow b$ means that $b = \phi(a)$

In a two-round game, effective policy space is $\phi^2(X)$.

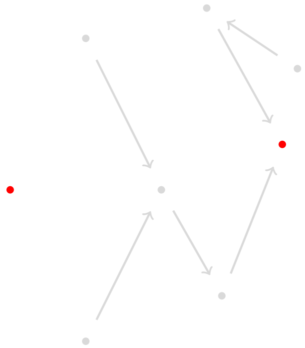


$$\phi(x) \equiv \arg\max_{y \succcurlyeq_M x} u_A(y)$$

$a \to b$ means that $b = \phi(a)$

Iterating operator eventually leads to fixed points.

$$\phi^T(X) = E \text{ for all } T \geq |X| - 1.$$



Lemma 1*

Under Generic Finite Alternatives,

$$f_T(x^0) = \{\phi^T(x^0)\} \text{ for every } T \text{ and } x^0.$$

All equilibria are outcome-equivalent to "greedy" one in which AS proposes ϕ(current default).

1. Greedy strategy implements same outcome if voters were myopic, as in McKelvey'76.
   Myopic voters compare $\phi^t(x^0)$ and $\phi^{t-1}(x^0)$.

2. Sophisticated voters reason backward, comparing $\phi^T(x^0)$ and $\phi^{T-1}(x^0)$.
   Hence, same coalition of voters support all on-path proposals.

3. Transitions need not be gradual: iff $\phi^T(x^0)$ unimprovable, $\exists$ eqm that jumps straight there.

Theorem 1

Suppose the collective choice problem satisfies Generic Finite Alternatives.

The collective choice problem is Manipulable.

$$\Updownarrow$$

Agenda setter obtains her favorite policy in every eqm for every initial default, if # of rounds exceeds $|X| - 1$.

Other Voting Rules:

- Consider general voting rule, modeled as collection $\mathcal{D} \subseteq 2^N$ of winning coalitions.
- Replace manipulability with $\forall x \notin X_A^*$, $\exists$policy y and winning coalition $D \in \mathcal{D}$ such that $y \succ_A x$ and $y \succ_i x$ for every $i \in D$.

Theorem 1

Suppose the collective choice problem satisfies Generic Finite Alternatives.

The collective choice problem is Manipulable.

$$\Updownarrow$$

Agenda setter obtains her favorite policy in every eqm for every initial default, if # of rounds exceeds $|X| - 1$.

Potential Issues:

- Manipulability is neither full-measure nor zero-measure in $\mathbb{R}^{|X| \times (n+1)}$.
- The number of rounds $\to \infty$ as $|X| \to \infty$.

We address both issues in Theorems 2 and 3 (in different ways).

Discretizing a Continuous Policy Space

We now consider a general policy space X satisfying Manipulability.

One perspective: Continuous X is an idealization and *actual policy choice is discrete*.

Start with any manipulable problem and study generic fine discretizations thereof.

- Discretization may fail manipulability.
- Horizon length for approximate dictatorial power that is uniform across discretizations.

Generic Grids and Thin Individual Indifference

Definition. A generic ε-grid is a finite subset $X_\varepsilon \subseteq X$ for which $\max_{x \in X} d(x, X_\varepsilon) < \varepsilon$, and the preferences of players restricted to X_ε are strict.

We consider collective choice problems that admit generic ε-grids for every sufficiently small $\varepsilon > 0$.

This is equivalent to a mild (and commonly assumed) condition on indifference curves being **thin**.

Definition. A collective choice problem satisfies Thin Indifference if $I_i(x) \backslash \{x\}$ has empty interior for every player i and policy x.

Note: $I_i(x) = \{y \in X : y \sim_i x\}$ is player i's indifference curve going through policy x.

Theorem 2 (in words)

Suppose the collective choice problem satisfies Thin Individual Indifference.

The collective choice problem is Manipulable.

$$\Updownarrow$$

Agenda setter obtains within δ of highest payoff in sufficiently fine grids ($\varepsilon < \varepsilon_\delta$) and sufficiently long horizons ($T \geq T_\delta$).

Comments:

- Manipulability is imposed on the ambient policy space but may be violated on the grid.
- Agenda setter obtains within δ of highest payoff in X, not merely that on the grid.
- The horizon T_δ depends on δ, but not the fineness/choice of the grid.

Theorem 2

Suppose the collective choice problem satisfies Thin Individual Indifference.

The collective choice problem is Manipulable.

$$\Updownarrow$$

For every $\delta > 0$, $\exists\, \varepsilon_\delta > 0$ and $T_\delta \in \mathbb{N}$ such that if

(a) policies are restricted to any generic ε-grid X_ε with $\varepsilon < \varepsilon_\delta$, and

(b) there are $T \geq T_\delta$ rounds,

then $\forall$ initial defaults $x^0 \in X_\varepsilon$, and in any equilibrium, AS's payoff is at least

$$\max_{x \in X} u_A(x) - \delta.$$

What about Continuous Policy Spaces?

