A Supplementary Appendix (Not For Publication)

This Supplementary Appendix contains (i) proofs that are omitted from the main text, and (ii) statements and proofs of additional results that are described informally in the main text (Section A.2).

A.1 Omitted Proofs

Proof of Theorem 1 on p. 14 continued. Here, we prove that in the finite horizon ($\bar{t} < \infty$), the first proposer captures the entire surplus in every SPE. We denote by h^t as the full history of proposers, realized state, proposals, and voting decisions prior to the realization of uncertainty in period t. The argument proceeds by backward induction:

Base Case: At $t = \bar{t}$: for every realization of uncertainty, $s^{\bar{t}}$, the proposer $P(s^{\bar{t}})$ keeps the entire dollar in every SPE.

Inductive Step: Suppose in period $t+1$, for every realization of uncertainty, s^{t+1}, the proposer $P(s^{t+1})$ keeps the entire dollar in every SPE. We argue that in period t, for every state s^t, the proposer $P(s^t)$ must keep the entire dollar in every SPE. Observe that if the proposer $P(s^t)$ offers ϵ to every other player, any player in $L(s^t)$ has a strict incentive to accept this proposal. Therefore, for every $\epsilon > 0$, the proposer's SPE payoff is bounded below by $1-(n-1)\epsilon$, which establishes that in equilibrium, the proposer $P(s^t)$ obtains the entire dollar. □

Proof of Theorem 2 on p. 19. We begin by describing the system of equations used to solve for $\underline{w}$ and $\overline{w}$. Consider continuation values in the beginning of period t, prior to recognition and information revelation, and player i such that player i expects to not be recognized in period t. It follows from a recursive calculation that

$$\underline{w} = \left(\frac{d}{n}\right)\delta\underline{w} + \left(\frac{n-d}{n}\right)\left(\frac{d(q-d)}{(n-1)(n-d)} + \frac{(n-d-1)(q-1-d)}{(n-1)(n-d-1)}\right)\delta\overline{w},$$

where $\frac{d}{n}$ is the probability that $i \in L(s)$; $\left(\frac{n-d}{n}\right)\left(\frac{d}{n-1}\right)\left(\frac{q-d}{n-d}\right)$ is the probability $i \notin L(s)$, $P(s) \in L(s)$ and that period-t proposer includes i in the winning coalition; and finally $\left(\frac{n-d}{n}\right)\left(\frac{n-d-1}{n-1}\right)\left(\frac{q-1-d}{n-d-1}\right)$ is the probability $\{i, P(s)\} \subset \mathcal{N}\backslash L(s)$ and the period-t proposer includes i in the wining coalition. Combining this equation with

$$d\underline{w} + (n-d)\overline{w} = 1 \tag{3}$$

yields the solutions in the text. Finally, the first proposer's expected share can be written as

$$\begin{aligned}
&\left(\frac{n-d}{n}\right)(1-d\delta\underline{w}-(q-d-1)\delta\overline{w})+\left(\frac{d}{n}\right)(1-(d-1)\delta\underline{w}-(q-d)\delta\overline{w})\\
&=\left(\frac{n-d}{n}\right)(1-\delta(1-(n-d)\overline{w})-(q-d-1)\delta\overline{w})+\frac{d}{n}(1-\delta(1-(n-d)\overline{w})+\delta\underline{w}-(q-d)\delta\overline{w})\\
&=\left(\frac{n-d}{n}\right)(1-\delta+\delta(n-q)\overline{w}+\delta\overline{w})+\frac{d}{n}(1-\delta+\delta(n-q)\overline{w}+\delta\underline{w})\\
&=1-\delta+\delta(n-q)\overline{w}+\frac{\delta}{n},
\end{aligned}$$

where the first line follows from the first proposer being a member of $L^1(s^0)$ with probability $\frac{d}{n}$; the second line uses (3); the third line simplifies the expression; and the fourth line uses (3) again. The derivative of the proposer's share with respect to d is

$$\frac{\delta(n-1)(n-q)(n-\delta q-(1-\delta))}{((n-1)n-d(\delta(n-q)+(n-1)))^2}>0$$

which implies that the first proposer's share is strictly increasing in d for $d<q$. □

Proof of Theorem 3 on p. 19. If $\alpha_p > \alpha_v$, then it follows that for sufficiently large n, $d_n > q_n$ in which case Theorem 1 implies that the first proposer captures the entire surplus. Suppose that $\alpha_p \leq \alpha_v$. By our earlier result, the first proposer's share is

$$\begin{aligned}
&1-\delta+\frac{\delta}{n}+\delta(n-q_n)\overline{w}_n\\
=&1-\delta+\frac{\delta}{n}+\frac{\delta(n-q_n)(n-1)(n-\delta d_n)}{n(n(n-1)-d_n(\delta(n-q_n)+n-1))}\\
=&1-\delta+\frac{\delta}{n}+\frac{\frac{\delta(n-q_n)}{n}\frac{(n-1)}{n}\frac{(n-\delta d_n)}{n}}{\frac{(n-1)}{n}-\frac{\delta d_n(n-q_n)}{n^2}-\frac{d_n}{n}+\frac{d_n}{n^2}}
\end{aligned}$$

Taking limits as $n\to\infty$, $q_n/n\to\alpha_v$, and $d_n/n\to\alpha_p$, we obtain

$$1-\delta+\frac{\delta(1-\alpha_v)(1-\delta\alpha_p)}{1-\delta\alpha_p(1-\alpha_v)-\alpha_p}=1-\frac{\delta(\alpha_v-\alpha_p)}{1-\delta\alpha_p(1-\alpha_v)-\alpha_p}.$$

□

Proof of Theorem 4 on p. 21. We first describe the function that we use as a lower bound on the amount a proposer must share with at least one other party. Consider the function $f:\Re\to\Re$ defined by $f(y)\equiv\frac{y-\hat{\delta}(\epsilon+\rho)}{\hat{\delta}(1-\epsilon-\rho)}$. Observe that f has a unique fixed point, namely $\hat{y}=\frac{\hat{\delta}(\epsilon+\rho)}{1-\hat{\delta}(1-\epsilon-\rho)}$. The function f is both strictly increasing and *expansive*: for each $y>\hat{y}$, an induction argument establishes that

$$f^k(y) - \hat{y} = \left(\frac{1}{\hat{\delta}(1-\epsilon-\rho)}\right)^k (y - \hat{y}).$$

Since $\hat{\delta}(1-\epsilon-\rho) < 1$, it follows that for each $y > \hat{y}$, there exists a finite $\overline{k}$ such that for every $k > \overline{k}$, $f^k(y) > 1$. We use this observation to prove this result.

Let the structural state in Stage 1 of period 0 be s^0 and consider $\overline{x}(s^0)$, the highest equilibrium share that the proposer $P(s^0)$ offers to any player other than herself. Suppose towards a contradiction that $\overline{x}(s^0) > \hat{y}$. Because $s^0 \in \mathcal{P}^0_{q,\epsilon}$, an argument identical to that of Theorem 1 implies that there exist a player i in $H(s^0) \bigcap L_\epsilon(s^0)$. Player i's continuation value $V_i(s^0)$ emerges from three events:

(i) he is recognized: the rents that he captures are bounded above by 1, and the probability of this event is bounded above by ϵ;

(ii) the realized structural state in period 1 is not in $\mathcal{P}_{q,\epsilon}$: his payoffs are bounded above by 1 and the probability of this event is bounded above by ρ;

(iii) the realized structural state in period 1 is in $\mathcal{P}_{q,\epsilon}$, and player i is not recognized: this event occurs with probability at least $1-\epsilon-\rho$, and his payoff is bounded above by the most that he receives in any structural state, s^1 in $\mathcal{P}_{q,\epsilon}$, denoted by $\tilde{x}_i(s^1)$.

