Sequential Veto Bargaining with Incomplete Information

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Veto bargaining prevalent in politics & organizations

- Legislatures send bills to Executives
- Executives need legislatures to confirm appointments
- Search committees put forward candidates for approval
- Boards of Directors require sign-off from shareholders

"If Congress returns the bill having appropriately addressed these concerns, I will sign it. For now, I must veto the bill."

Veto Bargaining

Veto bargaining: (bilateral) bargaining with single-peaked prefs and one-sided offers

- **Proposer** and **Vetoer**
- 1-dimensional policy

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Romer and Rosenthal (1978)

- TIOLI offer with complete information
- Proposer targets Vetoer precisely
  - Vetoer's ideal point affects outcome.
  - No vetoes.

This Paper

Canonical model omits two features:

- Proposer doesn't know Vetoer's ideal point
- Sequential proposals

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Does Proposer benefit from sequential proposals?

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→ *Proposer can learn from past rejections*

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Results

- Commitment payoff is achievable
- Proposer exploits leapfrogging
  → owes to single-peaked prefs
  → unlike usual monopolist
- Other equilibria can exist
  → with Coasian dynamics

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Model

Model: Bargaining over Policies

At each $t = 0, 1, \ldots$, Proposer proposes $a_t \in \mathbb{R}$ that Vetoer can accept or reject

Game ends when Vetoer accepts

If agreement is reached in period T, payoffs are

$$\delta^T u(a_T) \quad \text{and} \quad \delta^T u_V(a_T, v) \qquad \text{common } \delta \in [0, 1)$$

- Until agreement, flow utility from status quo, $a = 0$; normalize this utility to 0
- After agreement, flow utility from a_T
- So utility measured as gain over status quo

Model: Single-peaked preferences

Proposer's ideal point known to be 1.

Vetoer's ideal point is v, her type, which is private info.

We assume that it is common knowledge that $v \leq 1$.

Model: Solution Concept

We study Perfect Bayesian Equilibria.

Can interpret Vetoer as voting group, so long as Proposer only observes outcome, not vote profile.

Example

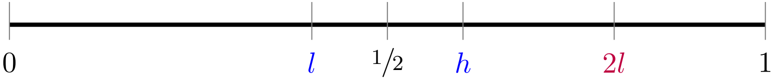
Two-Type Example

Proposer $u(a, v) = 1 - |1 - a|$

Vetoer $u_V(a, v) = v - |v - a|$

(constants normalize $u(0) = u_V(0, v) = 0$)

Vetoer type $v \in \{l, h\}$, with $0 < l < 1/2 < h < 2l$



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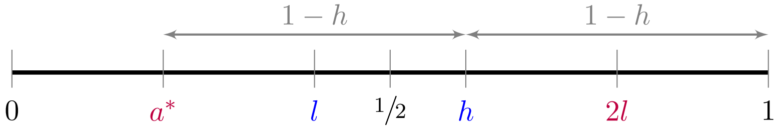

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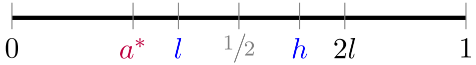
If $\Pr(h)$ is large enough, Proposer's optimal delegation set (deterministic static mechanism) is the separating menu $\{a^*, 1\}$, with h indiff between a^* and 1

(Otherwise, it is the pooling menu $\{2l\}$)

The Sequential Rationality Problem

With patient players in dynamic game, can Proposer (approx) get action 1 from type h and a^* from l?



Standard "skimming" recipe:

- Propose 1 at $t = 0$, which is accepted by h
- If rejected, propose a^* at $t = 1$, which is accepted by l

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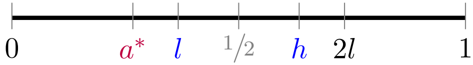

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But not an equilibrium!

