How to Sell Hard Information

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(all @ Penn State)

Why do we see informational intermediaries?

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Positives:

- Improve terms of trade.
- Alleviate agency problems.
- Facilitate assortative matching.

Negatives:

- Appropriate rents.
- Creates commitment problem
- Leads to excessive disclosure.

motivating question

How much of the gains from trade can an informational intermediary robustly capture, even if she adds no value?

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Robustness: intermediary designs game but cannot anticipate which equilibrium will be played.

Caution / worst-case scenario $\rightarrow$ adversarial eqm selection.

Agent plans to sell asset of market value $\theta \in \{0, 1\}$.

Competitive market: 2 risk-neutral buyers bid for asset.
Winner obtains θ.

Agent and buyers symmetrically informed about θ, and

$$Pr(\theta = 1) = \frac{1}{2}.$$

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If we thought about this world *without* an intermediary:

Bertrand competition $\Rightarrow$ each buyer bids $\frac{1}{2}$ for the asset.

Thus, Agent's full surplus is $1/2$.

role of the intermediary

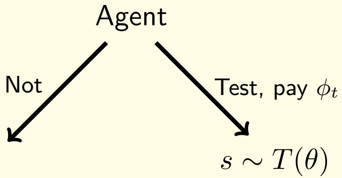
Intermediary chooses a test-fee structure:

- A test, $T : \{0, 1\} \to \Delta[0, 1]$.
- Scores are normalized so that a score of $s = E[\theta|s]$.
- The intermediary chooses a testing fee $\phi_T \in \mathbb{R}$, and a disclosure fee $\phi_D \in \mathbb{R}$.

If agent pays testing fee, he obtains score, and then chooses whether to pay disclosure fee to verifiably disclose score to market.

Market never directly observes whether the asset is tested.

extensive-form



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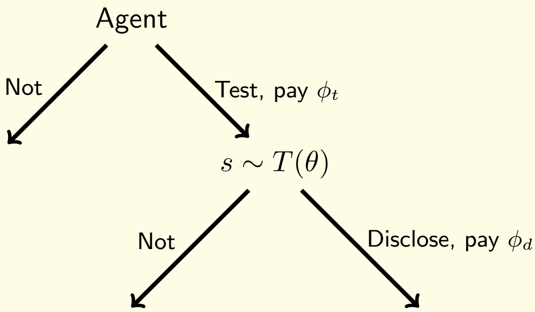

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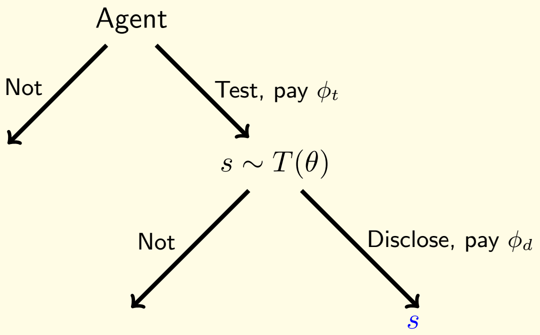

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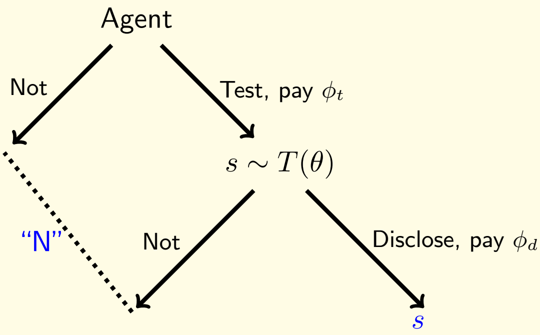

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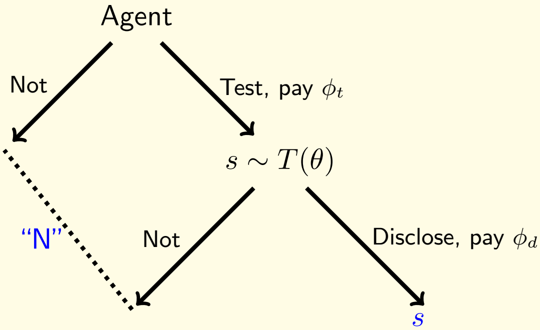


Market observes score s or nondisclosure N.