Combining the above implies that $V_i(s^0) \leq \epsilon + \rho + (1-\epsilon-\rho)\tilde{x}_i(s^1)$. Because the greatest share offered to any non-proposer, $\overline{x}(s^1)$ must exceed $\tilde{x}_i(s^1)$, and player i's discounted continuation value in state s^0 weakly exceeds $\overline{x}(s^0)$, it follows that

$$\frac{\overline{x}(s^0)}{\hat{\delta}} \leq V_i(s^0) \leq \epsilon + \rho + (1-\epsilon-\rho)\overline{x}(s^1)$$

or upon re-arranging that $\overline{x}(s^1) \geq f\left(\overline{x}(s^0)\right)$. Since f is strictly increasing and expansive, we are guaranteed that $f\left(\overline{x}(s^0)\right) > \overline{x}(s^0)$, which is greater than $\hat{y}$. Therefore, the same argument applies in state $s^1 \in \mathcal{P}_{q,\epsilon}$. Accordingly, there exists a sequence of states $\{s^t\}_{t\in\mathcal{T}}$ such that for each t, we have $\overline{x}(s^t) \geq f^t\left(\overline{x}(s^0)\right)$, and $\overline{x}(s^0) > \hat{y}$. Our earlier observation implies that if $\overline{t} = \infty$, a proposer eventually offers a share exceeding 1 to another player in some state, or if $\overline{t} < \infty$, a proposer in the final period offers a strictly positive share to another player. In both cases, we reach a contradiction. □

A Continuous-Time Version: The key recursive equation, translated from above, is

$$\bar{x}(s^t) \leq e^{-r\Delta}\left(1 - e^{-(\psi+\gamma)\Delta} + e^{-(\psi+\gamma)\Delta}\bar{x}(s^{t+\Delta})\right).$$

Taking differences and limits, and using L'Hopital's Rule,

$$\dot{x}(s^t) = \lim_{\Delta \to 0} \frac{x(s^{t+\Delta}) - x(s^t)}{\Delta} \geq \lim_{\Delta \to 0} \frac{\bar{x}(s^{t+\Delta})(1 - e^{-(r+\psi+\gamma)\Delta}) - e^{-r\Delta}(1 - e^{-(\psi+\gamma)\Delta})}{\Delta}$$
$$= (r + \psi + \gamma)\bar{x}(s^t) - (\psi + \gamma).$$

Therefore, if $\bar{x}(s^0) > \frac{\psi+\gamma}{r+\psi+\gamma}$, then $\dot{x}(s^0) > \epsilon$ for some $\epsilon > 0$. Since the above is true for $\dot{x}(s^t)$ at every t, eventually $\bar{x}(s^t)$ crosses 1, a contradiction.

Proof of Theorem 5 on p. 23. We redefine the cost of a coalition, $W_C(s)$: for a state s^t and coalition $C \subseteq \mathcal{N}$, let $W_C(s) \equiv \sum_{i \in C} u_i^{-1}(\delta_i V_i(s))$. Given that $V_i(s) \in [0, u_i(1)]$, $\delta_i \in (0, 1)$, $u_i(0) = 0$, and u_i is strictly increasing and continuous, we know that $W_C(s)$ is well-defined. Let

$$\underline{W}(s) \equiv \min_{\substack{C \subseteq \mathcal{N} \setminus \{P(s)\}, \\ C \cup \{P(s)\} \in \mathcal{D}}} W_C(s),$$

be the cost of the cheapest decisive coalitions for proposer $P(s)$, which are in the set

$$\mathbb{C}(s) \equiv \{C \subseteq \mathcal{N} \setminus \{P(s)\} : C \cup \{P(s)\} \in \mathcal{D} \text{ and } W_C(s) = \underline{W}(s)\}.$$

The proposals that involve creating such coalitions are

$$\mathcal{X}(s) \equiv \left\{x \in \mathcal{X} : \exists C \in \mathbb{C}(s) \text{ such that } u_i(x_i) = \delta_i V_i(s)\, \forall i \in C \text{ and } x_{P(s)} = 1 - \underline{W}(s)\right\}.$$

In an equilibrium, let $a(s)$ denote the (undiscounted) average of policies that are selected in the continuation after rejection of the proposal in state s. Because u_i is concave for each i and $\delta_i < 1$, we necessarily have $u_i(a_i(s)) > \delta_i V_i(s)$ for all i. Consequently, for any coalition C, we have $W_C(s) < \sum_{i \in C} a_i(s) \leq 1$. It follows that $1 - \underline{W}(s) > 0$, and hence that $\mathcal{X}(s)$ is non-empty.

No Delay: We first extend Lemma 1. Suppose there is a structural state s in $\mathcal{S}$ such that an equilibrium proposal offered with strictly positive probability, x', is rejected with strictly positive probability. Select some $x \in \mathcal{X}(s)$ and let $C \in \mathbb{C}(s)$ be the associated minimal winning coalition (excluding the proposer). Define a proposal x^ϵ for small $\epsilon \geq 0$ in which $u_i(x_i^\epsilon) = u_i(x_i + \epsilon)$ for every $i \in C$, $x_i^\epsilon = 0$ for every $i \notin C \cup \{P(s)\}$, and the proposer keeps $1 - \underline{W}(s) - (q-1)\epsilon$ for himself (which is feasible in light of the the fact that $1 - \underline{W}(s) > 0$). In the equilibrium, the proposal x^ϵ must be accepted by all members of C with probability 1 if $\epsilon > 0$. Because $\sum_{i \in \mathcal{N}} u_i^{-1}(\delta_i V_i(s)) <$

$\sum_{i\in\mathcal{N}} a_i(s) \leq 1$,

$$\begin{aligned}\underline{W}(s) + u^{-1}\left(\delta_{P(s)}V_{P(s)}(s)\right) &= \sum_{i\in C} u_i^{-1}\left(\delta_i V_i(s)\right) + u^{-1}\left(\delta_{P(s)}V_{P(s)}(s)\right)\\ &\leq \sum_{i\in\mathcal{N}} u_i^{-1}\left(\delta_i V_i(s)\right)\\ &< 1.\end{aligned}$$

Therefore, for sufficiently small $\epsilon > 0$, we have

$$x^{\epsilon}_{P(s)} = 1 - \underline{W}(s) - (q-1)\epsilon > u^{-1}\left(\delta_{P(s)}V_{P(s)}(s)\right).$$

Thus, conditional on x' being rejected, the proposer is discretely better off deviating to x^{ϵ} for sufficiently small $\epsilon > 0$. Conditional on x' being accepted, the proposer's share can be no greater than she obtains when offering x. Since proposal x' is rejected with strictly positive probability, she is strictly better off offering x^{ϵ} for sufficiently small $\epsilon > 0$. Therefore, no equilibrium offer x' can be rejected with strictly positive probability. □

Minimal Winning Coalition: Lemma 2 extends to this setting: if the proposer $P(s)$ chooses a policy outside $\mathcal{X}(s)$, then she can profitably deviate to such a policy (plus tiny additional payments to members of the minimal winning coalition) to obtain immediate agreement at a strictly lower cost.

An Additional Lemma: Recall that $\xi^i_P(s)$ is the equilibrium mixed action selected by proposer $P(s)$ at state s: for a proposal x in $\mathcal{X}$, let $\xi^i_P(s)(x)$ denote the equilibrium probability with which proposer $P(s)$ makes that proposal in state s. We prove an additional lemma for this setting bounding the continuation value for the coalition of losers.