- Sequential Rationality $\implies$ at $t = 1$ propose $2l$
- But anticipating $2l$, type h rejects 1 at $t = 0$

The Leapfrogging Solution

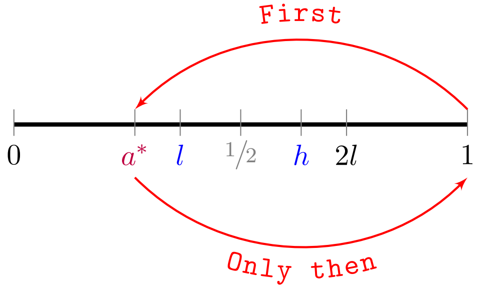
(Modulo discounting adjustments)

First propose a^*

- Accepted only by type l

Only thereafter propose 1 forever

- Accepted by type h



Key idea: By first securing agreement with l, sequential rationality no longer impels Proposer to moderate should h subsequently reject

Owes to single-peaked Vetoer prefs

$\rightarrow$ Futile in monopoly pricing; indeed, all equilibria there have skimming

A Non-Constructive Argument

Result (Two types)

Assume the optimal delegation set has separation: $\{a^*, 1\}$. When players are patient, Proposer can achieve approximately the delegation payoff (or better).

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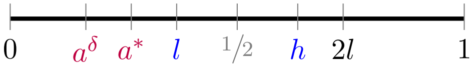
Proof (assuming equilibrium existence):



Let $a^\delta \equiv \delta a^*$ be lowest action s.t. h is indifferent between a^δ today and 1 tomorrow.

- If Proposer proposes a^δ in first period, l accepts and h rejects. After rejection of a^δ, Proposer believes $\Pr(h) = 1$ and proposes 1 forever.
- If Proposer proposes $a \neq a^\delta$ in first period, play some continuation eqm.
- In first period, Proposer chooses an optimal proposal.

So either Proposer uses $(a^\delta, 1)$ on path, or follows another path that is even better.

$\rightarrow$ Leapfrogging is a high-payoff option

Equilibrium Construction

An equilibrium construction is more involved

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Resolved by Proposition 1: for large δ, for any prior we explictly construct an eqm that achieves approx Proposer's optimal delegation payoff

→ whether the leapfrogging option is exercised immediately depends on the prior

Construction distinguishes three cases:

(a) Skimming. $\Pr(h)$ low: skimming approximates the pooling outcome, which is optimal.

(b) Leapfrogging. $\Pr(h)$ moderate: on path offers $(a^{\delta}, 1)$.

(c) Delayed leapfrogging. $\Pr(h)$ high: first offer 1; in second period mix between leapfrogging and skimming. Type h mixes in the first period to justify Proposer's indifference.

→ only difference across cases is path Proposer initiates; leapfrogging is always an option

Wrap-up of Example

Example illustrates why leapfrogging works

and how it delivers a high payoff by weakening seq rationality constraint

Limitations of example, beyond specificity

- are there equilibria that attain even higher or lower Proposer payoffs?
- optimal delegation payoff is a high benchmark, but why that one?

  $\rightarrow$ commitment in dynamic game?

General Analysis

Payoffs and Types

Proposer's $u(a)$ is (weakly) concave with a unique maximum at 1 (normalize $u(0) = 0$)

Vetoer's $u_V(a, v) \equiv -(a - v)^2 + v^2$ (normalized so that $u_V(0, v) = 0$)

- Single-crossing expectational differences (SCED); Kartik, Lee, Rappoport (2019)
- Interval choice: set of types willing to accept any offer is an interval

Vetoer's type $v \sim F \in \mathcal{F}$

- $\mathcal{F}$: CDFs with density bounded away from 0 and ∞ on an interval support
- Denote support of F by $[\underline{v}, \overline{v}]$
- $\overline{v} \leq 1$ (for simplicity)

Auxiliary Static Problem

Auxiliary static mechanism design problem:

$$\mathcal{S} \equiv \{m : [\underline{v}, \overline{v}] \to \Delta(\mathbb{R}) \text{ s.t. IC and IR}\}$$

(+ integrability; finite mean and variance lotteries)

$$U(F) \equiv \max_{m \in \mathcal{S}} \int u(m(v)) \mathrm{d}F(v)$$

Proposer's optimum

- Stochastic mechanisms are allowed
- This problem studied by Kartik, Kleiner, Van Weelden (2021)

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Assumption (Interval delegation is optimal)

An interval delegation set $[c^*, 1]$ solves Proposer's static problem.

- Simple, deterministic mechanism
- $m(v) = 0$ for $v < c^*/2$; $m(v) = c^*$ for $v \in (c^*/2, c^*)$; $m(v) = v$ for $v > c^*$
- KKVW derive sufficient conditions: e.g., f logconcave and u linear-quadratic

An Upper Bound

Why is the static problem relevant to our dynamic game?