Prior results considered **finite** policy spaces, either directly or as discretizations of ambient space.
Analysis exploited strict preferences.

Both voter and AS indifference introduce complications.

One (standard) resolution is to assume that ties are broken in AS's favor.

Our analysis uses a weaker notion:

> *Tie-breaking is Non-Capricious if whenever $x \sim_i y$, voter i always votes in favor (or against) the proposal whenever x is the continuation outcome from the proposal passing and y is the continuation outcome from the proposal failing.*

Theorem 3

The collective choice problem is Manipulable.

$$\Updownarrow$$

Agenda setter obtains within δ of highest payoff with sufficiently long horizons ($T \geq T_\delta$) for every initial default in any MPE with non-capricious tie-breaking.

Theorem 3

The collective choice problem is Manipulable.

$$\Updownarrow$$

Agenda setter obtains within δ of highest payoff with sufficiently long horizons ($T \geq T_\delta$) for every initial default in any MPE with non-capricious tie-breaking.

Divide-the-Dollar example: With NC tie-breaking, get **exact** result with $T = 3$.
With **capricious** tie-breaking, approx. result may fail.

Example

Taking Stock

Main finding is (informally) that:

Manipulability ⇔ AS obtains her favorite policy in every equilibrium (given sufficiently many rounds).

We formally established this under different technical conditions:

- **Theorem 1**: Exact result if X is finite & prefs are strict.
- **Theorem 2**: Approximate result for discretized general problems. (+ bounded # rounds)
- **Theorem 3**: Approximate result for general problems in class of equilibria. (+ address indifference)

Real-time Agenda Control + Manipulable Preferences

$$\Downarrow$$

Dictatorial Power

Why does Manipulability hold in Distributive & Spatial Politics?

1. Model & Manipulability

2. Main Results

3. Distributive Politics

4. Spatial Politics

5. Commitments, Procedures, and Deadlines

6. Conclusion

Distribution Problems

Definition

A collective choice problem is a Distribution Problem if it satisfies for every x and i:

1. Scarcity:

2. Transferability:

Distribution Problems

Definition

A collective choice problem is a Distribution Problem if it satisfies for every x and i:

1. Scarcity: If player i is not getting her highest payoff, then either
   (a) $\exists$ player j who is not getting his worst, or (b) There is a strong Pareto improvement.
2. Transferability:

Distribution Problems

Definition

A collective choice problem is a Distribution Problem if it satisfies for every x and i:

1. Scarcity: If player i is not getting her highest payoff, then either
   (a) $\exists$ player j who is not getting his worst, or (b) There is a strong Pareto improvement.

2. Transferability: If $u_i(x) > \underline{u}_i$, then $\exists$ policy y such that $u_j(y) > u_j(x)$ for all players $j \neq i$.

Distribution Problems

Definition

A collective choice problem is a Distribution Problem if it satisfies for every x and i:

1. Scarcity: If player i is not getting her highest payoff, then either
   (a) $\exists$ player j who is not getting his worst, or (b) There is a Pareto improvement.
2. Transferability: If $u_i(x) > \underline{u}_i$, then $\exists$ policy y such that $u_j(y) > u_j(x)$ for all players $j \neq i$.

Proposition

Every Distribution Problem is Manipulable.

Distribution Problems

Definition

A collective choice problem is a Distribution Problem if it satisfies for every x and i:

1. Scarcity: If player i is not getting her highest payoff, then either
   (a) $\exists$ player j who is not getting his worst, or (b) There is a Pareto improvement.
2. Transferability: If $u_i(x) > \underline{u}_i$, then $\exists$ policy y such that $u_j(y) > u_j(x)$ for all players $j \neq i$.

Proposition

Every Distribution Problem is Manipulable.

Proof: Suppose the policy is Pareto efficient.

- If AS isn't getting her favorite, then, by Scarcity, $\exists$ voter i who's getting better than his worst.
- By Transferability, can find strict improvement for AS and all voters $j \neq i$ at expense of voter i.

Distribution Problems

Definition

A collective choice problem is a Distribution Problem if it satisfies for every x and i:

1. Scarcity: If player i is not getting her highest payoff, then either
   (a) $\exists$ player j who is not getting his worst, or (b) There is a Pareto improvement.
2. Transferability: If $u_i(x) > \underline{u}_i$, then $\exists$ policy y such that $u_j(y) > u_j(x)$ for all players $j \neq i$.

Proposition

Every Distribution Problem is Manipulable for every veto-proof voting role.

Proof: Suppose the policy is Pareto efficient.

- If AS isn't getting her favorite, then, by Scarcity, $\exists$ voter i who's getting better than his worst.
- By Transferability, can find strict improvement for AS and all voters $j \neq i$ at expense of voter i.

Distributive Politics: Implications

Proposition

Every Distribution Problem is Manipulable for every veto-proof voting rule.