Bertrand competition $\Rightarrow p(s) = E[\theta|s] = s$ and $p_N = E[\theta|N]$.

But $E[\theta|N]$ depends on the agent's equilibrium strategy.

the problem

Intermediary chooses a test-fee structure, (T, ϕ), to maximize revenue guarantee.

Standard mechanism design:

$$\max_{\text{test-fee structure}} \quad \max_{\text{equilibria}} \quad \text{Revenue}$$

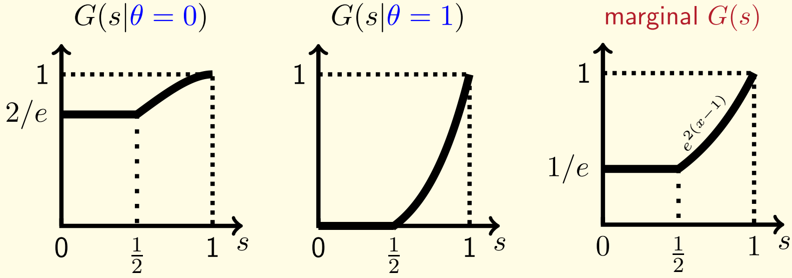
Our problem:

$$\max_{\text{test-fee structure}} \quad \min_{\text{equilibria}} \quad \text{Revenue}$$

$$\max_{\text{test-fee structure}} \; \min_{\text{equilibria}} \; \text{Revenue}$$

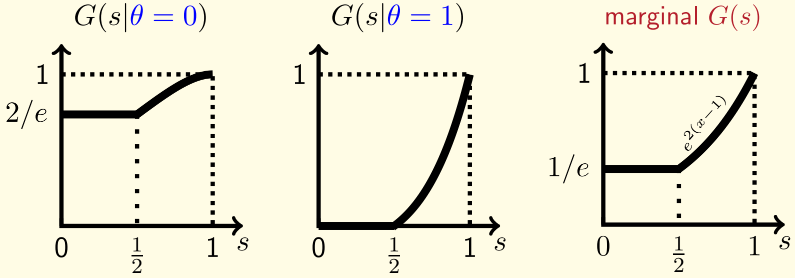
- Intermediary cannot fully anticipate which eqm the asset market settles on.
- If a test-fee structure has a favorable equilibrium that extracts almost all surplus, then it has an adversarial equilibrium that extracts almost no surplus.
- The worst equilibrium for the intermediary is Pareto preferred by all market participants to any other equilibrium.

robustly optimal test-fee structure



- Recall that scores are unbiased $(s = E[\theta|s])$.
- Atom on $s = 0$, and exponential CDF on $\left[\frac{1}{2}, 1\right]$.

robustly optimal test-fee structure



- Recall that scores are unbiased ($s = E[\theta|s]$).
- Atom on $s = 0$, and exponential CDF on $\left[\frac{1}{2}, 1\right]$.
- Free testing but disclosure fee is $\approx \frac{1}{2}$.
- Unique eqm: asset always tested, scores above $\frac{1}{2}$ disclosed, and $p_N \equiv E[\theta|\text{Non-disclosure}] = 0$.

goal of next few slides

Explain why this is robustly optimal.

- For talk, assume that testing fee is 0.
- Find revenue guarantee with a fully revealing test.
- See why noise improves revenue guarantee.
- Argue that robustly optimal test has exponential form.

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$\theta = 0$	$1/2$	0
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Favorable selection: Focus on a).
Adversarial Selection: Worry about b).