Lemma (Upper bound on Proposer's payoff)

Proposer's payoff from any strategy, given a Vetoer best response, is at most $U(F)$.

Invoking an auxiliary static problem is familiar from seller-buyer bargaining

Here, absent transfers, important that static problem allows for stochastic mechanisms

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Proof idea:

- Time-stamped allocation $(a, t) \mapsto$ static lottery $(a \text{ w.pr. } \delta^t;\ 0 \text{ w.pr. } 1 - \delta^t)$
- Payoff equivalent for Proposer and all Vetoer types
- Because Vetoer is playing a best response, resulting static mechanism is IC and IR

Lemma $\implies$ refer to $U(F)$ as commitment payoff (at least upper bound on)

Main Result

Theorem (Commitment payoff is achievable)

Assume an eqm exists for all δ and beliefs in $\mathcal{F}$.

When players are patient, $\exists$ eqm with Proposer payoff approx. his commitment payoff.

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- Unless $c^* = 1$ ("*no compromise*"), sequential proposals strictly better than single TIOLI
- Lack of commitment does not hurt Proposer, given his favorite eqm
- Non-Coasian: if $0 < 2\underline{v} < c^*$, Coasian dynamics suggest compromising down to $2\underline{v}$
  - → not seq rational to stop at c^* when there are pos-surplus types for whom c^* is unacceptable
  - → note that $\underline{v} > 0$ is the "gap case"
- Proof uses leapfrogging option; we do not construct a commitment-payoff eqm

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Proof ideas:

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  - $\rightarrow$ Uses SCED and interval delegation structure

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- If belief is $F_{[\underline{v},c^*]}$, use option to leapfrog to obtain commitment payoff (Lemma 3)
  - $\rightarrow$ Option to follow path of first offering 0 and then c^* forever
  - $\rightarrow$ If all types below $c^*/2$ accept first offer 0, then c^* is an optimal second offer by Lemma 1 (static mech is upper bound) and Lemma 2, given that it is accepted by all remaining types

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  - $\rightarrow$ If all types below $c^*/2$ accept first offer 0, then c^* is an optimal second offer by Lemma 1 (static mech is upper bound) and Lemma 2, given that it is accepted by all remaining types
- Use induction to extend from $F_{[\underline{v},c^*]}$ to $F_{[\underline{v},\overline{v}]}$, using SCED & applying Lemmas 1 and 2

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Full Delegation: interval delegation set $[2\max\{0, \underline{v}\}, 1]$

- Vetoer gets much discretion; if $\underline{v} = 0$, every Vetoer type gets her first best
- Proposer only minimally exploiting his bargaining power

  $\rightarrow$ Caveat: full delegation can sometimes be an optimal mechanism

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Proposition (Coasian dynamics)

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- Resolves eqm existence
- Construction adapts "dynamic programming" arguments from seller-buyer analyses
- But single-peakedness necessitates some differences
- When $\underline{v} > 0$, have to deter low-offer deviations (leapfrogging could be tempting!)

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$\rightarrow$ Norms can matter in veto bargaining: requires sequentiality and incomplete info

Literature
&
Conclusion

Related Literature

Sequential veto bargaining

- Romer & Rosenthal 1979; Cameron 2000; Rosenthal & Zame 2019; Chen 2021
- Cameron & Elmes 1995; Evdokimov 2022

Coase Conjecture in classic seller-buyer setting: FLT 1985; GSW 1985; AD 1989

Non-Coasian logic in other seller-buyer settings

- Board & Pycia 2014; Tirole 2016
- Wang 1998; Hahn 2006; Inderst 2008

Conclusion

Bilateral bargaining over policy: single-peaked preferences

Proposer is uncertain of Vetoer's ideal point, and can make sequential proposals

Takeaway #1: Leapfrogging behavior: first secure agreement with low types

- High type cannot use rejection to mimic low types
- This allows Proposer to *credibly* extract surplus from high types

Takeaway #2: Commitment payoff can be achieved

Takeaway #3: Other equilibria can exist, including a skimming equilibrium. Norms matter.

Thank you!