Lemma 3. *Consider a time period $t < \bar{t}$ and a structural state s. The following relates costs of coalitions across periods:*

$$W_{L(s)}(s) \leq \hat{\delta}\int_{\mathcal{S}} \underline{W}(s')d\mu\left(s' \mid s\right).$$

Proof. Observe that

$$\begin{aligned}W_{L(s)}(s) &= \sum_{i\in L(s)} u_i^{-1}\left(\delta_i V_i(s)\right)\\ &\leq \sum_{i\in L(s)} u_i^{-1}\left(\hat{\delta}V_i(s)\right) \qquad (4)\\ &\leq \hat{\delta}\sum_{i\in L(s)} u_i^{-1}(V_i(s)),\end{aligned}$$

where the equality follows from the definition of $W_C(s)$, the first inequality follows from $\delta_i \leq \hat{\delta}$, and the second inequality follows from $u_i^{-1}(\cdot)$ being a convex function and $u_i^{-1}(0) = 0$.

Consider a player i in $L(s)$. This player is recognized with probability 0 tomorrow. In other words, given s, for each feasible continuation structural state tomorrow, player i is distinct from tomorrow's proposer. Therefore, player i can only expect to obtain strictly positive payoffs tomorrow in a structural state s' in which the proposer $P(s')$ makes an offer that offers a strictly positive share to player i. In that contingency, he obtains a utility that equals his discounted continuation value, namely $\delta_i V_i(s')$. Therefore, for every player i in $L(s)$,

$$V_i(s) = \int_{\mathcal{S}} \delta_i V_i(s') \sum_{x \in \mathcal{X}(s')} \mathbf{1}_{x_i > 0}\, \xi_P^i(s')(x) d\mu\left(s' \mid s\right). \tag{5}$$

Substituting (5) into (4) yields

$$\begin{aligned}
W_{L(s)}(s) &\leq \hat{\delta} \sum_{i \in L(s)} u_i^{-1} \left(\int_{\mathcal{S}} \delta_i V_i(s') \sum_{x \in \mathcal{X}(s')} \mathbf{1}_{x_i > 0}\, \xi_P^i(s')(x) d\mu\left(s' \mid s\right) \right) \\
&\leq \hat{\delta} \sum_{i \in L(s)} \delta_i u_i^{-1} \left(\int_{\mathcal{S}} V_i(s') \sum_{x \in \mathcal{X}(s')} \mathbf{1}_{x_i > 0}\, \xi_P^i(s')(x) d\mu\left(s' \mid s\right) \right) \\
&\leq \hat{\delta} \int_{\mathcal{S}} \sum_{i \in L(s)} \delta_i u_i^{-1} \left(V_i(s') \sum_{x \in \mathcal{X}(s')} \mathbf{1}_{x_i > 0} \xi_P^i(s')(x) \right) d\mu\left(s' \mid s\right) \\
&\leq \hat{\delta} \int_{\mathcal{S}} \sum_{i \in \mathcal{N} \setminus P(s')} \delta_i u_i^{-1} \left(V_i(s') \sum_{x \in \mathcal{X}(s')} \mathbf{1}_{x_i > 0} \xi_P^i(s')(x) \right) d\mu\left(s' \mid s\right), \\
&\leq \hat{\delta} \int_{\mathcal{S}} \sum_{x \in \mathcal{X}(s')} \xi_P^i(s')(x) \sum_{i \in \mathcal{N} \setminus P(s')} \mathbf{1}_{x_i > 0} \delta_i u_i^{-1}(V_i(s')) d\mu\left(s' \mid s\right), \\
&= \hat{\delta} \int_{\mathcal{S}} \sum_{x \in \mathcal{X}(s')} \xi_P^i(s')(x) \underline{W}(s') d\mu\left(s' \mid s\right), \\
&= \hat{\delta} \int_{\mathcal{S}} \underline{W}(s') d\mu\left(s' \mid s\right).
\end{aligned}$$

in which the first line is the substitution, the second line uses the convexity of $u_i^{-1}(\cdot)$ and $u_i^{-1}(0) = 0$, the third line interchanges the sum and integral and applies Jensen's Inequality, the fourth line uses the fact that if s' is a feasible continuation from s, $L(s)$ is a subset of $\mathcal{N} \setminus P(s')$, the fifth line rearranges terms by interchanging summation and applies the convexity of $u_i^{-1}(\cdot)$, the sixth line uses that by definition, for each x in $\mathcal{X}(s')$, $\sum_{i \in \mathcal{N} \setminus P(s')} \mathbf{1}_{x_i > 0} \delta_i V_i(s') = \underline{W}(s')$, and the seventh line uses the generalized Lemma 2 to note that $\sum_{x \in \mathcal{X}(s')} \xi_P^i(s')(x) = 1$. □

We now prove the theorem by contradiction. Suppose the state in Stage 1 of period 0 is s^0, and that a policy proposed with positive probability in which the proposer $P(s^0)$ offers a strictly positive amount, x, to another player, in which case $\underline{W}(s^0) \geq x$. Since the bargaining process exhibits one-period decisive predictability, there exists a set of coalition partners C that excludes $P(s^0)$ such that $C \bigcup \{P(s^0)\}$ is in $\mathcal{D}$, and C is a subset of $L^1(s^0)$. By definition, $\underline{W}(s^0) \leq W_C(s^0)$ and by monotonicity, $W_C(s^0) \leq W_{L(s^0)}(s^0)$. Therefore, $W_{L(s^0)}(s^0)$ must be no less than x. Lemma 3 implies that there must exist a structural state s^1 such that $\underline{W}(s^1) \geq x/\hat{\delta}$. Since $\underline{W}(s^1)$ is defined to be the cost of the cheapest decisive coalition partners for proposer $P(s^1)$, the same argument as above implies that $W_{L(s^1)}(s^1)$ must also be no less than $x/\hat{\delta}$. Therefore, by induction, there exists a sequence of states $\{s^t\}_{t\in\mathcal{T}}$ such that for each t, $s \in \mathcal{S}$, and $\underline{W}(s^t) \geq \frac{x}{\hat{\delta}^t}$. If $\bar{t} = \infty$, $\hat{\delta} < 1$ implies that $\underline{W}(s^t)$ eventually exceeds $\sum_{i\in\mathcal{N}} u_i(1)$, which is beyond the range of feasible payoffs; if $\bar{t} < \infty$, the same argument implies that the proposer at $\bar{t}$ does not appropriate the entire surplus in some state $s^{\bar{t}}$. In both cases, we have reached a contradiction. □

Proof of Theorem 6 on p. 23. Observe that if $k = q$, Theorem 6 follows from Lemma 2: any proposal in which a proposer offers a strictly positive amount to a non-veto player is not in $\tilde{\mathcal{X}}^t(s^t)$.