Proposition

Consider a Distribution Problem with thin indifference curves, the following hold:

1. If the voting rule is a quota rule with $q < n$, the agenda setter obtains u_A^* in every non-capricious equilibrium regardless of the initial default if there are at least $\lceil n/(n-q) \rceil$ rounds.
2. If the voting rule is veto-proof, the above holds if there are at least n rounds.

Distributive Politics: Broader Implications

AS achieves exact dictatorial power in any problem by bundling policies with transfers / pork.

Pork greases wheels $\implies$ manipulability $\Rightarrow$ AS obtains favorite policy without making payments.

Chosen policy need not maximize total surplus.

Pork-Barrel Politics:

- Suppose there are public projects that involve benefits and costs.
- Agenda setter may maximize total benefits while offloading all costs on others.

1. Model & Manipulability
2. Main Results
3. Distributive Politics
4. Spatial Politics
5. Commitments, Procedures, and Deadlines
6. Conclusion

Manipulability is Generic in Spatial Politics

Suppose $X = \mathbb{R}^d$ where $d \geq 3$ and player i's preferences are $u_i(x) = -\frac{1}{2}||x - x_i^*||^2$.

Manipulability is Generic in Spatial Politics

Suppose $X = \mathbb{R}^d$ where $d \geq 3$ and player i's preferences are $u_i(x) = -\frac{1}{2}||x - x_i^*||^2$.

Theorem

This problem is manipulable for a full-measure and open-dense set of $(x_A^*, x_1^*, \ldots, x_n^*) \in \mathbb{R}^{d(n+1)}$.

Genericity condition: when restricted to any 3 policy dimensions, no 4 ideal points are coplanar.

Manipulability is Generic in Spatial Politics

Suppose $X = \mathbb{R}^d$ where $d \geq 3$ and player i's preferences are $u_i(x) = -\frac{1}{2}||x - x_i^*||^2$.

Theorem

This problem is manipulable for a full-measure and open-dense set of $(x_A^*, x_1^*, \ldots, x_n^*) \in \mathbb{R}^{d(n+1)}$.

Genericity condition: when restricted to any 3 policy dimensions, no 4 ideal points are coplanar.

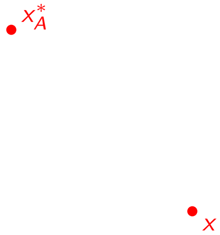
We will give proof for $d = 3$. Condition is then:

- No 4 ideal points are coplanar.
- ($\implies$) No 3 ideal points are colinear.

What We'll Prove

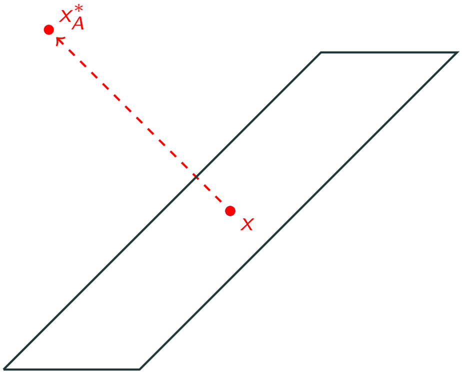
If $X = \mathbb{R}^3$ and no 4 ideal points are coplanar, this problem is manipulable:

x_A^* is only unimprovable policy.

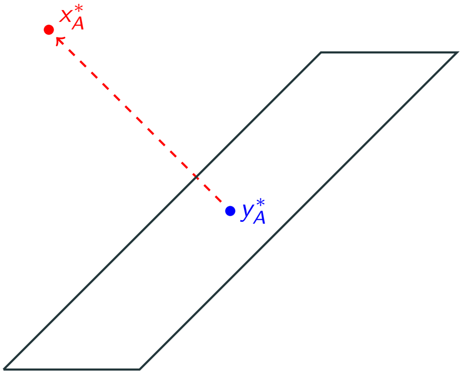


x is initial default and x_A^* is AS's favorite.

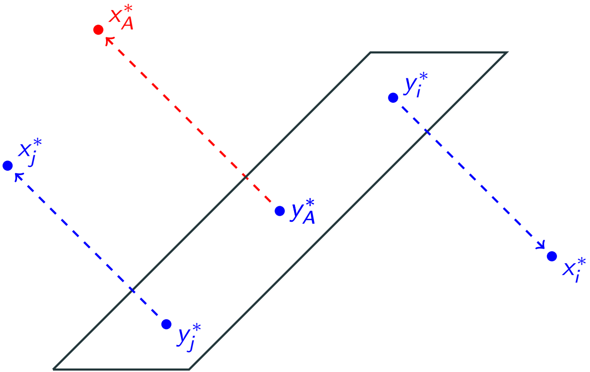
We want to show that x is improvable: $\exists z$ such that $z \succ_A x$ and $z \succ_M x$.



x_A^*
x

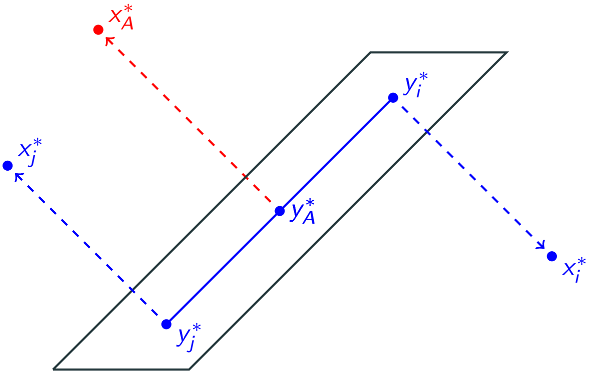


$x = y_A^*$, AS's constrained ideal point on the plane.