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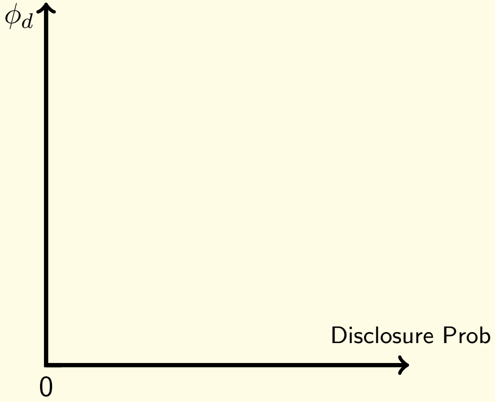
By charging fees $\nearrow \frac{1}{2}$, Revenue Guarantee $\simeq \frac{1}{4}$.


ϕ_d
Disclosure Prob
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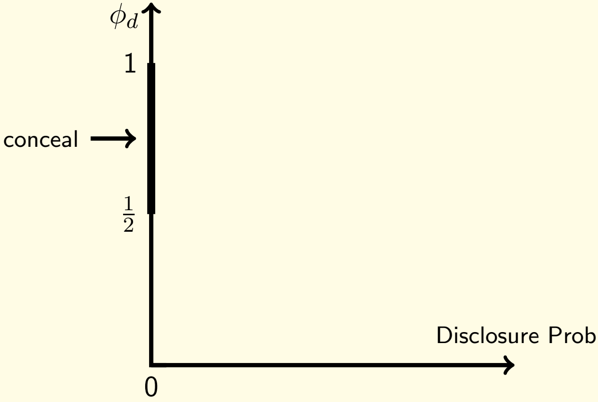
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conceal
$\frac{1}{2}$
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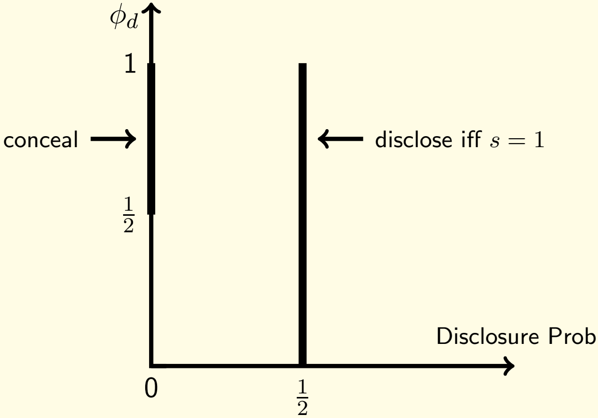
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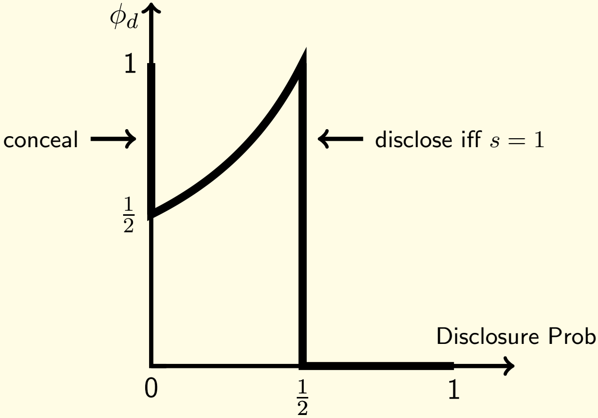
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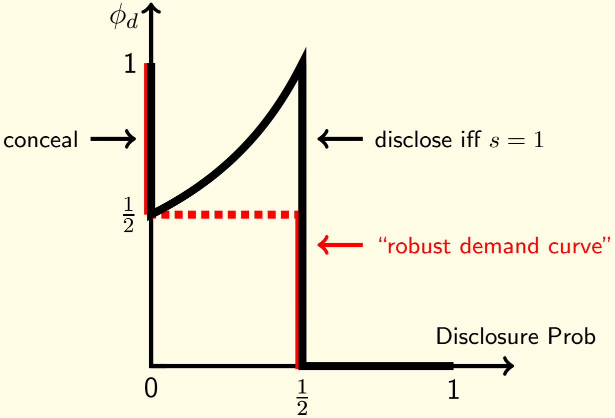
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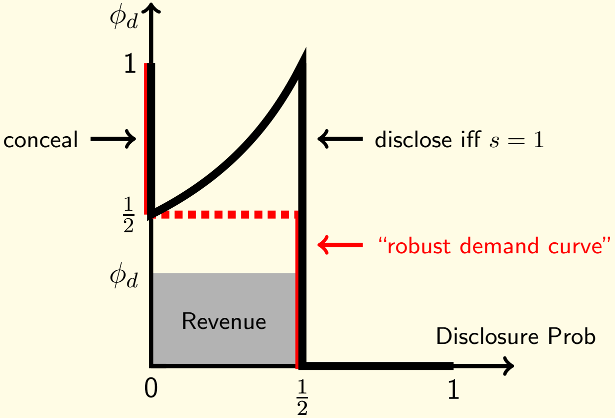