Now suppose that $k < q < n$: it must be that there are at least two non-veto players. Observe that for every state s^t, there exists $\tilde{x}^t(s^t)$ such that for every offer $x \in \mathcal{X}^t(s^t)$, $\tilde{x}^t(s^t) = \max_{i\notin(\{P(s)\}\cup\{1,\ldots,k\})} x_i$. Our claim is that for every $s^0 \in \mathcal{S}^0$, $\tilde{x}^0(s^0) = 0$. Suppose towards a contradiction that $\tilde{x}^0(s^0) > 0$. Consider the set of non-veto players whose support cannot be secured for shares less than $\tilde{x}^0(s^0)$:

$$\tilde{H}^0(s^0) \equiv \left\{i \in \{k+1,\ldots,n\}\backslash\{P(s^0)\} : \delta_i V_i^1(s^0) \geq \tilde{x}^0(s^0)\right\}.$$

$\tilde{H}^0(s^0)$ must have a cardinality of at least $n-(q-1)$ because otherwise proposer $P(s^0)$ would be able to form a coalition of veto and non-veto players without having to offer $\tilde{x}^0(s^0)$ to any player. Therefore, $\tilde{H}^0(s^0) \bigcap L^1(s^0)$ is non-empty. Therefore, there must exist some state s^1 such that player i is offered at least $\tilde{x}^0(s^0)/\delta_i$, which implies that $\tilde{x}^1(s^1) \geq \tilde{x}^0(s^0)/\hat{\delta}$. By induction (as before), there must then exist a state in which a proposer shares more than the entire surplus (if $\bar{t} = \infty$) or offers a strictly positive share in $\bar{t}$ (if $\bar{t} < \infty$), both of which are contradictions. □

Proof of Theorem 7 on p. 24. Because one-period predictability is perfect (of degree $n-1$), players today know the identity of tomorrow's proposer. We denote the identity of *tomorrow's* proposer anticipated today in state s by $P^1(s)$, distinguishing it from the identity of *today's* proposer, $P(s)$. At $\bar{t}$, the proposer $P(s^{\bar{t}})$ forms a minimal winning coalition with the $(n-1)/2$ other players who obtain the lowest amount from the default option: because majority improvements are possible,

$$x^D_{P(s^{\bar{t}})} < 1 - \min_{\substack{C\subseteq\mathcal{N}\backslash\left\{P(s^{\bar{t}})\right\},\\ |C|=(n-1)/2}} \sum_{j\in C} x^D_j.$$

Therefore, proposer $P(s^{\bar{t}})$ is strictly better off from the acceptance of this proposal than her disagreement payoff, and all players in C are indifferent between accepting and rejecting this proposal (and in equilibrium, they vote to accept it). Observe that regardless of the identity of $P(s^{\bar{t}})$, that proposer never includes any player from $(n+3)/2, \ldots, n$ in her minimal winning coalition. So if an agreement has not been reached previously, the continuation payoff at the beginning of period $\bar{t}$ for any player $i \notin \{1, \ldots, \frac{n+1}{2}\} \cup \{P(s^{\bar{t}})\}$ is 0.

Consider negotiations in the penultimate period, $\bar{t}-1$. There are two cases to consider:

1. $P^1(s^{\bar{t}-1}) > (n-1)/2$: If there is disagreement today, the next period proposer forms a minimal winning coalition with players $\{1, \ldots, (n-1)/2\}$. Therefore, all players in $\{(n+1)/2, \ldots, n\} \cap L(s^{\bar{t}-1})$ expects 0 payoffs in the event of disagreement today. There are $(n-1)/2$ players in this set. If $P(s^{\bar{t}-1}) \leq (n-1)/2$ or $P(s^{\bar{t}-1}) = P^1(s^{\bar{t}-1})$, then she can guarantee passage of a proposal in which she offers ϵ to each player in this set, and therefore, in equilibrium, she captures the entire surplus. Otherwise, proposer $P(s^{\bar{t}-1}) > (n-1)/2$, in which case, she can obtain the agreement of $(n-3)/2$ other players at no cost. She then includes player 1 and obtains $1 - x_1^D$.

2. $P^1(s^{\bar{t}-1}) \leq (n-1)/2$: If there is disagreement today, the next period proposer forms a minimal winning coalition with other players in $\{1, \ldots, (n+1)/2\}$. Therefore, all players in $\{(n+3)/2, \ldots, n\}$ expect 0 payoffs in the event of disagreement today. There are $(n-1)/2$ players in this set. If $P(s^{\bar{t}-1}) \leq (n+1)/2$, then she obtains the entire surplus in equilibrium. Otherwise, she can obtain the agreement of $(n-3)/2$ other players at no cost. If $1 \neq P^1(s^{\bar{t}-1})$, the proposer offers a share of x_1^D to player 1 and otherwise, she offers x_2^D to player 2.

Consider negotiations in period $\bar{t}-2$. All players anticipate, in equilibrium, that if there is disagreement today, the only players who may expect a strictly positive surplus are in $\{P^1(s^{\bar{t}-2}), 1, 2\}$, and the remaining $n-3$ players expect zero surplus. Proposer $P(s^{\bar{t}-2})$ captures the entire surplus if $n-3 \geq \frac{n+1}{2}$, which is equivalent to $n \geq 7$. Therefore, the proposer in period $\bar{t}-2$ captures the entire surplus. By induction, the first proposer captures the entire surplus in every SPE. □

An infinite horizon Game: Consider an infinite horizon game with a random deadline. The state ω encodes the deadline: the final period is $\bar{t}(\omega) < \infty$, which is an $\mathcal{F}$-measurable function. We assume that for all ω in Ω, $\bar{t}(\omega) \geq 2$, so it is common knowledge that negotiations proceed for at least three periods. Players receive information about the deadline through the signal received in period t, $\sigma^t(\omega)$, and based on this information, they form a partition over the states of nature.

Definition 4. *The deadline is* one-period predictable *if in each period t, and for all ω such that $\bar{t}(\omega) = t+1$ and ω' such that $\bar{t}(\omega') > t+1$, there does not exist a member of the partition $\mathcal{S}^t$ that contains both ω and ω'.*

One-period predictability of the deadline guarantees that players know, one-period in advance, as to whether the next period of negotiations is the final period. We view this as a modest requirement, ruling out settings in which players cannot anticipate a deadline even in the preceding period.

Theorem 10. *If the deadline is one-period predictable, there exists a pure strategy SSPE in which the first proposer captures the entire surplus.*

Proof. We construct a pure strategy SSPE. Suppose that the state is s^t.

1. For all $\omega \in s^t$, $\bar{t}(\omega) \in \{t, t+1\}$: the proposer $P(s)$ and other players follow the strategy profile outlined in Theorem 7 for the final and penultimate periods.

2. For all $\omega \in s^t$, $\bar{t}(\omega) > t+1$: the proposer $P(s)$ offers 0 to each player. Each player votes in favor of any proposal that assures her at least her continuation value, and otherwise rejects. In equilibrium, all players $L(s) \cap \{P^1(s^t), 1, 2\}$ vote to accept the proposal.

Observe that no player has any incentive to deviate in the proposing or voting stages. □

Proof of Theorem 8 on p. 26. Define a policy x proposed by player p to be *movable* in period t if $x_j \geq \frac{\delta}{1+\delta k}$ for each j in $A^t(p)$. We write $M^t(p)$ for the set of movable policies by player p in period t. Consider a strategy profile in which:

1. In every period t for which there is no proposal on the table, the proposer p^t offers $\frac{\delta}{1+\delta k}$ to each amender and 0 to all others.

2. When voting on a proposal in period t that has been moved by each amender in A^t, each player votes to accept the proposal unconditionally unless he is either the proposer p^{t+1} or an amender in $A^{t+1}(p^{t+1})$. The proposer in period $t+1$ votes to accept the proposal if and only if he obtains at least $\frac{\delta}{1+\delta k}$, and the amender votes to accept if and only if he obtains at least $\frac{\delta^2}{1+\delta k}$. Define a proposal to be *passable* if it satisfies these conditions.

3. In period t, if the proposal on the table is movable, then each amender moves the proposal. If it is neither movable nor passable, then assuming previous amenders have moved the proposal, each a_i^t offers an amendment to keep $\frac{1}{1+\delta k}$ for himself and share $\frac{\delta}{1+\delta k}$ with each amender in the set $A^{t+1}(a_i^t)$. In the case where the proposal is passable but not movable, let i' denote the last amender for whom the amount offered is strictly less than $\frac{\delta}{1+\delta k}$. For all $i \leq i'$, a_i^t offers the same amendment just described. For all $i > i'$ (if any), a_i^t moves the proposal.