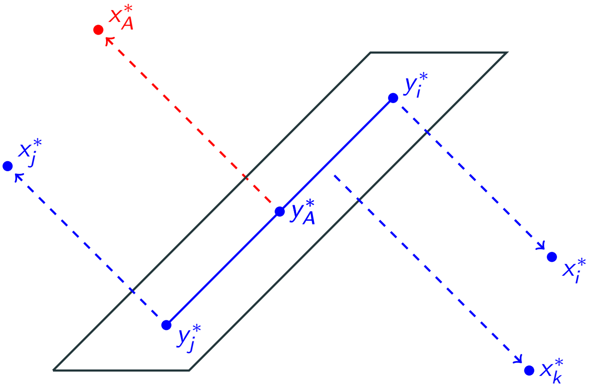


y_i^* and y_j^* are constrained ideal points for i and j.

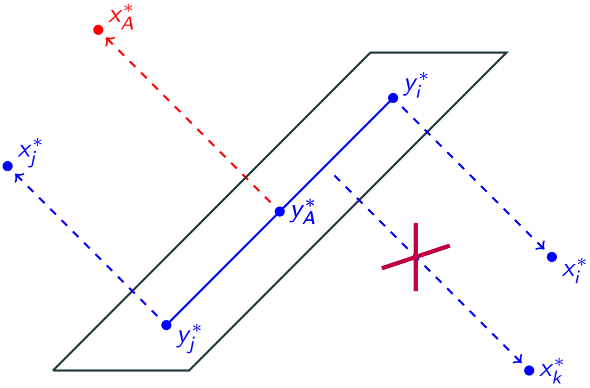
Can do this for all voters.



Claim 1: At most two constrained ideal points and y_A^* are collinear.

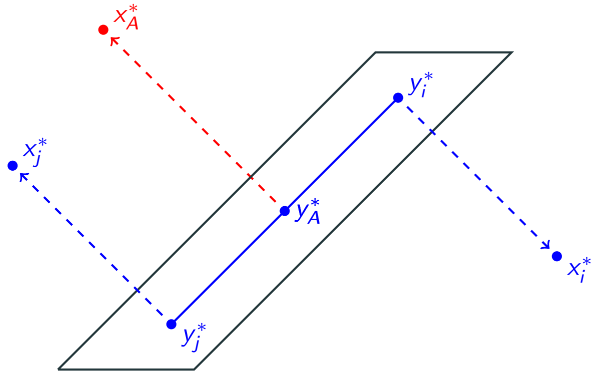


Suppose towards contradiction that this is true for a third voter k.



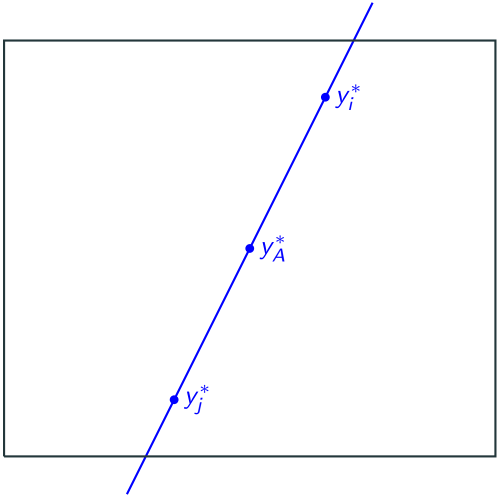
Suppose towards contradiction that this is true for a third voter k.

Then $\{x_A^*, x_i^*, x_j^*, x_k^*\}$ all lie on the same plane, violating genericity.



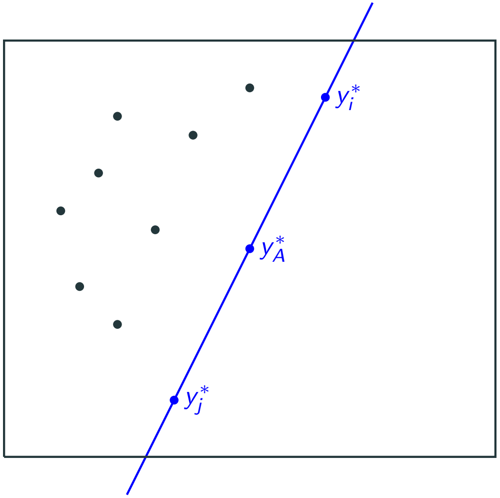
Claim 1: At most two constrained ideal points and y_A^* are collinear.