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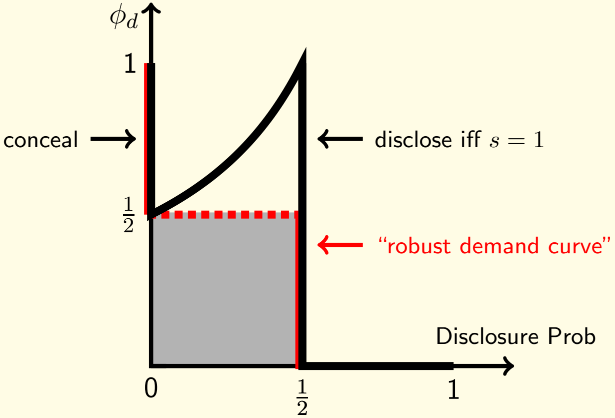

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If disclosure fee exceeds $1/2$, $\exists$ a fully concealing equilibrium.
$\implies$ The intermediary must price slightly below $1/2$.

In that case, unique eqm involves agent disclosing score $s = 1$, concealing $s = 0$, and non-disclosure price $p_N = 0$.

If there *were* an intermediate score $s \in \left(\frac{1}{2}, 1\right)$, the agent would be strictly willing to disclose such scores too.

But alas, no such scores exist in a fully revealing test.

Introducing intermediate score increases probability of disclosure without reducing disclosure fees.

This is why noise helps.

a noisy test

$Pr(\theta, s)$	$s = 0$	$s = 3/4$	$s = 1$
$\theta = 0$	$(1 - p)/2$	$p/2$	0
$\theta = 1$	0	$3p/2$	$(1 - 3p)/2$

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If $\phi_d \geq \frac{1}{2}$: the fully concealing equilibrium still exists.

But if $\phi_d < \frac{1}{2}$: $\exists$ an eqm in which the agent discloses $s \in \left\{\frac{3}{4}, 1\right\}$.

Probability of disclosure goes up from $\frac{1}{2}$ to $(1+p)/2$.

Pooling qualities into $s = 3/4$ clearly benefits intermediary.

But are there limits to how much pooling can be done?

Excessive pooling creates bad equilibrium:

Agent conceals $s \in \left\{0, \frac{3}{4}\right\}$, and discloses only $s = 1$.

Now if the agent does not disclose any score, he obtains

$$p_N = E[\theta|\text{Non-Disclosure}] = \frac{3p}{1+3p}.$$

This equilibrium has a low disclosure probability: $(1-3p)/2$.