4. When voting in period t between a proposal x proposed by player p and an amendment x' by player p', each player i votes for x if and only if

   - $x \in M^{t+1}(p)$ and $x' \in M^{t+1}(p')$, and $x_i > x'_i$,
   - or $x \in M^{t+1}(p)$ and $x' \notin M^{t+1}(p')$,
   - or $x \notin M^{t+1}(p)$, $x' \notin M^{t+1}(p')$, and i is in $A^{t+1}(p)$.

First, as a preliminary observation, we note that all movable proposals are passable. If $k \geq \frac{n-1}{2}$, then $p^t \cup A^t(p^t)$ has cardinality of at least $\frac{n+1}{2}$, so the current proposer and amenders can pass a proposal with no other support. According to the strategies, all members of that group will vote in favor of a movable proposal, so it is passable. If $k < \frac{n-1}{2}$, the set of players not in $p^{t+1} \cup A^{t+1}(p^{t+1})$ has cardinality of at least $\frac{n+1}{2}$, and can pass a proposal with no other support. According to the strategies, all members of that group will vote in favor of a movable proposal, so it is passable.

We prove that, for this strategy profile, no player has a profitable deviation for any history by considering each of the three roles separately: proposer, amender, and voter.

- **Proposer:** Suppose there is no offer on the table, so the proposer p^t must make an offer: any proposal that offers less than $\frac{\delta}{1+\delta k}$ to a player j in A^t is amended by that player and defeated. Since no proposal accepted in equilibrium in the continuation game offers a higher discounted expected payoff to the proposer p^t than $\frac{1}{1+\delta}$, he has no incentive to deviate to any proposal that offers less to amender j than $\frac{\delta}{1+\delta k}$. Of the proposals that are accepted in equilibrium, the equilibrium proposal maximizes the proposer's payoff.

- **Amender:** Suppose first that the current proposal on the table in period t is movable. The proposal is also passable, so moving it leads to its implementation (given continuation strategies), yielding a payoff of at least $\frac{\delta}{1+\delta k}$ for the amender. Amending the proposal cannot generate a strictly higher payoff for the amender given prescribed behavior in the continuation game.

  Next suppose the current proposal is neither movable nor passable. Moving the proposal results in implementation of some other policy one period hence, with an expected discounted payoff no greater than $\frac{\delta}{1+\delta k}$ (given continuation strategies). By proposing the amendment prescribed by the equilibrium strategies, the amender can achieve a discounted payoff of $\frac{\delta}{1+\delta k}$, which is (weakly) greater.

  Finally suppose the current proposal is passable but not movable. Amender $a^t_{i'}$ (where i' is defined in part 3 of the description of the equilibrium strategies) plainly has a strict incentive to amend the proposal by offering to keep $\frac{1}{1+\delta k}$ for himself and share $\frac{\delta}{1+\delta k}$ with each amender in the set $A^{t+1}(a^t_{i'})$ (given that this proposal will be implemented one period hence, and that no proposal more favorable to i' would be implemented). Anticipating this successful amendment, each amender i playing prior to i' has a strict incentive (by induction) to offer an analogous amendment. For $i > i'$, a^t_i can obtain an immediate payoff not less than $\frac{\delta}{1+\delta k}$ by moving the proposal (because subsequent amenders will move it and it is passable), and cannot obtain a greater discounted payoff by offering an amendment.

- **Voting Decisions:** By construction, players cast votes in favor of the alternative that yields their highest continuation payoff.

We now argue uniqueness of these SSPE payoffs. Using the argument in Proposition 4 of Baron and Ferejohn (1989), it follows that each evaluator and proposer have identical continuation values. Analogous to Theorem 1, all players who are not evaluators today must obtain 0 in an SSPE. Thus, in an SSPE, the proposer's share, v, must solve $v = 1 - \delta k v$, which generates the payoffs described in Theorem 8. □

Proof of Theorem 9 on p. 27. We exhibit the role that germaneness restrictions play by considering a particular extensive-form:

- If player p^0 is the first proposer, all $n-1$ others must move the proposal for it to be put to a vote.
- If any non-proposer amends it, then all vote between the amendment and the original proposal, and all other than p^0 must move it.
- If all move the proposal, then it is put to a vote. If rejected, then a new proposer is selected in period 1 without any germaneness restriction. If accepted, the proposal is accepted.

For this problem, we consider the following system of equations: for z in the unit simplex, and an original proposer i, let $w_k(z,i)$ satisfy for $k \neq i$:

$$z_k + w_k = \delta \left(z_k + \frac{\epsilon}{2} - \sum_{l \notin \{i,k\}} w_l \right)$$

It can be verified that this system of equations admits a unique solution. Consider z' such that $z'_k = 0$ for all $k \neq i$, then for every k,

$$w_k(z', i) = \frac{\delta\epsilon}{2(1+\delta(n-2))} \text{ for every } k \neq i$$

We construct an equilibrium in which each proposer proposes to keep the entire dollar, the first amender j amends that proposal to give $1 - \frac{\epsilon}{2}$ to the proposer, give w_k for every $k \notin \{i,j\}$, and keep the remainder for herself. On path, after the amendment, each amender moves the proposal, it's put to a vote, and is accepted.

Define a proposal x to be *passable* if

$$\left| \left\{ j : j \neq p^{t+1} \text{ and } x_j \geq \frac{\delta\epsilon}{2(1+\delta(n-2))} \right\} \right| \geq \frac{n+1}{2}.$$

Define an amendment x in period t to be *movable* if it is passable, and $x_j \geq z_j + w_j(z,i)$ for every $j \neq p^t$, and who hasn't moved the proposal already.

We describe the strategy profile in greater detail as follows:

1. Each proposer proposes to keep the entire surplus for herself.
2. When the proposer p^t proposes z, and this is the proposal on the table, an amender j amends it to offer $w_k(z,p^t)$ to players $k \notin \{j, p^t\}$, offers $z_{p^t} - \frac{\epsilon}{2}$ to the proposer, and keeps the rest for herself.

3. An amender moves an amendment on the table iff it is movable. If the amendment is not movable, then she amends it to offer $w_l(z', p^t)$ for every $l \notin \{k, p^t\}$, offers $1 - \frac{\epsilon}{2}$ to p^t, and keeps the rest to herself.

4. At the final voting stage, all players other than tomorrow's proposer who obtain at least $\frac{\delta\epsilon}{2(1+\delta(n-2))}$ vote in favor of the proposal.

5. At the voting stage between an amendment and the proposal on the table,
   - if both are movable: then a voter votes for whichever offers her a higher discounted continuation payoff, breaking ties in favor of the proposal on the table.
   - if the amendment is not movable, then a voter votes for the proposal on the table.

Below, we verify incentives at each of these stages.

Proposer: Since the proposer expects to lose $\frac{\epsilon}{2}$ for any proposal that she makes, a proposer has no gain from deviating.

First Amender: By following equilibrium strategies, the first amender j obtains a payoff of

$$\delta(z_j + \frac{\epsilon}{2} - \sum_{l \notin \{j, p^t\}} w_l) > \delta(z_j + w_j),$$

where the RHS is how much she obtains if she proposes something that is not movable. Any other deviation generates a lower surplus.

Future Amenders: Note that by amending, each amender k expects to obtain $\delta(z_k + \frac{\epsilon}{2} - \sum_{l \notin \{k, p^t\}} w_l)$ and thus, is willing to move any proposal that offers her at least that much.

Voting: By construction, players cast votes in favor of the alternative that yields the highest continuation payoff.