Claim 2: Either $y_i^* \neq y_A^*$ or $y_j^* \neq y_A^*$ (or both).

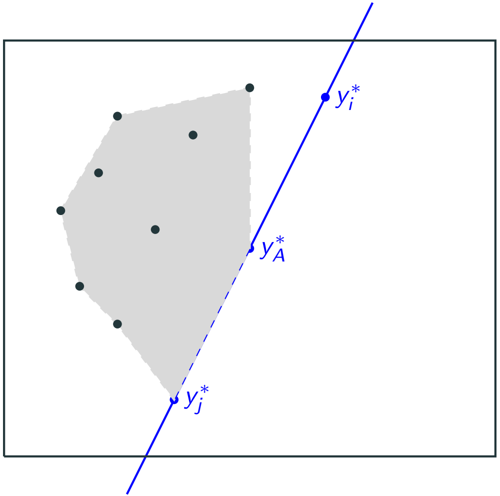


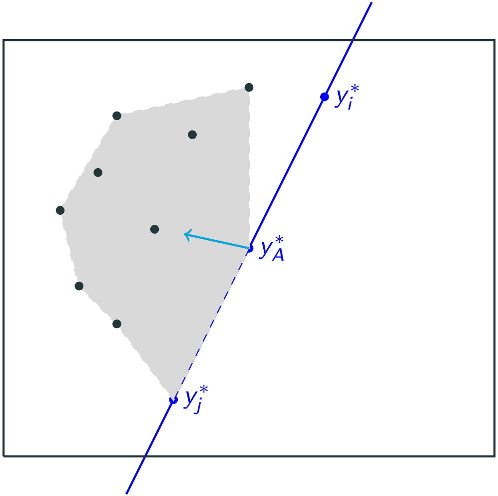
Let's look at the plane: at most 2 voter (constrained) ideal points on this line.

There are $(n-2)$ other (constrained) ideal points lurking.

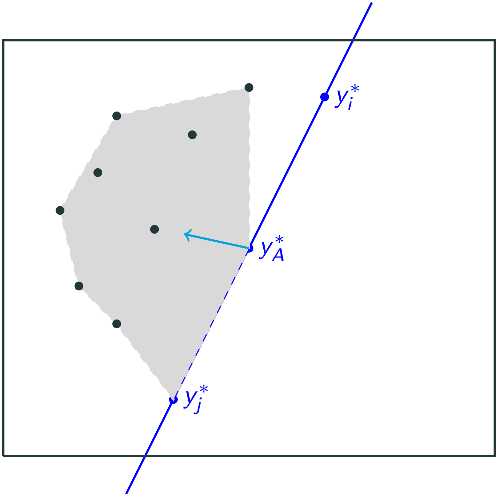


At least $(n-1)/2$ of the points lie above or below the line.


y_i^*
y_A^*
y_j^*

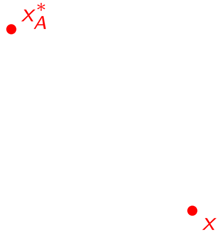


Moving in this direction makes all $(n-1)/2$ voters and voter j strictly better off.

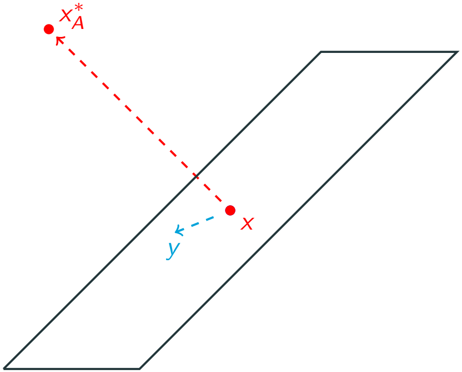


Moving in this direction makes all $(n-1)/2$ voters and voter j strictly better off.

Since y_A^* is AS's constrained ideal point, a small movement induces a second-order loss for her.

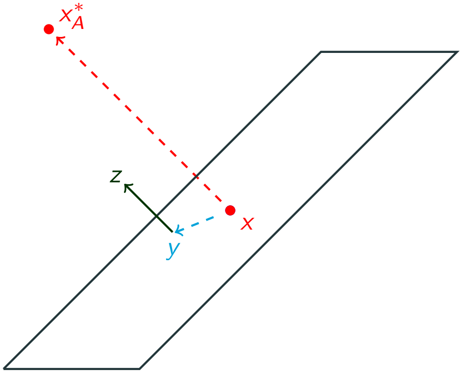


x is initial default and x_A^* is AS's favorite. We want to show that x is improvable.



We found a nearby y on the plane that makes $(n+1)/2$ voters strictly better off.

Moving from $x \to y$ induces only a second-order loss for agenda setter.



We found a nearby y on the plane that makes $(n+1)/2$ voters strictly better off.

Moving from $x \to y$ induces only a second-order loss for agenda setter.

Thus, we can find z such that $z \succ_A x$ and $z \succ_M x$.