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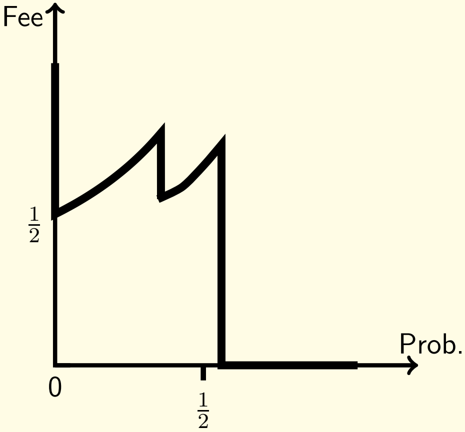
Intermediary would like to rule out this equilibrium by setting p so that if $s = \frac{3}{4}$, agent finds it profitable to deviate & disclose:

$$\frac{3}{4} - \phi_d > \frac{3p}{1+3p}$$

$\Rightarrow$ At $\phi_d \approx \frac{1}{2}$, this implies that $p \leq \frac{1}{9}$.

demand curve with 3 scores



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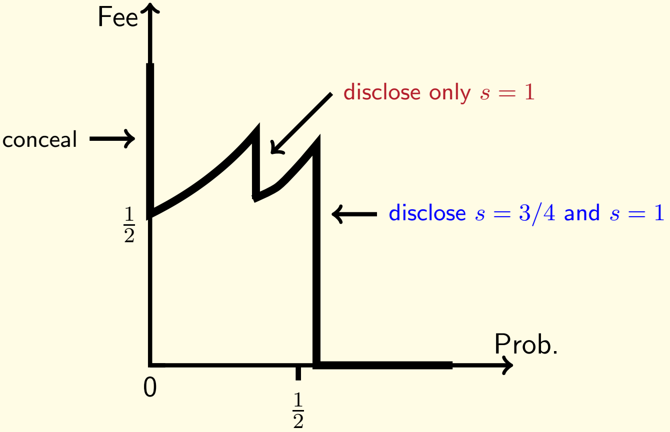

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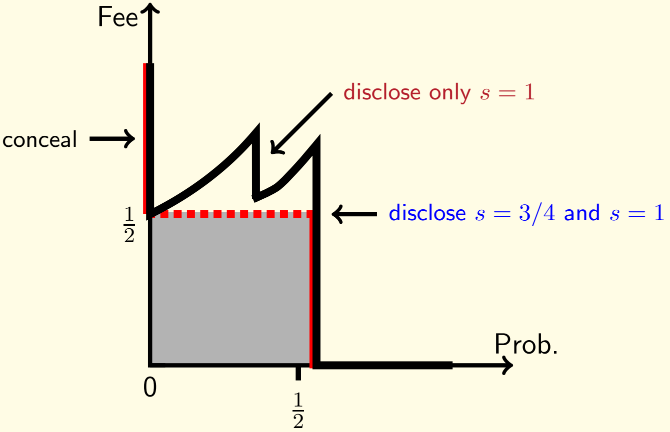

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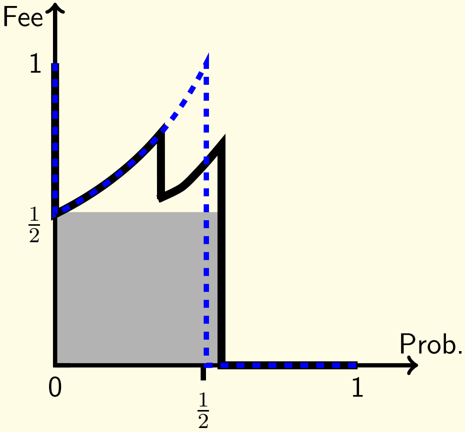
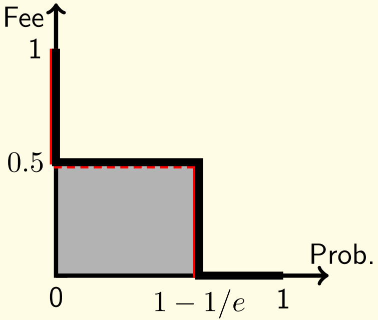


We see that pooling benefits the intermediary, but it has limits.

The intermediary's revenue is the largest rectangle that fits under the robust demand curve, and area above is "slack."

What if we could generate a rectangular demand curve?