□

A.2 Additional Results

A.2.1 A Durable Version of Imperfect Predictability (Section 5.3)

Here, we return to the alternative view of bargaining power to which our results on imperfect predictability apply. In the main text, we highlighted how in the model of Section 5.3, bargaining power is highly variable, and in that case, we derived a lower bound on how much the proposer captures when d players in each period may be excluded. This section shows that durability amplifies our results, and so transience offers a lower bound on the impact of predictability.

Particularly, following Simsek and Yildiz (2016), we consider a sequence of bargaining games in which the period between offers, Δ, converges to 0. At any point in time, each of $n-d$ players have strictly positive bargaining power, and share it equally, and d players have no bargaining power at all. The set of players who have bargaining power changes with the arrival of a Poisson shock whose rate is ζ. When that Poisson shock arrives, a new set of $n-d$ players is drawn independently from the past. In the time between shocks, the set of players with bargaining power remains unchanged.[29] Let $\underline{w}_\Delta$ be the expected payoff for a player who has no chance of proposing today and $\overline{w}_\Delta$ be the expected payoff for a player who has a chance of proposing today. It follows from a recursive calculation that $\underline{w}_\Delta$ equals

$$e^{-\zeta\Delta}(e^{-r\Delta}\underline{w}_\Delta) + (1-e^{-\zeta\Delta})\left(\frac{d}{n}e^{-r\Delta}\underline{w}_\Delta + \frac{n-d}{n}\left(\frac{d(q-d)}{(n-1)(n-d)} + \frac{(n-d-1)(q-1-d)}{(n-1)(n-d-1)}\right)e^{-r\Delta}\overline{w}_\Delta\right),$$

where the first term corresponds to payoffs in the absence of a Poisson shock, and the second term corresponds to the payoffs after a Poisson shock, derived exactly as in the proof of Theorem 2. Substituting $K = \frac{d(q-d)}{(n-1)(n-d)} + \frac{(n-d-1)(q-1-d)}{(n-1)(n-d-1)} < 1$, one obtains

$$\begin{aligned}
\underline{w}_\Delta &= e^{-\zeta\Delta}(e^{-r\Delta}\underline{w}_\Delta) + (1-e^{-\zeta\Delta})\left(\frac{d}{n}e^{-r\Delta}\underline{w}_\Delta + \frac{n-d}{n}Ke^{-r\Delta}\overline{w}_\Delta\right)\\
&= e^{-(\zeta+r)\Delta}\underline{w}_\Delta + (1-e^{-\zeta\Delta})\left(\frac{d}{n}e^{-r\Delta}\underline{w}_\Delta + \frac{Ke^{-r\Delta}}{n}(1-d\underline{w}_\Delta)\right)\\
&= \frac{(1-e^{-\zeta\Delta})Ke^{-r\Delta}}{n(1-e^{-(\zeta+r)\Delta} - \frac{d}{n}e^{-r\Delta}(1-e^{-\zeta\Delta})(1-K))},
\end{aligned}$$

where the first equality uses the definition of K, the second equality uses $(n-d)\overline{w}_\Delta + d\underline{w}_\Delta = 1$, and the third equality simplifies the expression. Notice that both the numerator and denominator involve terms that are vanishing to 0 as $\Delta \to 0$, and so L'Hopital's Rule yields

$$\begin{aligned}
\underline{w}_0 &= \frac{\zeta K}{n(\zeta + r - \frac{d}{n}\zeta(1-K))}\\
&= \frac{K}{n(1+\frac{r}{\zeta} - \frac{d}{n}(1-K))}.
\end{aligned}$$

Observe that $\frac{r}{\zeta}$ measures the effective durability of bargaining power, when one interprets player i's bargaining power as the probability of her being the proposer right before the proposer is selected. Taking the durability rate, $\frac{r}{\zeta}$, to equal 0 corresponds to our model studied in Section 5.3 in which power is highly transient (and the expression is identical once one imposes $\delta = 1$). By contrast, the limit as $\frac{r}{\zeta}$ diverges to ∞ corresponds to a setting where the current set of players with bargaining power persists forever.

[29] To be clear on the timing: first, a proposer is recognized, then the Poisson shock strikes specifying the set of $n-d$ players who share bargaining power until the next Poisson shock, then the proposer makes a proposal, and then all vote on that proposal.

Because $\underline{w}_0$ is decreasing in $\frac{r}{\zeta}$, and therefore $\overline{w}_0$ is increasing in that same term, our results highlight how durable bargaining power amplifies the gap between strong players who have bargaining power and weak players who do not.

A.2.2 Inequity Aversion (Section 6.4)

In each period, each player has a $1/n$ probability of being the proposer, independently of the past, and the period-$(t+1)$ proposer is revealed in period t. Consider a SSPE of this environment: it is straightforward to establish that agreement is immediate and offers are made to a minimal winning coalition. Suppose that in equilibrium the proposer p^t offers y to each member of his coalition, and includes player $i \neq p^{t+1}$ in his coalition. Player i's payoff from accepting the equilibrium proposal is

$$U = y - \frac{\alpha}{n-1}(1 - qy) - \frac{\beta}{n-1}(n-q)y, \tag{6}$$

in which the first term is player i's selfish payoff, the second term is her loss from disadvantageous inequality with respect to the proposer, and the third is her loss from advantageous inequality. By contrast, her payoff from rejecting the equilibrium proposal is

$$\delta \left[\frac{1}{n}(-\frac{\alpha}{n-1}) + \frac{n-1}{n} \left\{ \frac{q-1}{n-1} U + \left(\frac{n-q}{n-1} \right) \left(-\frac{\alpha}{n-1} \right) \right\} \right]. \tag{7}$$

The first term represents the payoff from being *excluded* from the coalition in the event that she is identified in period $t+1$ as the period-$(t+2)$ proposer (which occurs with probability $1/n$).[30] The second term represents the complementary event: with probability $(q-1)/(n-1)$, proposer p^{t+1} includes player i in her coalition and offers her U, and with probability $(n-q)/(n-1)$, she is excluded, in which case she obtains the exclusion payoff. Because player i must be just indifferent between accepting and rejecting a proposal, equating (6)-(7) implies that

$$U = -\frac{n-q+1}{n-\delta(q-1)} \left[\frac{\delta\alpha}{n-1} \right]$$
$$y = \frac{\alpha n (1-\delta)}{((n-1) + \alpha q - \beta(n-1))\,(n - \delta(q-1))}$$

The above computations imply the following corollary.

Corollary 1. *As $\delta \to 1$, the first proposer captures the entire surplus in every MPE.*

A.2.3 Political Maneuvers

Our results extend seamlessly to environments in which players can maneuver for bargaining power or otherwise influence the selection of future proposers. We model a setting in which in each period t,

[30] Observe that with inequity averse preferences, the payoff from receiving a share of 0, independently of how others divide the surplus, is $-\frac{\alpha}{n-1}$.

prior to the arrival of information and the selection of a proposer, each player i (potentially including a Chair, denoted $i = 0$, in addition to the negotiators) chooses a (potentially) costly maneuver m_i^t from some set M_i, and that the entire history of maneuvers up to that point (in addition to past random shocks and proposers) influences recognition in period t. These maneuvers may represent payments made to the Chair, or the Chair deciding the order of proposers (or taking actions so as to influence the recognition process).

Suppose that each legislator $i = 1, \ldots, n$ and the Chair, $i = 0$, can choose (potentially) costly maneuvers m_i in each period from some set M_i that has persistent effects on recognition. We describe in order the timing of maneuvers, the recognition rule, the payoff relevant state, the appropriate predictability conditions and our formal results.