Necessity of ≥ 3 Policy Dimensions

Single dimension:

- Euclidean prefs $\Longrightarrow \exists$ Median Voter whose ideal point x^*_{med} is a Condorcet Winner.
- Hence, all policies between x^*_A and x^*_{med} are unimprovable.

Two-dimensional case:

- **Fact:** *Manipulability fails whenever* $x^*_A \notin CH(\{x^*_1, \ldots, x^*_n\})$.
- Contrasts with McKelvey's (1976) Chaos Theorem: $\succ_M$ *is globally intransitive iff* $d \geq 2$.
- The set of unimprovable policies is a line segment (measure-0), but equilibrium dynamics force policies onto this line.

Spatial Politics: Implications

Theorem*

Spatial Politics with Euclidean prefs is (generically) Manipulable $\iff$ $d \geq 3$ policy dimensions.

$\implies$ *AS can generate Manipulable problem by bundling policy issues.*

Faced with 2D policy decision, AS can obtain her favorite policy by introducing a third policy dimension to deliberations — even if that third dimension is "settled" (i.e., AS already obtains favorite policy in that dimension).

What We Did:

Real-time agenda control without commitment in an amendment agenda with a finite horizon.

What happens if each of these is modified?

Value of Commitment: Example

Suppose z is initial default.

AS can achieve w with fixed slate $(w, \mathbf{y})$.
Not sequentially rational.

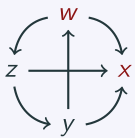
Without commitment, AS achieves only x.

A Non-Manipulable Problem

AS's Prefs

$$w \longleftarrow x \longleftarrow y \longleftarrow z$$

Majority Relation



$$a \rightarrow b \equiv b \succ a$$
$$a \equiv \text{unimprovable}$$

Adjournment Provisions

Many common legislative procedures involve cloture rules:

- Closed-rule bargaining or, equivalently, successive/Euro-Latin agendas
  - deliberations adjourn with implementation of **first proposal that passes**
- Open-rule bargaining
  - deliberations adjourn early if **(only) the current default is "moved"**

Real-time agenda control without commitment renders all of these procedures outcome-equivalent.

A Procedural Equivalence Result

Theorem

Suppose the Collective Choice Problem satisfies Generic Finite Alternatives and the generalized amendment procedure is rich.

For any game with T rounds and initial default policy x^0, the unique eqm outcome is $\phi^T(x^0)$.

The Role of Deadlines

Agenda embodies a dynamic procedure to solve static or time-indexed collective choice problem.

- Players negotiate over policy that prevails at a given calendar date τ.
- Each round of bargaining takes at least $\Delta > 0$ units of time.
- At most $T = \lfloor \tau / \Delta \rfloor$ rounds of deliberation.

Even if deadline were uncertain, our results apply so long as deadline is sufficiently predictable.

- Distribution Problems: only 3 rounds of predictability are needed for exact dictatorial power.
- Generally, AS obtains within δ of maximal payoff given T_δ rounds of predictability.

An Infinite-Horizon Model (Anesi-Siedmann'14)

No terminal round: Game ends only if AS proposes prevailing default option or amendment is rejected.

Suppose policy z is initial default.

Claim: Agenda setter achieves only y.

Logic: Voters predict that if x or w become default option, then w is implemented.

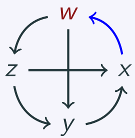
As $y \succ_M w$, voters reject moves from y to x.

Perpetual Reconsideration

AS's Prefs

$w \longleftarrow x \longleftarrow y \longleftarrow z$

Majority Relation



$a \rightarrow b \equiv b \succ a$

Manipulability ✓

Horizon Comparisons

Theorem

Suppose the collective choice problem satisfies Generic Finite Alternatives.

Then exactly one of the following two statements holds:

1. For some initial default, the agenda setter:
   (a) the agenda setter strictly prefers $2 \leq T < \infty$ rounds to a single round, and
   (b) the agenda setter strictly prefers a single round to the infinite horizon.
2. For all initial defaults, the agenda setter is equally well off across all three protocols.

Implications: Non-monotonicity + Strategic benefit of deadlines.

1. Model & Manipulability
2. Main Results
3. Distributive Politics
4. Spatial Politics
5. Commitments, Procedures, and Deadlines
6. Conclusion

Summary

New model of real-time agenda control *without commitment*.

Main finding: AS has dictatorial power $\iff$ problem is Manipulable.

- Holds under broad class of legislative procedures & voting rules

Manipulability is satisfied in canonical distributive & spatial models.

- AS may strategically create Manipulability by using pork/transfers or bundling policy issues.

The Commitment Benchmark

Back

AS commits to a strategy in the dynamic game (including horizon T).

Note: this allows for flexible proposals, unlike the literature's models of fixed agendas.

The Commitment Benchmark

Back

AS commits to a strategy in the dynamic game (including horizon T).

Note: this allows for flexible proposals, unlike the literature's models of fixed agendas.