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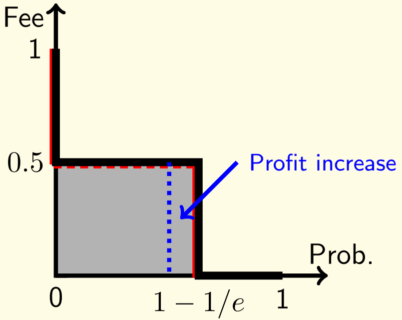

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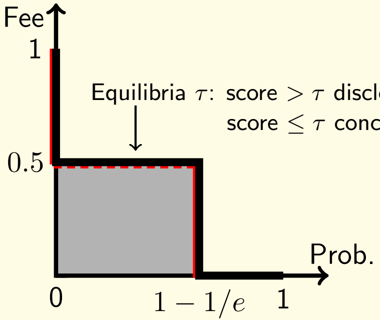

$\exists\ [\underline{s}, \overline{s}]$ such that for every $\tilde{s} \in [\underline{s}, \overline{s}]$, $\exists$ an equilibrium where $\tilde{s}$ is the threshold score:

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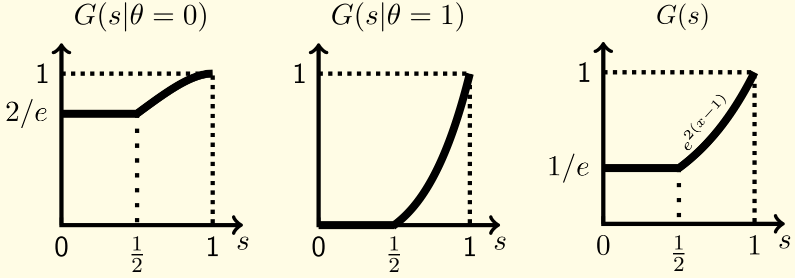
$\implies G(\tilde{s}) = \alpha e^{\tilde{s}/\phi_d}$ for some constant α.



The robustly-optimal test-fee structure features:

- Free testing but disclosure fee $\approx \frac{1}{2}$.
- Unique equilibrium: asset is always tested, agent discloses $s \geq \frac{1}{2}$, and nondisclosure price is 0.
- Maximal revenue guarantee is $\frac{1}{2}\left(1 - \frac{1}{e}\right)$.

summary

Several features emerge of robustly optimal tests:

1. Intermediary uses option value to tempt the agent.
2. Pooling benefits intermediary but has limits.
3. Those limits $\implies$ exponentially distributed scores.

Even though info has no social value, intermediary robustly gains.

But she must leave agent with some rents.

Our general analysis considers any CDF F on $[\underline{\theta}, \overline{\theta}]$.

Bayes-Plausibility

$\Downarrow$

G is generated by a test if and only if
G is a mean-preserving contraction of F.

Thus, we look across such test-fee structures (G, ϕ_t, ϕ_d).

revenue guarantees vs. full surplus

$$R_M \equiv \sup_{(G,\phi)} \inf_{\sigma \in \Sigma(G,\phi)} Rev(\sigma; G, \phi).$$

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We compare above with full informational surplus, $R_F \equiv E[\theta] - \underline{\theta}$.

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Thm. If a test-fee structure has a favorable equilibrium with revenue $\approx R_F$, it has an adversarial equilibrium with revenue ≈ 0.

To find maximal revenue guarantee, we show:

1. Suffices to consider test-fee structures in which the agent has the asset tested with probability 1 in every equilibrium.
2. Characterize adversarial eqm in terms of score thresholds.
3. Formulate relaxed problem that has the same value & whose solution is limit for those of original problem.

option value as a carrot

Intermediary must break eqm in which agent refrains from testing.

Consider a strategy profile in which the agent does not take the test, and the market believes this.

Since, in eqm, the agent never discloses his score,

$$p_N = E[\theta|\text{Non-disclosure}] = E[\theta].$$

Nondisclosure is treated as no news rather than bad news.