Timing: At the beginning of period t, players engage in political maneuvers. Each player i simultaneously chooses an action variable m_i^t from the feasible set of maneuvers, M_i, a non-empty and compact subset of a Euclidean space. We write $M \equiv M_0 \times \ldots \times M_n$. The selected vector of maneuvers in period t is $m^t = (m_0^t, ..., m_n^t)$, which is observed by all players. We let $h_m^t = (m^0, \ldots, m^t)$ denote the full history of maneuvers up to and including that of period t.

After the maneuvers are selected, players proceed to the **Information and Recognition** stage described in Section 4. We let H_m^t denote the set of possible histories of maneuvers up to and including those of time t, and $H_m = \bigcup_{t \in \mathcal{T}} H_m^t$ denote the set of all possible histories of maneuvers. The period t recognition rule is represented by a deterministic function $\widetilde{P}^t : H_m^t \times H_P^{t-1} \times \Omega \to \mathcal{N}$ in which H_P^{t-1} is the set of possible proposer histories, and Ω is the state space. Because the state of nature and the history of maneuvers recursively determine the entire sequence of proposers, we can write the recognition rule more compactly as $P^t : H_m^t \times \Omega \to \mathcal{N}$. After the revelation of information and recognition, the proposer p^t proposes a policy in $\mathcal{X}$ and others vote in a fixed sequential order. The proposal is implemented if and only at least q players (including the proposer) vote in favor.

Payoffs: We augment each legislator's payoff in Section 4 with that from maneuvers; substantively we assume that no legislator has any interest in prolonging negotiations because he intrinsically enjoys the process of political maneuvering. For a history $h_m^t \in \mathcal{H}_m$, let $v_i : \mathcal{H}_m \to \Re$ represent player i's costs from that history incurred at time t. If offer x is accepted at time t, legislator i's payoff is

$$u_i\left(x, t, h_m^t\right) = \delta^t x_i - \sum_{\tau=0}^{t} \delta_i^\tau v_i\left(h_m^\tau\right).$$

We assume that for all t, and all $h_m^t \in \mathcal{H}_m$, $v_i\left(h_m^t\right) \geq 0$. Thus, maneuvering is (potentially) costly, and prolonging negotiations cannot be motivated by the desire for further maneuvering. For many applications, it suffices to consider a special case of v_i in which the only dimension of the history of maneuvers, h_m^t, that is costly at time t is the current individual maneuver, m_i^t. However, our results also accommodate settings in which the cost of maneuvering is affected by the maneuvers of others and one's own past maneuvers.

For the Chair's preferences, we write

$$u_0\left(x,t,h_m^{(t)}\right) = \delta_0^t W(x) - \sum_{\tau=0}^{t} \delta_0^\tau v_0\left(h_m^\tau\right),$$

in which $W(x)$ represents her payoffs from a policy x. We make no restrictions on v_0.

Markov Perfect Equilibria: We augment our description of structural states and equilibria to account for the possibilities for maneuvering. In the maneuvering stage of period t, let $\tilde{s}_M^t \equiv (h_m^{t-1}, s^{t-1})$ denote all past maneuvers and all that is known after period $t-1$ about future recognition. We write $\tilde{s}_P^t \equiv (h_m^t, s^t)$ as the state at the proposal stage, in which both the maneuvers and information revealed at period t are included. Let S_M^t denote the set of possible states for the maneuvering stage of period t. We let $S_{P,i}^t$ denote the collection of all states for the proposal stage consistent with player i being the proposer. An MPE is an SPE in which each player's equilibrium strategy can be written as a sequence of functions $\left(\xi_M^{i,t}, \xi_P^{i,t}, \xi_V^{i,t}\right)_{t\in\mathcal{T}}$ such that $\xi_M^{i,t} : S_M^t \to \Delta M_i$ is player i's randomization over maneuvers in period t in structural state $\tilde{s}_M^t$, $\xi_P^{i,t} : S_{P,i}^t \to \Delta X$ is player i's randomization over proposals when recognized in period t in structural state $\tilde{s}_P^t$, and $\xi_V^{i,t} : S_P^t \times \mathcal{X} \to \Delta\{\text{yes}, \text{no}\}$ is player i's randomization whether to vote in favor of the proposal in period t in structural state s_P^t.

Predictability: Using the above notation, we can extend our notions of predictability to account for political maneuvers. If the profile of maneuvers at $t+1$ is m^{t+1}, then the sequence of signals identify that the member of the partition $\mathcal{S}^t$ that ω is in is s^t, then player i is recognized at $t+1$ if and only if ω is in

$$\Omega_i^M(h_m^t, m^{t+1}, s^t) \equiv \left\{\omega \in s^t : P^{t+1}\left((h_m^t, m^{t+1}), \omega\right) = i\right\},$$

which has probability $r_i^M(h_m^t, s^t, m^{t+1}) \equiv \mu(\Omega_i^M(h_m^t, m^{t+1}, s^t)|s^t)$. A player is a *loser conditional* on m^{t+1} in structural state $s_P^t = (h_m^t, s^t)$ if in period $t+1$, he is definitely not the proposer if the period-$t+1$ profile of maneuvers is m^{t+1}:

$$L_C^{t+1}(s_P^t, m^{t+1}) \equiv \left\{i \in \mathcal{N} : r_i^M(s_P^t, m^{t+1}) = 0\right\}.$$

The player is an *unconditional loser* if he is not the proposer regardless of m^{t+1}:

$$L_U^{t+1}(s_P^t) \equiv \bigcap_{m^{t+1}\in M} L_C^{t+1}(s_P^t, m^{t+1}).$$

We offer two distinct notions of predictability.

Definition 5. *The bargaining process exhibits one-period* ***unconditional predictability*** *of degree* **d** *if* $\left|L_U^{t+1}(s_P^t)\right| \geq d$ *for all* s_P^t *in* S_P^t *and* t *in* $\mathcal{T}$.

Definition 6. *The bargaining process exhibits one-period* ***conditional predictability*** *of degree* **d** *if* $\left|L_C^{t+1}(s_P^t, m^{t+1})\right| \geq d$ *for all* s_P^t *in* S_P^t, $m^{t+1} \in M$, *and* t *in* $\mathcal{T}$.

With conditional predictability, the players are able to rule out d legislators in period t when they can predict the maneuvers in period $t+1$. Unconditional predictability is stronger (and implies conditional predictability) as the players need not predict the maneuvers played in period $t+1$ to rule out d legislators from being the proposer. The following describes the implications of each condition.

Theorem 11. *If the bargaining process exhibits one-period unconditional (respectively conditional) predictability of degree* q*, the proposer selected at* $t = 0$ *captures the entire surplus in every (respectively every pure strategy) MPE.*

Proof. For every state s_P^t in S_P^t, let $V_i^{t+1}(s_P^t)$ denote the expected continuation value of player i before Stage 1 of the next period, after the rejection of an offer in state s_P^t, and excluding maneuvering costs that have already been incurred (at period t or before). Lemmas 1 and 2 extend to this setting immediately, so in every MPE proposal is accepted with probability 1.

Case 1: *Unconditional Predictability of Degree* q: Constructing $\overline{x}(s_P^0)$ and $H^0(s_P^0)$ as in the proof of Theorem 1, it follows that $H^0(s_P^0) \bigcap L_U^1(s_P^0)$ is non-empty. Consider a generic player i in $H(s^0) \bigcap L^1(s^0)$. For a generic player i in $H^0(s_P^0) \bigcap L_U^1(s_P^0)$, his continuation value is a combination of offers that he receives in states in S_P^1 and maneuvering costs that he incurs in period 1. Since maneuvering can be only costly, it must be that there exists some structural state s_P^1 in S_P^1 such that the associated proposer offers player i at least $\frac{\overline{x}(s_P^0)}{\delta_i} \geq \frac{\overline{x}(s_P^0)}{\hat{\delta}}$, which implies that $\overline{x}^1(s_P^1) \geq \frac{\overline{x}(s_P^0)}{\hat{\delta}}$. Induction (as before) implies that there exists a state in which a proposer shares more than the entire surplus (if $\overline{t} = \infty$) or offers a strictly positive share in the final period (if $\overline{t} < \infty$), both of which are contradictions.