Definition

Policy y is reachable from x if $\exists$ a sequence $\{a^k\}_{k=0}^{K}$ such that

$$y = a^K \succ_M a^{K-1} \succ_M \ldots \succ_M a^0 = x.$$

Proposition

If AS has commitment power, she obtains her favorite policy that's reachable from x^0.

Prediction coincides with classic results for "binary voting trees" (e.g., Farquharson 1969; Miller 1977).

Divide-the-Dollar with NC Tie-Breaking

Back to Theorem 3

Setting. $X = \Delta^{n+1}$ and $u_i(x) = x_i$. For simplicity, focus on the three-voter $(n = 3)$ case. Assume WLOG that $x_1^0 \geq x_2^0 \geq x_3^0$, and that $x_3^0 > 0$.

Divide-the-Dollar with NC Tie-Breaking

Back to Theorem 3

Setting. $X = \Delta^{n+1}$ and $u_i(x) = x_i$. For simplicity, focus on the three-voter ($n = 3$) case. Assume WLOG that $x_1^0 \geq x_2^0 \geq x_3^0$, and that $x_3^0 > 0$.

MPE with NC Tie-Breaking.
Voters always break ties in favor of proposal. AS proposes $\hat{\phi}(x)$ when default is x, where

$$\hat{\phi}(x) := \text{Policy in which AS} \begin{cases} \text{extracts share from richest voter,} \\ \text{breaks ties toward lower-index voters.} \end{cases}$$

Divide-the-Dollar with NC Tie-Breaking

Back to Theorem 3

Setting. $X = \Delta^{n+1}$ and $u_i(x) = x_i$. For simplicity, focus on the three-voter ($n = 3$) case. Assume WLOG that $x_1^0 \geq x_2^0 \geq x_3^0$, and that $x_3^0 > 0$.

MPE with NC Tie-Breaking.
Voters always break ties in favor of proposal. AS proposes $\hat{\phi}(x)$ when default is x, where

$$\hat{\phi}(x) := \text{Policy in which AS} \begin{cases} \text{extracts share from richest voter,} \\ \text{breaks ties toward lower-index voters.} \end{cases}$$

One-round game $\quad \rightarrow \hat{\phi}(x^0) = (0, x_2^0, x_3^0, 1 - x_2^0 - x_3^0).$

Divide-the-Dollar with NC Tie-Breaking

Back to Theorem 3

Setting. $X = \Delta^{n+1}$ and $u_i(x) = x_i$. For simplicity, focus on the three-voter $(n = 3)$ case. Assume WLOG that $x_1^0 \geq x_2^0 \geq x_3^0$, and that $x_3^0 > 0$.

MPE with NC Tie-Breaking.

Voters always break ties in favor of proposal. AS proposes $\hat{\phi}(x)$ when default is x, where

$$\hat{\phi}(x) := \text{Policy in which AS} \begin{cases} \text{extracts share from richest voter,} \\ \text{breaks ties toward lower-index voters.} \end{cases}$$

One-round game $\rightarrow \hat{\phi}(x^0) = (0, x_2^0, x_3^0, 1 - x_2^0 - x_3^0)$.

Two-round game $\rightarrow \hat{\phi}^2(x^0) = (0, 0, x_3^0, 1 - x_3^0)$.

Divide-the-Dollar with NC Tie-Breaking

Back to Theorem 3

Setting. $X = \Delta^{n+1}$ and $u_i(x) = x_i$. For simplicity, focus on the three-voter ($n = 3$) case.
Assume WLOG that $x_1^0 \geq x_2^0 \geq x_3^0$, and that $x_3^0 > 0$.

MPE with NC Tie-Breaking.
Voters always break ties in favor of proposal. AS proposes $\hat{\phi}(x)$ when default is x, where

$$\hat{\phi}(x) := \text{Policy in which AS} \begin{cases} \text{extracts share from richest voter,} \\ \text{breaks ties toward lower-index voters.} \end{cases}$$

One-round game $\rightarrow \hat{\phi}(x^0) = (0, x_2^0, x_3^0, 1 - x_2^0 - x_3^0)$.

Two-round game $\rightarrow \hat{\phi}^2(x^0) = (0, 0, x_3^0, 1 - x_3^0)$.

Three-round game $\rightarrow \hat{\phi}^3(x^0) = (0, 0, 0, 1) = x_A^*$.

AS obtains exactly her favorite policy in $T = 3$ rounds.

Divide-the-Dollar with Capricious Tie-Breaking

Back to Theorem 3

Setting. $X = \Delta^{n+1}$ and $u_i(x) = x_i$. For simplicity, focus on the three-voter ($n = 3$) case. Assume WLOG that $x_1^0 \geq x_2^0 \geq x_3^0$, and that $x_3^0 > 0$.

Divide-the-Dollar with Capricious Tie-Breaking

Back to Theorem 3

Setting. $X = \Delta^{n+1}$ and $u_i(x) = x_i$. For simplicity, focus on the three-voter ($n = 3$) case. Assume WLOG that $x_1^0 \geq x_2^0 \geq x_3^0$, and that $x_3^0 > 0$.