Possible deviation: pays ϕ_t & discloses score s iff $s - \phi_d > E[\theta]$.

Upside with no downside: buyers expect nondisclosure.

$$\text{Equilibrium Payoff} = E[\theta]$$

$$\text{Disclose } s \Rightarrow \text{Payoff} = s - \phi_d.$$

$$\text{Nondisclosure} \Rightarrow \text{Payoff} = E[\theta].$$

This deviation is profitable if

$$E[\theta] < -\phi_t + \int_{\underline{\theta}}^{\overline{\theta}} \underbrace{\max\{E[\theta], s - \phi_d\}}_{\text{Option Value}} dG \qquad \text{(Participation)}$$

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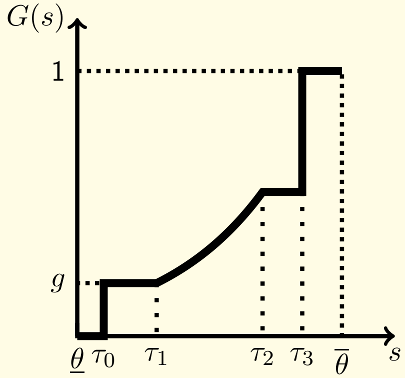
$$\text{Nondisclosure} \Rightarrow \text{Payoff} = E[\theta].$$

This deviation is profitable if

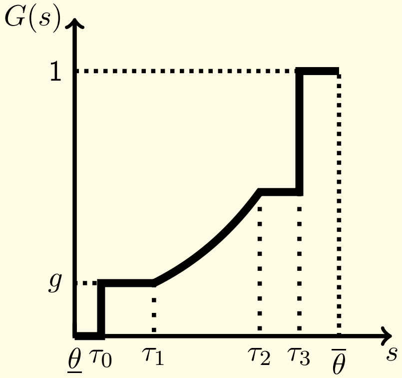
$$E[\theta] < -\phi_t + \int_{\underline{\theta}}^{\overline{\theta}} \underbrace{\max\{E[\theta], s - \phi_d\}}_{\text{Option Value}} dG \qquad \text{(Participation)}$$

Lemma. $P \Rightarrow$ asset is tested with probability 1 in every eqm.
$\neg P \Rightarrow \exists$ eqm in which the asset is tested with probability 0.

main result



main result



- There exists a robustly optimal test that looks like above.
- Disclosure fee is always strictly positive
- Testing fee is sometimes strictly positive. (e.g., log-concave).

extensions

#1: Charging only testing fee $\Leftrightarrow$ Suffices to use binary test.

#2: If tests are costly for intermediary, and that cost is weakly increasing in Blackwell informativeness, identical conclusions hold.

#3: If intermediary can offer multiple pieces of evidence and let agent choose (e.g., partial disclosure), robustly optimal to use a "single evidence" test-fee structure.

what we did

Our analysis studies the endogenous generation of hard information by a revenue-maximizing intermediary.

Intermediary tempts agent through option value.

Once market expects agents to buy and disclose info, it treats nondisclosure prejudicially.

Intermediary maximizes revenue guarantee using noise and disclosure fees.

Hard Information / Evidence / Certification:

- Unraveling: Grossman'81; Milgrom'81
- Obstacles: Jovanovic'82; Dye'85; Matthews & Postlewaite'85.
- Intermediation: Lizzeri'99; DeMarzo, Kremer, & Skrzypacz'19

Adversarial equilibrium selection:

- Full implementation: large classical lit, building on Maskin'99
- More recently: BBM'17; Du'18; Dworczak & Pavan'20; Inostroza & Pavan'20; Halac, Kremer, & Winter'20; Halac, Lipnowski, & Rappoport'20; Mathevet, Perego, & Taneva'20; Morris, Oyama, & Takahashi'20.

Designing distributions:

- Roesler & Szentes (2017), Ortner & Chassang (2018), Condorelli & Szentes (2020)
- Shishkin (2020)

Thank you!