Case 2: *Conditional Predictability of Degree* q: Construct $\overline{x}(s_P^0)$ and $H^0(s_P^0)$ as in the proof of Theorem 1. The state for maneuvers in period 1, $s_M^1 = (h_m^0, s_P^0)$, which is identical to s_P^0. Since the MPE is in pure strategies, there is a profile of maneuvers m^1 that is chosen in s_M^1 that is perfectly predictable in state s_P^0. Since the bargaining process exhibits predictability of degree q, it follows that $|L_C^1(s^0, m^1)| \geq q$. Since $H^0(s_P^0)$ must have a cardinality of at least $n - (q-1)$, $H^0(s_P^0) \bigcap L^C(s^0, m^1)$ is non-empty. It follows exactly as in the argument above that there exists some structural state s_P^1 in S_P^1 such that $\overline{x}^1(s_P^1) \geq \frac{\overline{x}(s_P^0)}{\hat{\delta}}$. Induction, as before, implies a contradiction. □

Finally, we note that the example that we discuss in which the Chair selects proposers is that in which $M_0 = \mathcal{N}$, $\theta^t(\omega) = \theta$ for all t and ω, and $P^t(h_m^t, \theta^t) = m_0^t$. This is a bargaining process that satisfies conditional predictability of degree $n-1$, and so in every pure strategy MPE, the first proposer captures the entire surplus.

A.2.4 Private Learning

In opaque legislative institutions, a legislator may not know who else has or hasn't been able to access power brokers, but in many cases, she may know if she has been unable to do so. We show that a qualitatively similar (but weaker) result applies when players privately learn about bargaining power.

Consider a canonical probability space $(\Omega, \mathcal{F}, \mu)$ (where Ω is the state space, $\mathcal{F}$ is a σ-algebra, and μ is a probability measure) encompassing all uncertainty pertaining to the bargaining process, and let $\omega \in \Omega$ denote the generic state of nature. For every $t \in \mathcal{T}$, define $h_P^t \equiv (p^\tau)_{\tau \in \mathcal{T}, \tau \leq t}$ as the history of proposers, and let H_P^t denote the set of possible proposer histories. The recognition rule is a sequence of functions $\widetilde{P}^t : H_P^{t-1} \times \Omega \to \mathcal{N}$, where $\widetilde{P}^t$ governs the selection of p^t, the proposer in period t. Of course, for a process of that type, the state of nature recursively determines the entire sequence of proposers. Hence we can rewrite the recognition rule more compactly as a stochastic process $(P^t)_{t \in \mathcal{T}}$, where each P^t is $\mathcal{F}-$measurable and maps Ω to $\mathcal{N}$.

In stage 1 of period t, the players commonly observe a signal σ^t, where $\sigma^t(\omega)$ is $\mathcal{F}$-measurable. For each t, we can represent the information structure induced by the stochastic processes $(\sigma^\tau, P^\tau)_{\tau \in \mathcal{T}, \tau \leq t}$ as a partition, $\mathcal{S}^t$, of the state space Ω. The partition identifies states of nature that generate exactly the same signals and history of proposers through period t. Formally, $\mathcal{S}^t$ satisfies two requirements: (i) it partitions Ω and therefore $\bigcup_{s^t \in \mathcal{S}^t} s^t = \Omega$; and (ii) for each $s^t \in \mathcal{S}^t$, $\{\omega, \omega'\} \subset s^t$ if and only if $\sigma^\tau(\omega) = \sigma^\tau(\omega')$ and $P^\tau(\omega) = P^\tau(\omega')$ for every $\tau \leq t$.

In addition to the public signal σ^t, each player i observes a private signal σ_i^t. For each t, the information structure generates a partition $\tilde{\mathcal{S}}_i^t$ for player i in which $\tilde{s}_i^t \in \tilde{\mathcal{S}}_i^t$ is a generic member of player i's partition. The information structure is common knowledge. Let a structural state be $\tilde{s}^t = (\tilde{s}_1^t, \ldots, \tilde{s}_n^t)$, which encapsulates the information possessed by each player, and let $\tilde{\mathcal{S}}^t$ be the set of possible structural states. Being that players have private information and there are no proper sub-games, we restrict attention to Perfect Bayesian Equilibria. We consider a similar solution-concept to that before: proposal equilibrium strategies by player i condition on her information $\tilde{s}_i^t$, and voting strategies by player i condition on information $\tilde{s}_i^t$ and the proposal on the table. We call such an equilibrium a Markov Perfect Equilibrium.

Let $\tilde{r}_i^{t+1}(\tilde{s}^t) \equiv \mu\left(\widehat{\Omega}_i^{t+1}(\tilde{s}_i^t) \mid \tilde{s}_i^t\right)$. The *privately informed losers* are those players who have 0 probability of being the proposer at $t+1$ conditional on all that is known at the proposal stage in period t: $L^{t+1}(s^t) \equiv \left\{i \in \mathcal{N} : \tilde{r}_i^{t+1}(\tilde{s}^t) = 0\right\}$.

Definition 7. *The bargaining process exhibits* ***one-period private predictability of degree*** **d** *if* $|L^{t+1}(\tilde{s}^t)| \geq d$ *for all* $\tilde{s}^t$ *in* $\tilde{\mathcal{S}}^t$ *and* t *in* $\mathcal{T}$.

One-period private predictability of degree d requires that each of at least d players privately learns in period t as to whether she has a strictly positive probability of being the proposer in period $t+1$. Unlike one-period predictability of degree d, the identity of these d players is not commonly known. For a non-unanimous q voting rule, it is straightforward to construct an MPE in which the first proposer captures the entire surplus: suppose that each proposer offers to take all of the surplus, and each privately informed loser votes in favor of each proposal. No player has any incentive to deviate from this profile. Thus, the following result applies.

Corollary 2. *If the bargaining process exhibits one-period private predictability of degree q, then there exists an MPE where the proposer selected at $t = 0$ captures the entire surplus.*

In fact, a stronger result applies for the finite horizon, with a restriction on off-path beliefs for each player. We assume that a player who privately learns that she is a loser maintains those beliefs, regardless of the proposal made by the proposer.[31] Because the information that each player privately learns about whether she is a loser is definitive, we call this PBE with this system of beliefs to be a *Definitive PBE.*

Theorem 12. *If $\bar{t} < \infty$, and the bargaining process exhibits one-period predictability of degree q, then in every Definitive PBE, the proposer selected at $t = 0$ captures the entire surplus.*

Proof. We proceed by backward induction. Observe that the final proposer captures the entire surplus in every PBE. Suppose that in period $t + 1$, the next proposer keeps the entire surplus in every Definitive PBE. Then if the proposer today offers ϵ to every other player, every private loser has a strict incentive to accept such an offer, and it necessarily passes. Therefore, a lower bound for the proposer's payoff is $1 - (n - 1)\epsilon$, which establishes that in equilibrium, today's proposer captures the entire surplus. □

[31] In principle, a player who privately learns that she is a loser may have different beliefs about her recognition probability tomorrow in the event of a deviation by a proposer (since such deviations occur with 0 probability).