MPE with Capricious Tie-Breaking.

Voters always break ties in favor of proposal $\iff$ it's final or penultimate round.

AS proposes $\hat{\phi}(x)$ when default is x, where

$$\hat{\phi}(x) := \text{Policy in which AS } \begin{cases} \text{extracts share from richest voter,} \\ \text{breaks ties toward lower-index voters.} \end{cases}$$

Divide-the-Dollar with Capricious Tie-Breaking

Back to Theorem 3

Setting. $X = \Delta^{n+1}$ and $u_i(x) = x_i$. For simplicity, focus on the three-voter ($n = 3$) case.
Assume WLOG that $x_1^0 \geq x_2^0 \geq x_3^0$, and that $x_3^0 > 0$.

MPE with Capricious Tie-Breaking.

Voters always break ties in favor of proposal $\iff$ it's final or penultimate round.

AS proposes $\hat{\phi}(x)$ when default is x, where

$$\hat{\phi}(x) := \text{Policy in which AS} \begin{cases} \text{extracts share from richest voter,} \\ \text{breaks ties toward lower-index voters.} \end{cases}$$

One-round game $\quad \rightarrow \hat{\phi}(x^0) = (0, x_2^0, x_3^0, 1 - x_2^0 - x_3^0)$.

Divide-the-Dollar with Capricious Tie-Breaking

Back to Theorem 3

Setting. $X = \Delta^{n+1}$ and $u_i(x) = x_i$. For simplicity, focus on the three-voter ($n = 3$) case.
Assume WLOG that $x_1^0 \geq x_2^0 \geq x_3^0$, and that $x_3^0 > 0$.

MPE with Capricious Tie-Breaking.

Voters always break ties in favor of proposal $\Longleftrightarrow$ it's final or penultimate round.

AS proposes $\hat{\phi}(x)$ when default is x, where

$$\hat{\phi}(x) := \text{Policy in which AS} \begin{cases} \text{extracts share from richest voter,} \\ \text{breaks ties toward lower-index voters.} \end{cases}$$

One-round game $\rightarrow \hat{\phi}(x^0) = (0, x_2^0, x_3^0, 1 - x_2^0 - x_3^0)$.

Two-round game $\rightarrow \hat{\phi}^2(x^0) = (0, 0, x_3^0, 1 - x_3^0)$.

Divide-the-Dollar with Capricious Tie-Breaking

Back to Theorem 3

Setting. $X = \Delta^{n+1}$ and $u_i(x) = x_i$. For simplicity, focus on the three-voter ($n = 3$) case.
Assume WLOG that $x_1^0 \geq x_2^0 \geq x_3^0$, and that $x_3^0 > 0$.

MPE with Capricious Tie-Breaking.

Voters always break ties in favor of proposal $\Longleftrightarrow$ it's final or penultimate round.

AS proposes $\hat{\phi}(x)$ when default is x, where

$$\hat{\phi}(x) := \text{Policy in which AS} \begin{cases} \text{extracts share from richest voter,} \\ \text{breaks ties toward lower-index voters.} \end{cases}$$

One-round game $\rightarrow \hat{\phi}(x^0) = (0, x_2^0, x_3^0, 1 - x_2^0 - x_3^0)$.

Two-round game $\rightarrow \hat{\phi}^2(x^0) = (0, 0, x_3^0, 1 - x_3^0)$.

Three-round game: **No** proposals pass b/c at least two voters get 0 upon both passage & rejection.
AS can't "bribe" voters with $\varepsilon > 0$ shares b/c they'll be extracted in future!

Divide-the-Dollar with Capricious Tie-Breaking

Back to Theorem 3

Setting. $X = \Delta^{n+1}$ and $u_i(x) = x_i$. For simplicity, focus on the three-voter ($n = 3$) case.
Assume WLOG that $x_1^0 \geq x_2^0 \geq x_3^0$, and that $x_3^0 > 0$.

MPE with Capricious Tie-Breaking.

Voters always break ties in favor of proposal $\Longleftrightarrow$ it's final or penultimate round.

AS proposes $\hat{\phi}(x)$ when default is x, where

$$\hat{\phi}(x) := \text{Policy in which AS} \begin{cases} \text{extracts share from richest voter,} \\ \text{breaks ties toward lower-index voters.} \end{cases}$$

One-round game $\rightarrow \hat{\phi}(x^0) = (0, x_2^0, x_3^0, 1 - x_2^0 - x_3^0)$.

Two-round game $\rightarrow \hat{\phi}^2(x^0) = (0, 0, x_3^0, 1 - x_3^0)$.

Three-round game: **No** proposals pass b/c at least two voters get 0 upon both passage & rejection.
AS can't "bribe" voters with $\varepsilon > 0$ shares b/c they'll be extracted in future!
$\Longrightarrow$ By induction, AS's payoff is $\leq 1 - x_3^0$ even as $T \rightarrow \infty$.